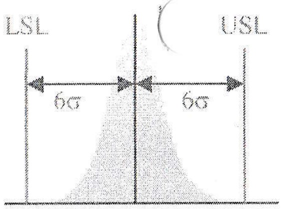

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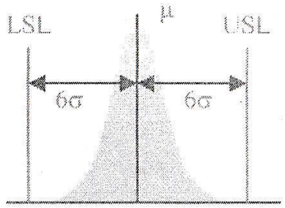
Aliasing

Lean Sigma
Black Belt Training



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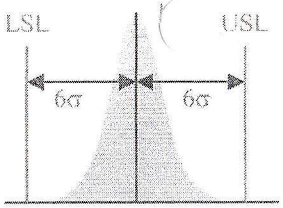
Module Objectives

- Understand Aliasing and the use of the defining relation
- Understand design generators and how they can be used to generate alternate fractions of a fractional factorial design
- Learn to use the concept of Aliasing to your benefit



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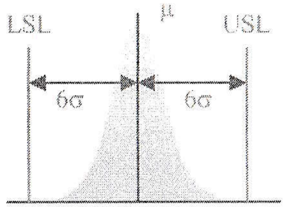
Remember Full Factorials

Consider the following 2^3 Full Factorial Design:

I	A	B	C	Y
1	-1	-1	-1	*
1	1	-1	-1	*
1	-1	1	-1	*
1	1	1	-1	*
1	-1	-1	1	*
1	1	-1	1	*
1	-1	1	1	*
1	1	1	1	*

Quick Review:

- How many degrees of freedom are associated with this 8 run design?
- What are the effects that you can estimate with this design?
- Which interaction effect is most unlikely to occur?



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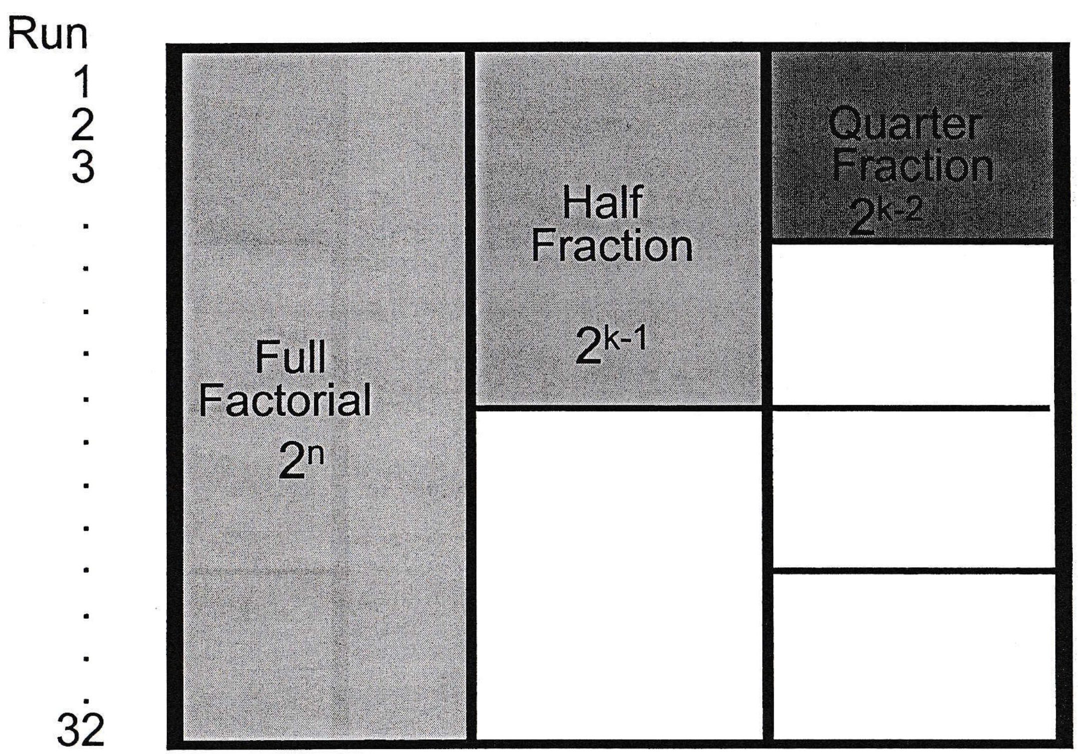
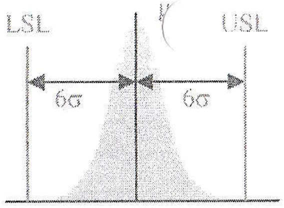
Remember Fractional Factorials

- **Fractional Factorials** only test a fraction of the Full Factorial
- These designs are based on the *Sparsity of Effects principle* (higher order Interactions are unlikely to occur)





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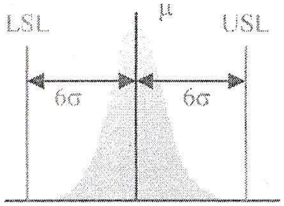
What is Aliasing?

