Minitab®

SPC

LESSON: Quality Methods - Introduction to Control Charts

Statistical Process Control (SPC), Basics of Control Charts, OC Curve, Power, ARL

Dr. Diane Evans

ROSE-HULMAN INSTITUTE OF TECHNOLOGY

Shewhart Control Charts

Walter A. **Shewhart** proposed the concept of a **control chart** while working at Bell Telephone in **1924**

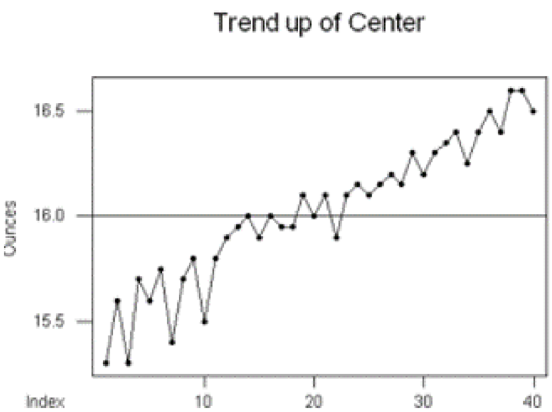
A control chart is a diagram for monitoring process data to differentiate between **special cause** variation and **chance cause** variation. Special cause variation signals the need to take corrective action in a process.

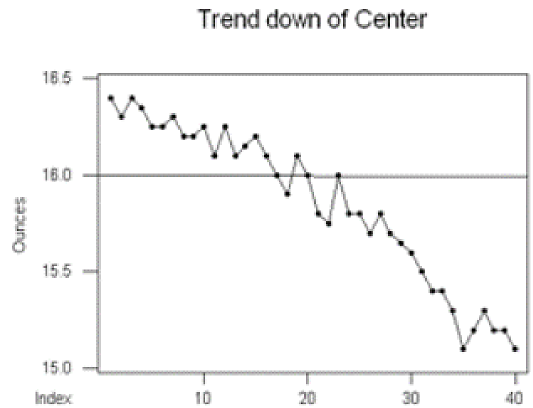
Language:

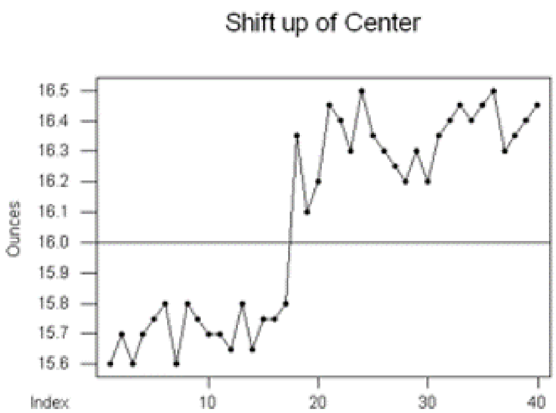
- Special cause = assignable cause, non-random
- Chance cause = natural, inherent, common, or random cause

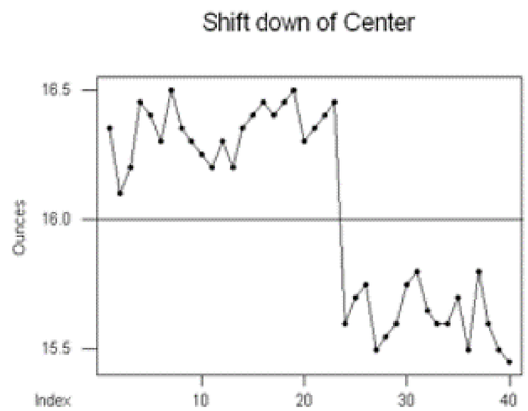
What is Process Data?

