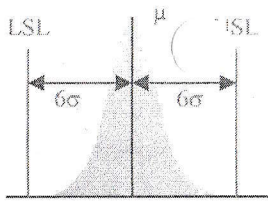


Centerpoints

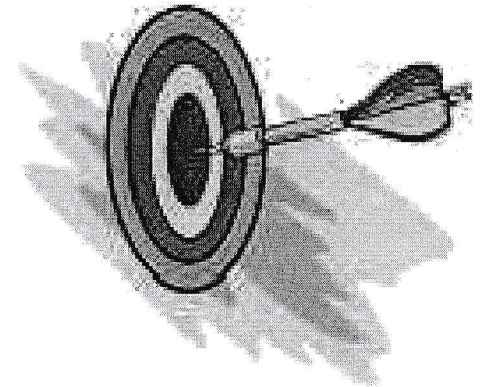
Lean Sigma Black Belt Training



Centerpoints

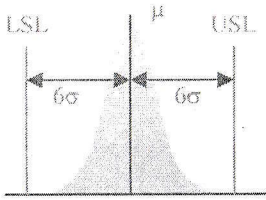
- **What Are Centerpoints**

- Centerpoints are added runs or conditions created in a DOE by setting the factor to a point half way between the high and low levels indicated by the DOE design
- Centerpoints are indicated by a coded value of 0



- **Why Use Centerpoints**

- Centerpoints are used as an efficient way to test for curvature in the design, without adding a large number of experimental runs
- Centerpoints can be used to test repeatability of a response variable at a set condition

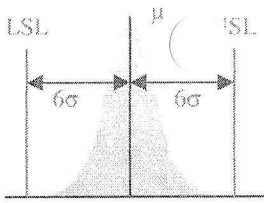


Centerpoints

- **When To Use Centerpoints**

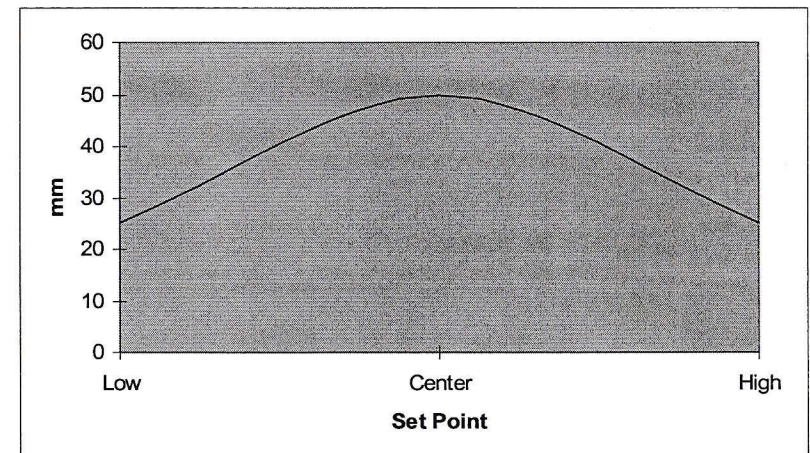
- It is a good idea to use centerpoints when repeating or duplicating runs would be expensive. This allows the repeatability of the process to be examined.
- It is a good idea to use centerpoints when using a bold inference space.





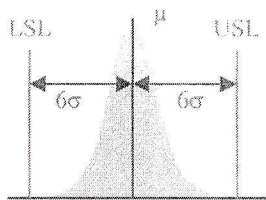
Curvature

- When using a 2 Level design there is always a risk of non-linear responses
- What if this situation occurs?



- How would you learn this problem exists?
- How will predictions be affected?
- What might you do in this situation?

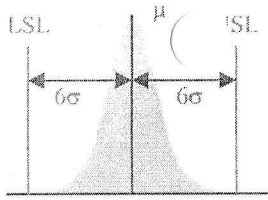




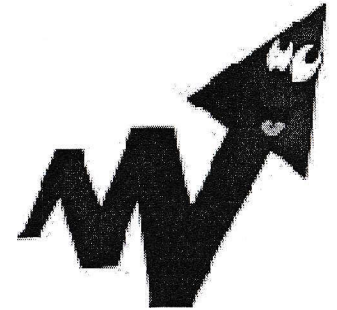
Design Matrix With Centerpoints

StdOrder	RunOrder	CenterPt	Blocks	Bake Time	Bake Temp
1	2	1	1	6	375
2	4	1	1	10	375
3	1	1	1	6	450
4	3	1	1	10	450
5	5	0	1	8	412.5
6	6	0	1	8	412.5
7	8	0	1	8	412.5
8	7	0	1	8	412.5