- The unique effect of a factor or interaction can not be separated from the unique effect of another factor or interaction because both columns have the *same* sign at each experimental run

- Remember, this occurs when we work with Fractional Factorial Designed Experiments



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Remember Fractional Factorials

We need to investigate three factors, but due to the cost of experimental runs we are constrained to only 4 runs instead of 8.

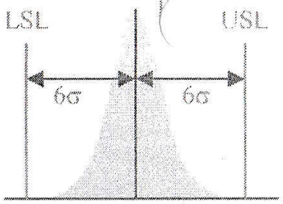
The following table was completed for a new 4 run design (1/2 fraction or 2^{3-1})?

I		A	B	AB	Y
1	-1	-1	1	*	
1	1	-1	-1	*	
1	-1	1	-1	*	
1	1	1	1	*	

I	A	B	C= AB	AC	BC	ABC	Y
1	-1	-1	1				*
1	1	-1	-1				*
1	-1	1	-1				*
1	1	1	1				*

Assignment:

- How many degrees of freedom are associated with this 4 run design?
- Complete the table for the 1/2 fraction.
- Which effects are aliased with each other?



The Defining Relation

- We generated this design by assigning Factor C to the AB column, therefore, the **<u>Design Generator</u>** is

  C = AB

The **<u>Defining Relation</u>** for this design is found by the following method:

C = AB

C(C) = AB(C), where CxC = I

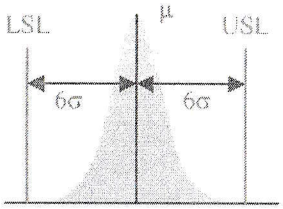
I = ABC

Note: Any column multiplied by itself will always yield a column of +1's which equals the identity column (I).

Since I = ABC is positive, this is considered the principal fraction.

<u>Assignment:</u>

- What is the resolution of this design?
- What does the defining relation tell us about the ABC interaction column in the design matrix?



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Designing the Fractional Factorial

To determine the aliasing structure of this design?

$I = ABC$

$I(A) = ABC(A) \quad A = BC$

$L_A = A + BC$

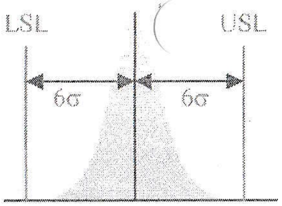
$I(B) = ABC(B) \quad B = AC$

$L_B = B + AC$

$I(C) = ABC(C) \quad C = AB$

$L_C = C + AB$

I	A	B	C= AB	AC	BC	ABC	Y
1	-1	-1	1	-1	-1	1	*
1	1	-1	-1	-1	1	1	*
1	-1	1	-1	1	-1	1	*
1	1	1	1	1	1	1	*



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Alternate Fractional Designs

Suppose that we had chosen the other 1/2 fraction (also called the *alternate* or *complementary* fraction)?

For the complementary fraction, the **Defining Relation** for the design is:

$$I = -ABC$$

The **Design Generator** is:

$$C = -AB$$

I	A	B	C= -AB	AC	BC	ABC	Y
1	-1	-1	-1	1	1	-1	*
1	1	-1	1	1	-1	-1	*
1	-1	1	1	-1	1	-1	*
1	1	1	-1	-1	-1	-1	*

$I(E) = E(A\cancel{BC})$

Alternate Fractional Designs

AB BC
AC BE
AD ~~AE~~

What is the aliasing structure of the alternate fraction?

$I = (\cancel{BA}ABC)$

$AE \Rightarrow A(\cancel{A}BC)$

I = -ABC

I(A) = -ABC(A) A = -BC

L'_A = A - BC

I(B) = -ABC(B) B = -AC

L'_B = B - AC

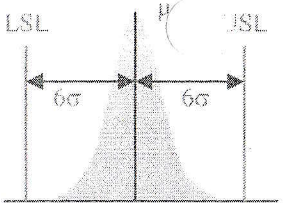
I(C) = -ABC(C) C = -AB

L'_C = C - AB

I	A	B	C= -AB	AC	BC	ABC	Y
1	-1	-1	-1	1	1	-1	*
1	1	-1	1	1	-1	-1	*
1	-1	1	1	-1	1	-1	*
1	1	1	-1	-1	-1	-1	*

ABC BCD ~~AB~~ AB

	A	B	C	D	E	F
1	+	+	+	+	+	+
2	−	+	+	+	−	+
3	+	−	+	+	−	−
4	−	−	+	+	+	−
5	+	+	−	+	−	−
6	−	+	−	+	+	−
7	+	−	−	+	−	+
8	−	−	−	+	−	+
9	+	+	+	−	+	−
10	−	+	+	−	−	−
11	+	−	+	−	−	+
12	−	−	+	−	+	+
13	+	+	−	−	−	+
14	−	+	−	−	+	+
15	+	−	−	−	+	−
16	−	−	−	−	−	−



Considering Aliasing During Analysis

We need to investigate six factors, but due to a large order backlog and cost, the Coating Business Unit Manager will allow us to run only 16 runs.