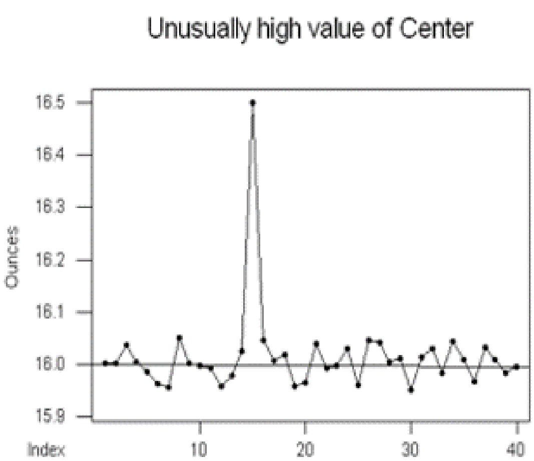
Trends in Process Data

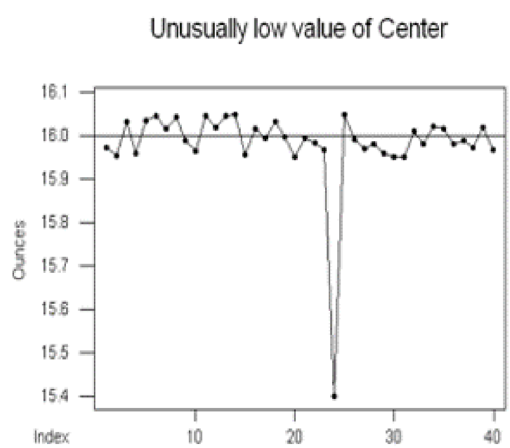


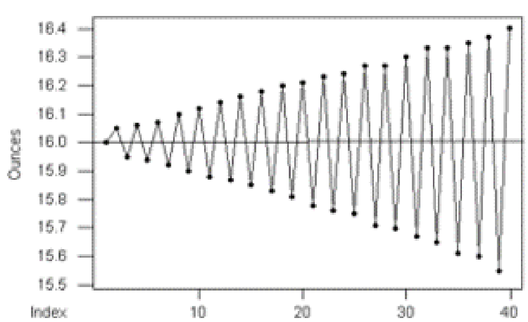


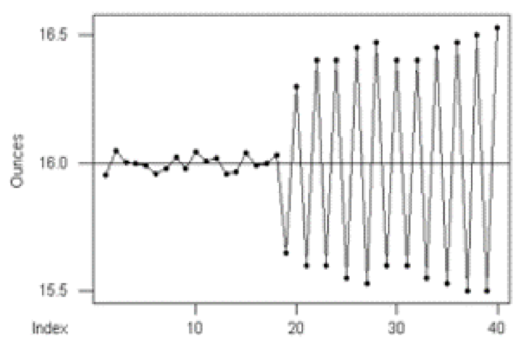












Special Causes in an Industrial Setting

- Operator absent
- Operator falls asleep
- Poor adjustment of equipment
- Faulty controllers
- Machine malfunctions
- Computer crashes
- Poor batch of raw material
- Power surges

Common Causes

- Poor design
- Poor maintenance of machines
- Lack of clearly defined standard operating procedures (SOP's)
- Poor working conditions,e.g. lighting, noise, dirt, temperature, ventilation
- Machines not suited for the job
- Substandard raw materials
- Measurement error
- Vibration in industrial processes
- Ambient temperature and humidity (Crapo?)
- Insufficient training
- Normal wear and tear
- Variability in settings
- Computer response time

> Deming: 85% of problems on the factory floor occur because of top management's errors and common cause variation.

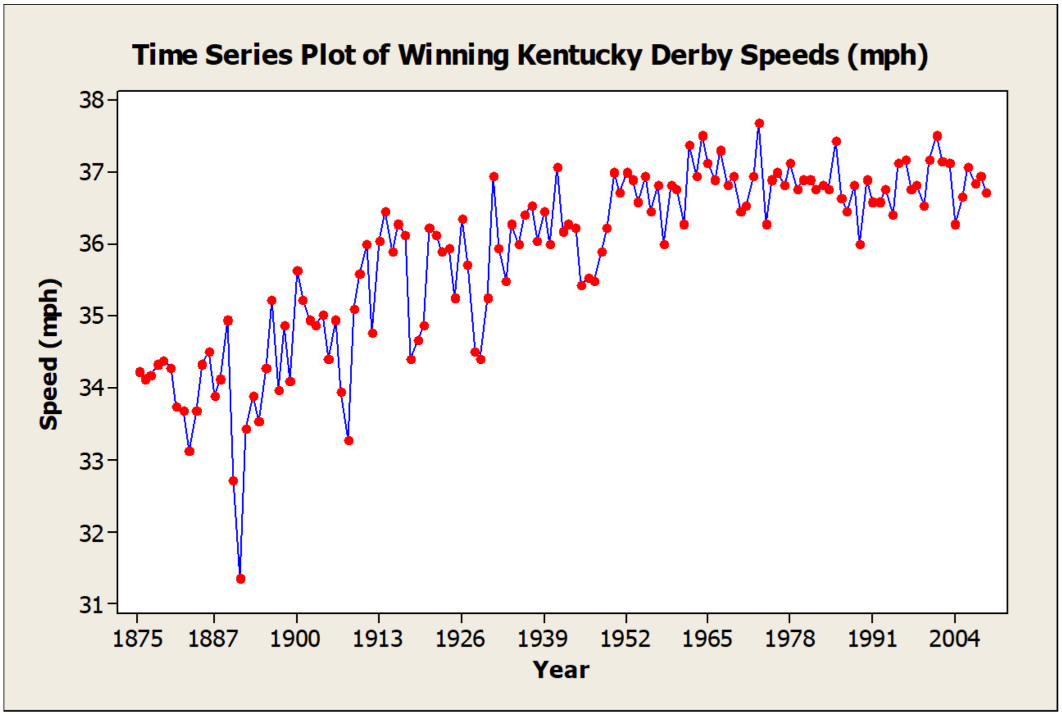
Better Example of Variation Types: My drive to school