- Note Centerpoints listed as 0's —
- Note levels are midpoints of high and low level setpoints. (10 + 6 = 16, 16 / 2 = 8 min) —

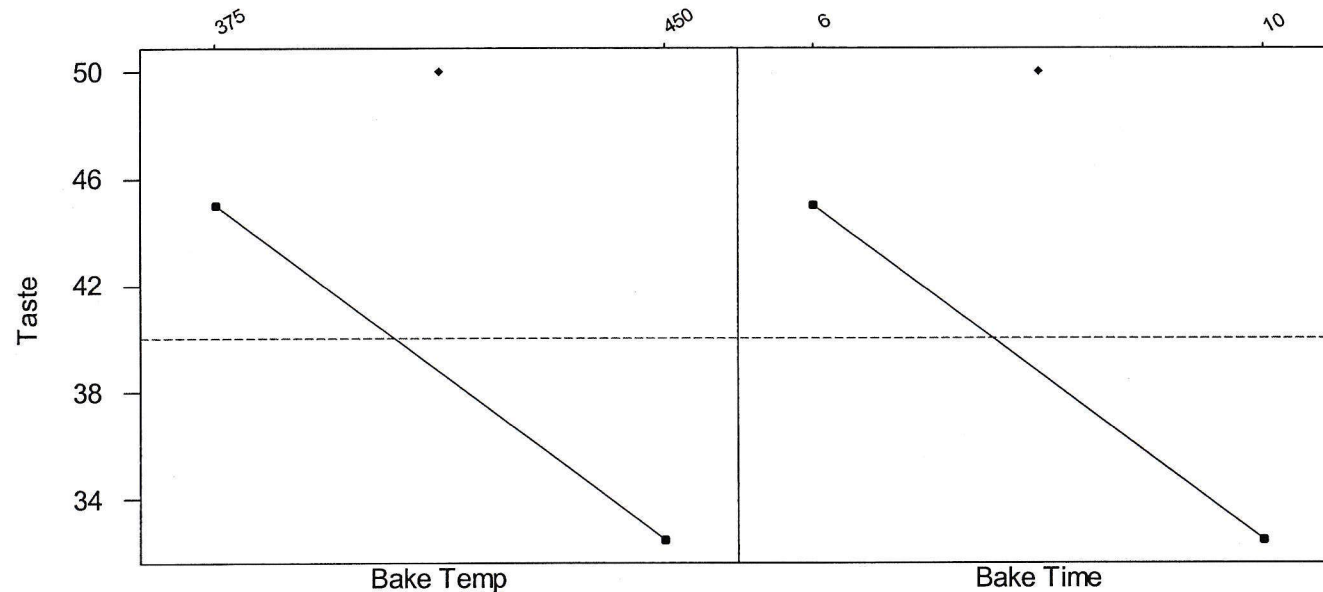


Analyzing DOE's with Centerpoints

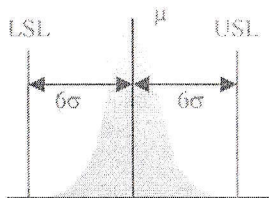


Main Effects Plot (data means) for Taste

♦ Centerpoint



- Graphical
 - Is there curvature?
 - If there is no curvature where should the centerpoint fall on the main effects line?



Analyzing DOE's with Centerpoints

StdOrder	RunOrder	CenterPt	Blocks	Bake Time	Bake Temp	Taste	Texture	Appearance
1	2	1	1	6	375	45	40	40
2	4	1	1	10	375	45	40	45
3	1	1	1	6	450	45	45	45
4	3	1	1	10	450	20	20	20
5	5	0	1	8	412.5	50	50	50
6	6	0	1	8	412.5	50	50	50
7	8	0	1	8	412.5	50	50	50
8	7	0	1	8	412.5	50	50	50

- Analytical

- Calculate the average of the factorial responses. Y_f

$$Y_f = 45 + 45 + 45 + 20 = 155$$

$$155 / 4 = 38.75$$

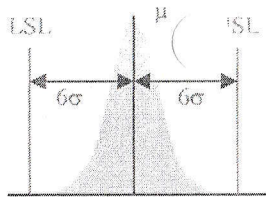
- Calculate the average of the centerpoints. Y_c

$$Y_c = 50 + 50 + 50 + 50 = 200$$

$$200 / 4 = 50.00$$

- Calculate the difference in averages. $Y_f - Y_c$

$$38.75 - 50.00 = -11.25$$



Analyzing DOE's with Centerpoints

- Statistical Significance

- To find out if the amount of curvature is statistically significant, we can use Analysis of Variance (ANOVA).