- What design would you choose to meet both the Black Belt and Coating Business Manager's needs? $\frac{1}{4}$ Factorial

- Create the design matrix for the above experiment assuming that the design generators are E = ABC and F = BCD.

6

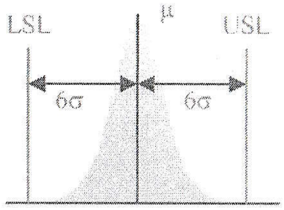
$2^6 = 32$

$2^{6-2} = 16$

2^{6-2}

E = ABC

F = BCD



Considering Aliasing During Analysis

We need to investigate six factors, but can only afford 16 runs instead of 64. The **Design Generators** are

E = ABC and F = BCD.

The **Defining Relation** for this design can be determined as follows:

1. E(E) = ABC(E) which simplifies to I = ABCE
2. F(F) = BCD(F) which simplifies to I = BCDF
3. Multiply the two defining relations together to get the complete defining relation:

   $ABCE(BCDF) = AB^2C^2DEF = ADEF$

(ABCE)(BCDE)

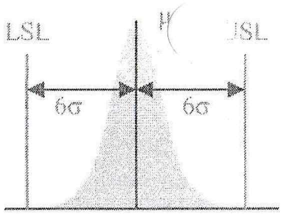
AB AC AD AE
B^2 BC BD BE
BC C^2 CD CE

wordling which is Resolution

Therefore, the complete **Defining Relation** for this design is:

$$I = ABCE = BCDF = ADEF$$

Since all defining words are positive, this is called the principle fraction.



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Considering Aliasing During Analysis

What is the aliasing structure of this design?

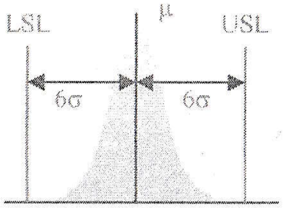
$$I = ABCE = BCDF = ADEF$$

$$I(A) = ABCE(A) = BCDF(A) = ADEF(A)$$

$$A = BCE = ABCDF = DEF$$

$$L_A = A + BCE + DEF + ABCDF$$

A = BCE = DEF = ABCDF
B = ACE = CDF = ABDEF
C = ABE = BDF = ACDEF
D = BCF = AEF = ABCDE
E = ABC = ADF = BCDEF
F = BCD = ADE = ABCEF
AB = CE = ACDF = BDEF
AC = BE = ABDF = CDEF
AD = EF = BCDE = ABCF
AE = BC = DF = ABCDEF
AF = DE = BCEF = ABCD
BD = CF = ACDE = ABEF
BF = CD = ACEF = ABDE
ABD = CDE = ACF = BEF
ACD = BDE = ABF = CEF



Considering Aliasing During Analysis

Suppose that we had chosen another 1/4 fraction instead of the principal fraction?

For the complementary fraction, the **Defining Relation** for the design is:

$$I = -ABCE = BCDF = -ADEF$$

The **Design Generators** are:

$E = -ABC$ and $F = +BCD$

Questions:

- How would you define all four fractions of the full factorial design?

Considering Aliasing During Analysis

What is the aliasing structure of the alternate fraction?

$$I = -ABCE = BCDF = -ADEF$$

$$I(A) = -ABCE(A) = BCDF(A) = -ADEF(A)$$

$$A = -BCE = ABCDF = -DEF$$

$$L_A = A - BCE - DEF + ABCDF$$

D = AB
E = AC
F = BC
G = ABC

D(D) = AB   I = ABD
E(E) = AC   I = ACE
F(F) = BC   I = BCF
G(G) = ABCG   I = ABCG

A = - BCE = - DEF = ABCDF
B = - ACE = CDF = - ABDEF
C = - ABE = BDF = - ACDEF
D = - AEF = BCF = - ABCDE
E = - ABC = - ADF = BCDEF
F = - ADE = BCD = - ABCEF
AB = - CE = ACDF = - BDEF
AC = - BE = ABDF = - CDEF
AD = - EF = ABCF = - BCDE
AE = - BC = - DF = ABCDEF
AF = - DE = ABCD = - BCEF
BD = CF = - ABEF = - ACDE
BF = CD = - ABDE = - ACEF
ABD = ACF = - BEF = - CDE
ABF = ACD = - BDE = - CEF