After a quarter of tracking driving times, I have established that it takes me ~18 minutes each day to get from home to school in the morning (when leaving @ 8:05 a.m.)

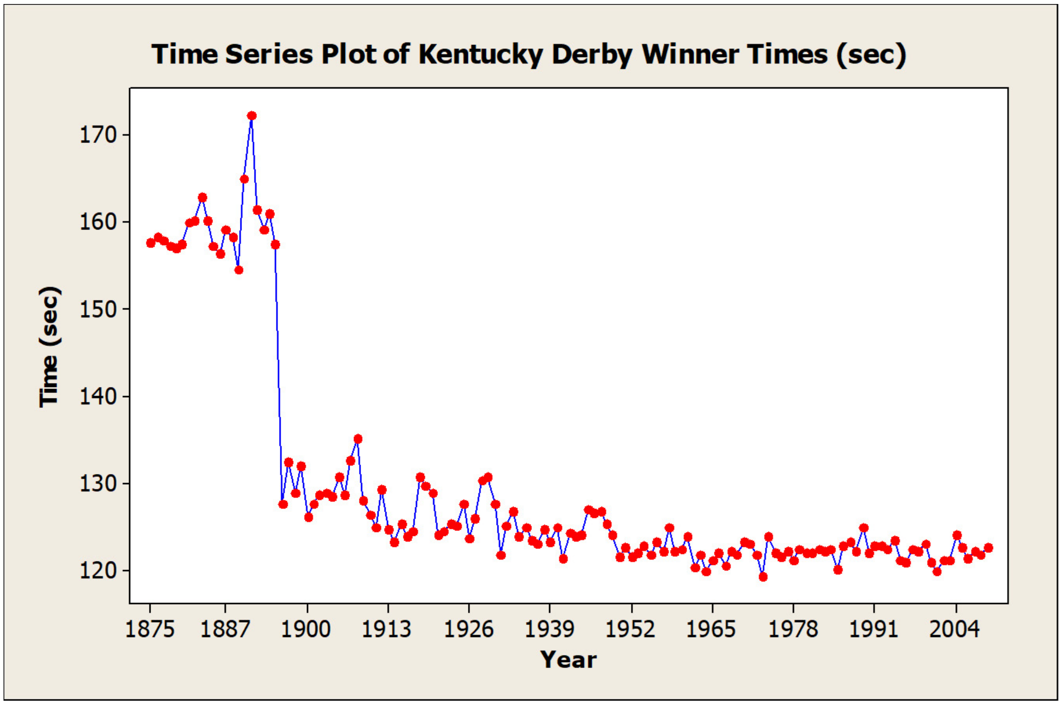
Of course, it doesn't take exactly 18 minutes every day. Sometimes it takes 17 minutes and sometimes it takes 19.2 minutes; variation is inherent in this process. Rarely it may take 30 minutes or more.

What could possibly cause a change from this average of 18 minutes a day?
Common Causes? Special Causes?