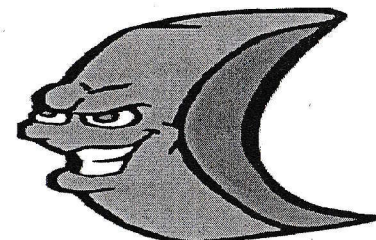
- The Sum of Squares for curvature is calculated as:

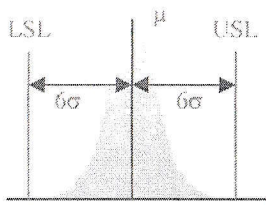
$$SS_{\text{Curvature}} = \frac{n_f n_c (\bar{y}_f - \bar{y}_c)^2}{n_f + n_c}$$

- The Mean Square (MS) is $SS_{\text{Curvature}} / \text{DF}$ (Degrees of Freedom)

- The F value is then $MS_{\text{Curvature}} / MS_{\text{Error}}$

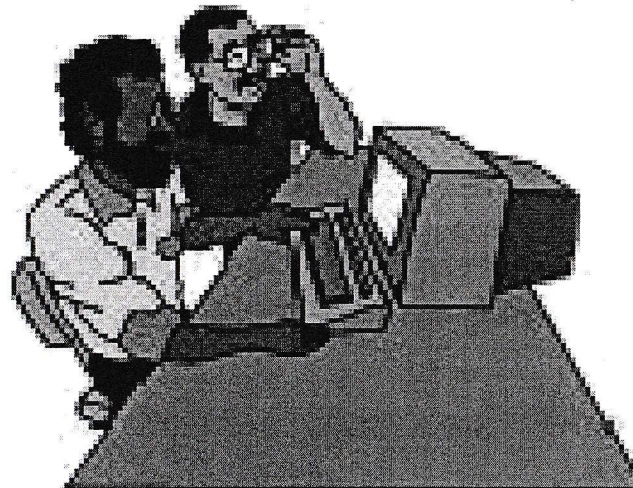
- The F Value is then compared to an F critical value at a given alpha level or Minitab will calculate a P value.

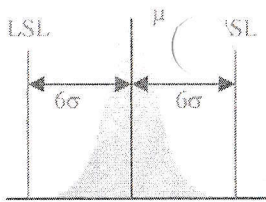




Analyzing DOE's with Centerpoints

- You can choose to analyze the design without the centerpoints.
 - Analyze Factorial Design > Terms
 - Deselect (Do not include centerpoint as term in model.





ANOVA Analysis

Estimated Effects and Coefficients for y (coded units)

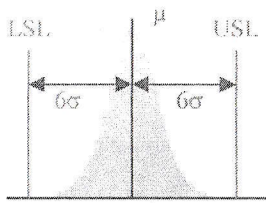
Term	Effect	Coef	StDev Coef	T	P
Constant		40.4444	0.06231	649.07	0.000
time	1.5500	0.7750	0.09347	8.29	0.000
temp	0.6500	0.3250	0.09347	3.48	0.018
time*temp	-0.0500	-0.0250	0.09347	-0.27	0.800

Analysis of Variance for y (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	2	2.82500	2.82500	1.41250	40.42	0.001
2-Way Interactions	1	0.00250	0.00250	0.00250	0.07	0.800
Residual Error	5	0.17472	0.17472	0.03494		
Curvature	1	0.00272	0.00272	0.00272	0.06	0.814
Pure Error	4	0.17200	0.17200	0.04300		
Total	8	3.00222				

$$SS_{\text{Curvature}} = \frac{n_f n_c (\bar{y}_f - \bar{y}_c)^2}{n_f + n_c}$$

interpretation?

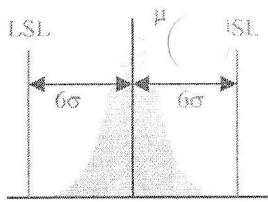


ANOVA Analysis

- If the experiment is rerun and the presence of curvature is added.
- See how the analysis changes.

Analysis of Variance for y (coded units)						
Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	2	2.8250	2.82500	1.41250	0.75	0.518
2-Way Interactions	1	0.0025	0.00250	0.00250	0.00	0.972
Residual Error	5	9.3747	9.37472	1.87494		
Curvature	1	9.2027	9.20272	9.20272	214.02	0.000
Pure Error	4	0.1720	0.17200	0.04300		
Total	8	12.2022				

Significant!
Indication that Non-linearities Exist



Summary

- A “neutral” setting of experimental factors can be included to provide additional response insight.
- These neutral settings are called centerpoints
- Use centerpoints to test for curvature
- Use centerpoints to test repeatability