Kentucky Derby Speed of Winning Horse



Kentucky Derby Time of Winning Horse



Control Chart Elements

The values of the variable (measurement data) or attribute (count data) are plotted on the vertical axis over time (as in a time series chart)

The horizontal axis represents the ordered subgroups or samples

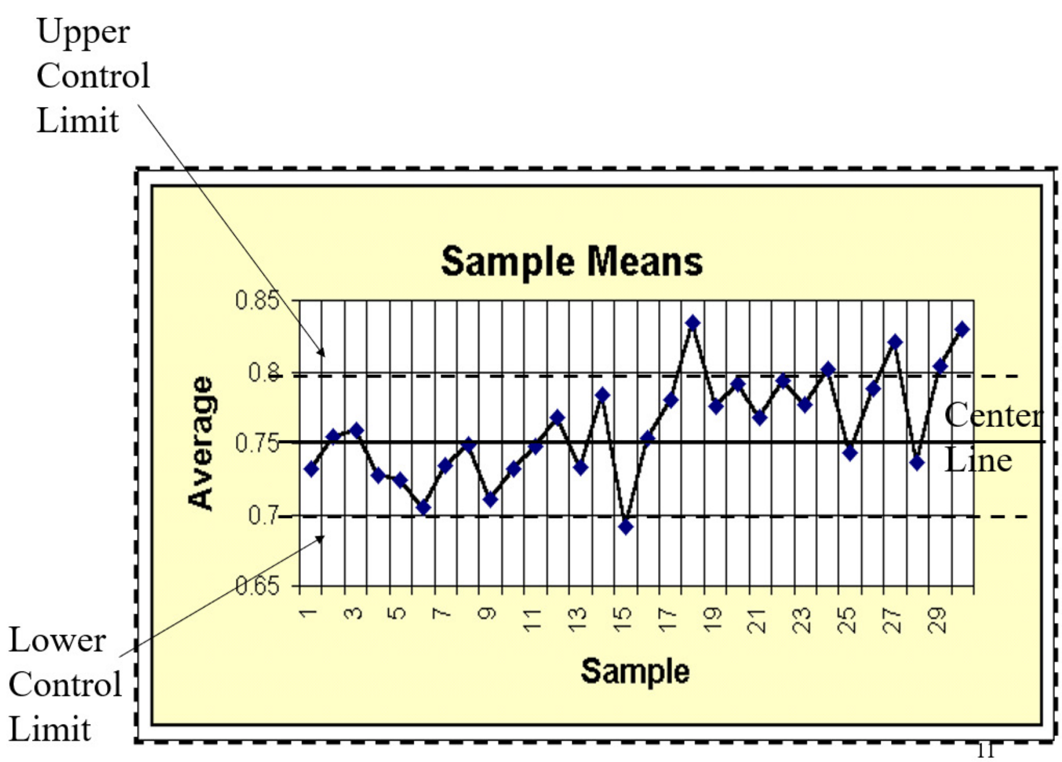
3 major lines are indicated on a control chart:

- Center line (CL)
- Upper and lower control limits (UCL, LCL)

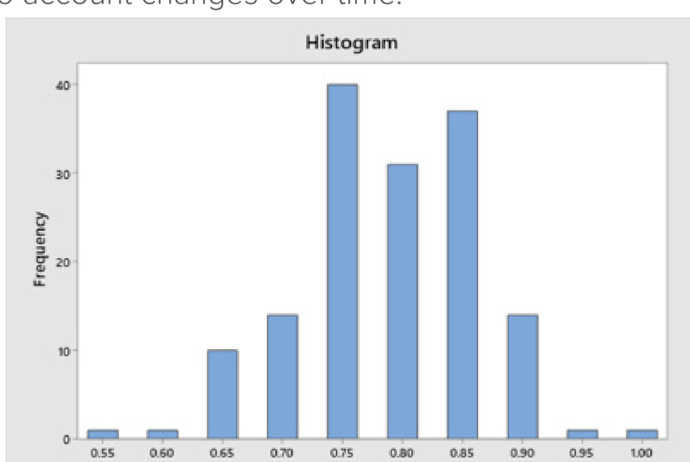
Evaluate if the process is "in control"

- Are points beyond the control limits?
- Are there unusual non-random patterns in the data?

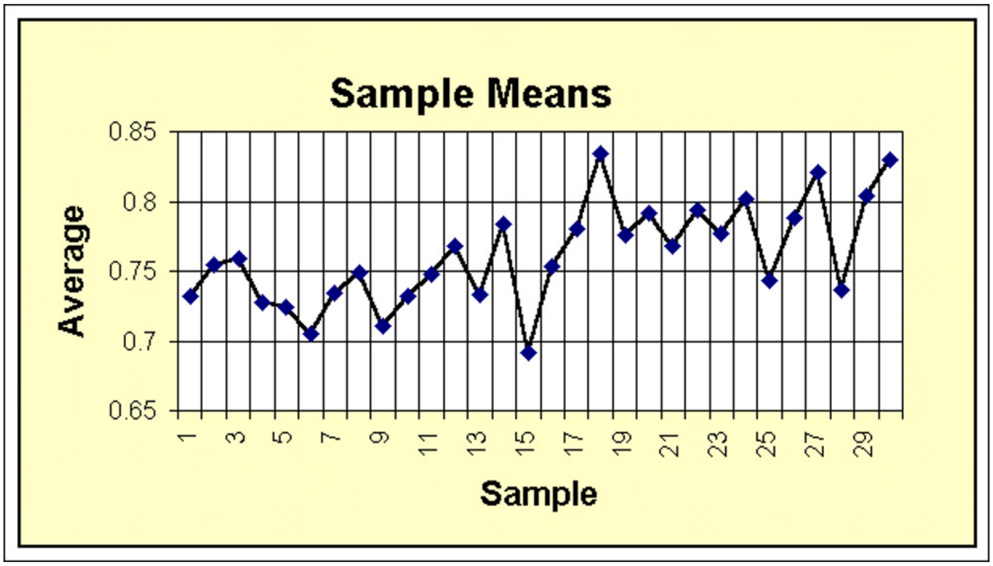
Typical Control Chart



Histograms do not take into account changes over time.



See Wheeler's article "Why We Keep Having 100-Year Floods"
Control charts can tell us when a process changes



Control Charts Can Indicate

1. **When** to take **corrective action**
2. **What type** of **corrective action** to take, which can be noted by the observed patterns on a control chart
3. **When to leave a process alone**; recall the Funnel Experiment
4. Means for **quality improvement**
5. **How well** a process meets customer specifications (capability analysis)

Key Terminology: Process is "In Control"

In general, a process is "in control" if the data stays within **3** standard deviations of the process mean and **displays random patterns.**

There may exist common cause variation in the process – some variation is inherent in any process (lack of clearly defined machine instructions, supplier's raw materials, work conditions).

To be in control, a process **must not have any special cause variation**; it must be common cause variation only.

Deming believed **85% of variation is due to common causes** (which can't be eliminated by the workers on the floor).

Commonly Used Control Charts

Variable (measurement) data (e.g., heights, times, temperatures, ...)

- $\overline{X}$-charts (or written Xbar charts) and R-charts
- $\overline{X}$-charts and s-charts

Charts for individuals and moving ranges (I-MR charts)

Attribute (count) data (e.g., number of defects per item, number of defectives in a lot)

- For “defectives” (**p-chart, np-chart**)
- For “defects” (**c-chart, u-chart**)

Some Key Ideas

Control charts track both process mean and process variation

- **Both must be in control (why??)**

Generally control charts do not track individuals

- **Why not?? When is it reasonable to use individual control charts?**

Continually test – do the charts suggest any reason to conclude the statistic has changed?

Dr. Franklin (at Eli Lilly) quote:
“Control charts are like toddlers”

Errors in Making Inferences from Control Charts

Type I Error (a) – inferring process is out of control when it is actually in control

Type II Error (b) – inferring process is in control when it is actually out of control

Reality - Unknown	No Mean Shift	Mean Shift
Mean Shift	Type II Error β	Right
No Mean Shift	Right	Type I Error α

Our Inference or Conclusion

Example 1. The diameter of cotter pins X produced by an automatic machine is our variable on interest.

Based on historical data, the process average diameter is **μ = 15 mm** with a process standard deviation of **σ = 0.8 mm**. If subgroups of size **n = 4** are randomly selected from the process:

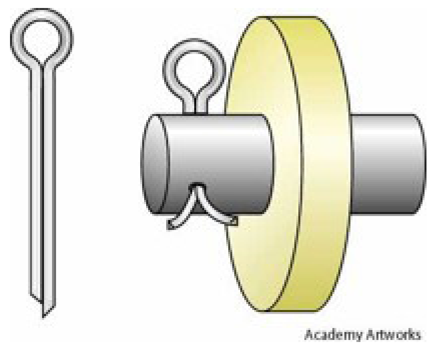
1. Find the 1s and 2s control limits for the **average** diameter $\overline{X}$.

2. Find the 3s control limits for the **average** diameter $\overline{X}$.

3. What is the probability of a "false alarm" for a point above the UCL OR below the LCL (i.e., deciding the machine is out-of-control when it is in-control)?

4. If the process mean shifts to 14.5 mm, what is the probability of not detecting this shift on the first subgroup drawn after the shift? [Note: This is the probability of making a Type ____ Error .]

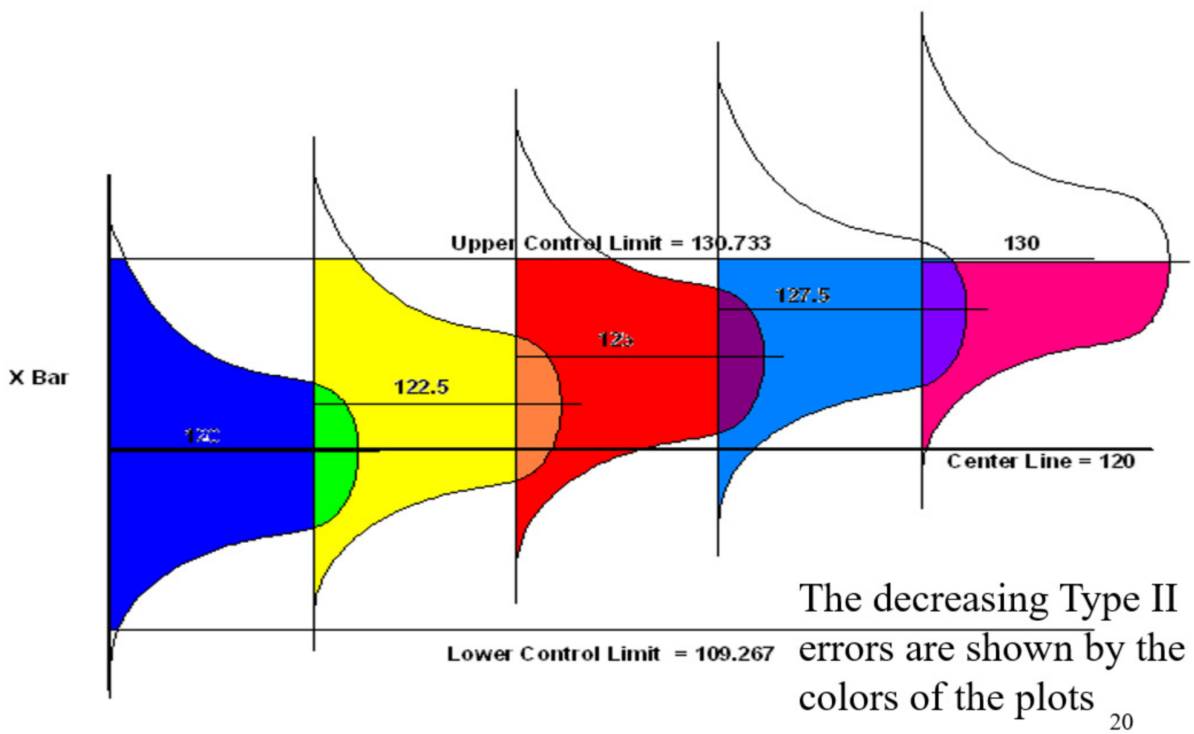
Cotter Pin (from Example 1):



The **Operating Characteristic Curve**, or **OC Curve**

- The probability of a Type II error changes as the process mean shifts
- The OC curve is a plot of the probability of a Type II Error versus the shifted process mean

Shifting Process Mean....



Example 1. (continued) The diameter of cotter pins: μ =15 mm, σ = 0.8 mm, n = 4

5. Construct the OC curve for values of the process mean starting at 15 mm and increasing.

We are determining the probability of committing a Type II Error as the process mean shifts upward

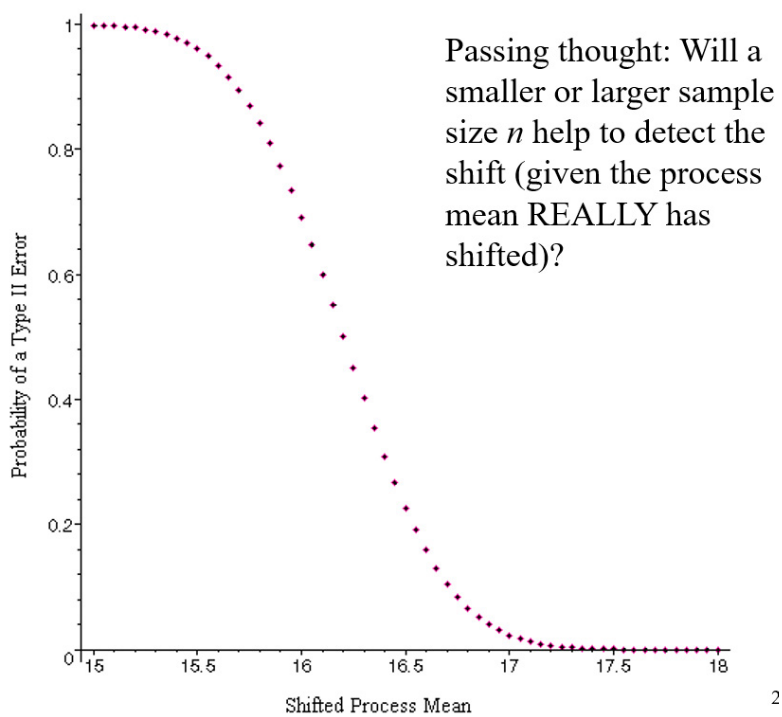
If the process mean shifts to 15.1, what's the probability of a Type II error?
P(LCL < $\overline{X}$ < UCL | m = 15.1) = 0.9964

If the process mean shifts to 15.2, what's the probability of a Type II error?
P(LCL < $\overline{X}$ < UCL | m = 15.2) = 0.9936

If the process mean shifts to 15.3, what's the probability of a Type II error?
P(LCL < $\overline{X}$ < UCL | m = 15.3) = 0.9877

Continuing to increase the mean by increments of 0.1 gives us:

P(LCL < $\overline{X}$ < UCL | m = 15.4) = 0.9772
P(LCL < $\overline{X}$ < UCL | m = 15.5) = 0.9599
P(LCL < $\overline{X}$ < UCL | m = 15.6) = 0.9332
P(LCL < $\overline{X}$ < UCL | m = 15.7) = 0.8943
P(LCL < $\overline{X}$ < UCL | m = 15.8) = 0.8413
P(LCL < $\overline{X}$ < UCL | m = 15.9) = 0.7734



Type I and Type II Errors and control limits

If we widen the control limits (e.g., 3.5s), then Type I error ______________, but Type II error _______________.

If we narrow the control limits (e.g., 2s), then Type II error ______________, but Type I error ________________.

Inverse Relationship of errors as control limits are varied!!! As control limits increase: Type I ↓ and Type II ↑; opposite for decrease

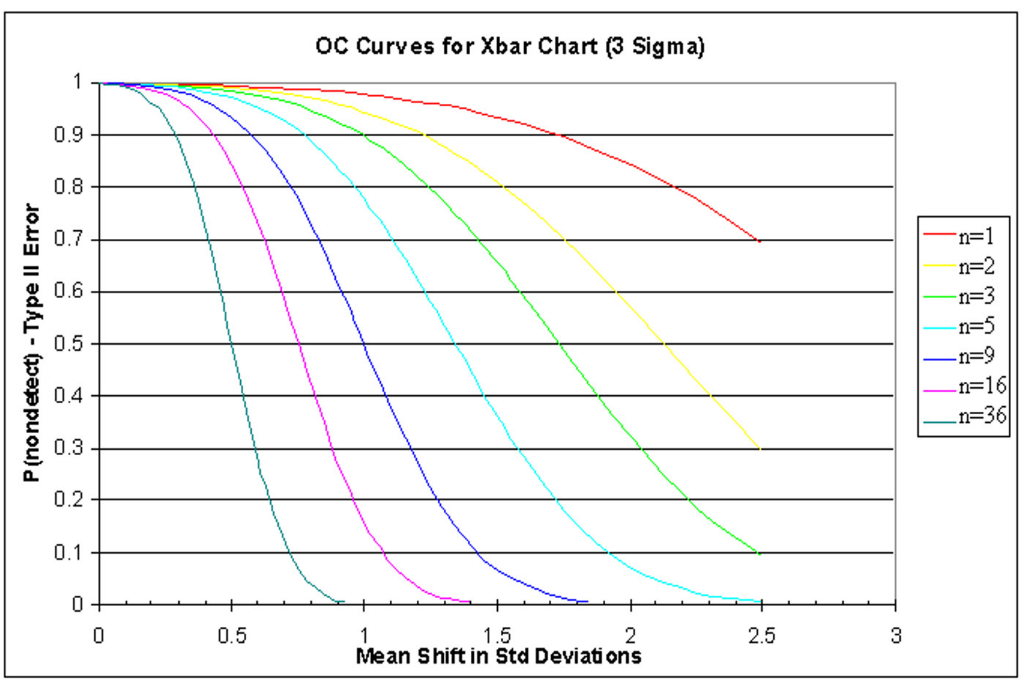
Issues of Type I and Type II Errors

Type I Errors

- Time and cost investigating no real problem.
- "Wild goose chase"
- Loss of confidence in quality activities
- Cranky engineers
- "Producer's Risk"

Type II Errors

- Impact and cost of undetected process shifts, where one BIG cost is UNHAPPY CUSTOMERS
- Fearful floor workers scared to report Type I errors (don't want cranky engineers), and so Type II errors occur
- "Consumer's Risk"



Decrease Type II errors with larger sample size n

Type II Errors and Power of a Test

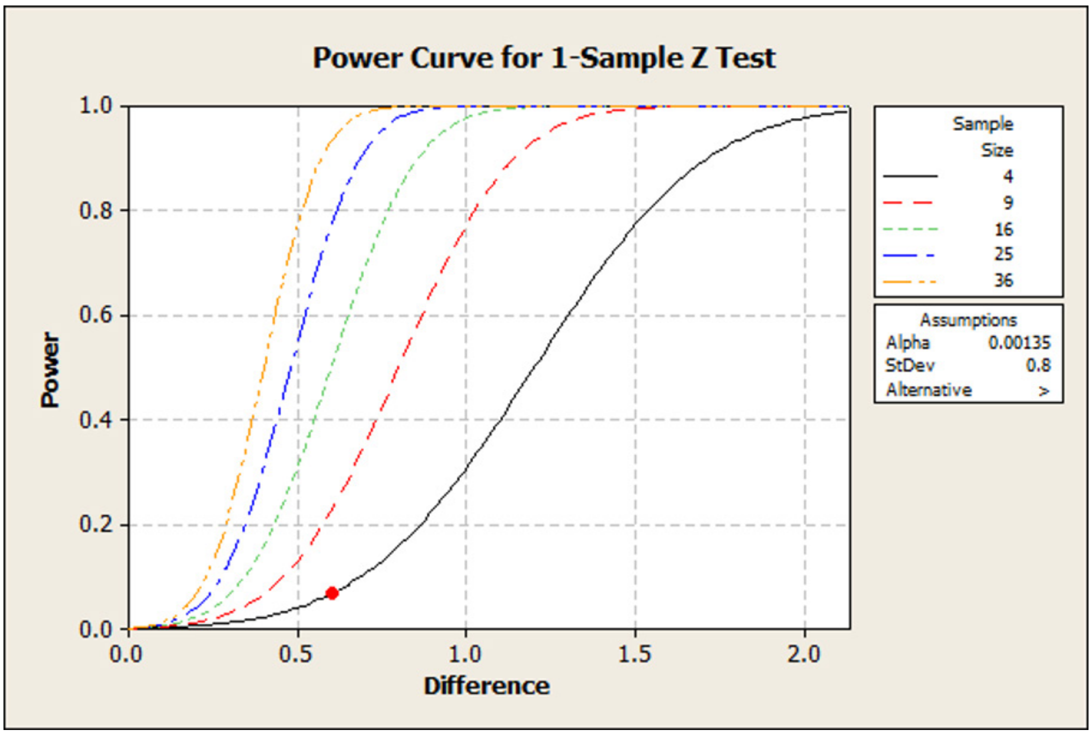
Reminder: The probability of not rejecting the null hypothesis (process is in control) when the alternative hypothesis is true (process is not in control) is a Type II Error b.

The complement of this probability (1-b), the probability of correctly rejecting the null hypothesis when the alternative hypothesis is true, is called the **power** of the test.

Type II errors decrease & the **power of a test increases** with **increasing sample sizes n.**

Only problem: Increase in sample size n comes at a price: a higher sampling cost.

Power of Test for Example 1: Cotter Pin Diameters

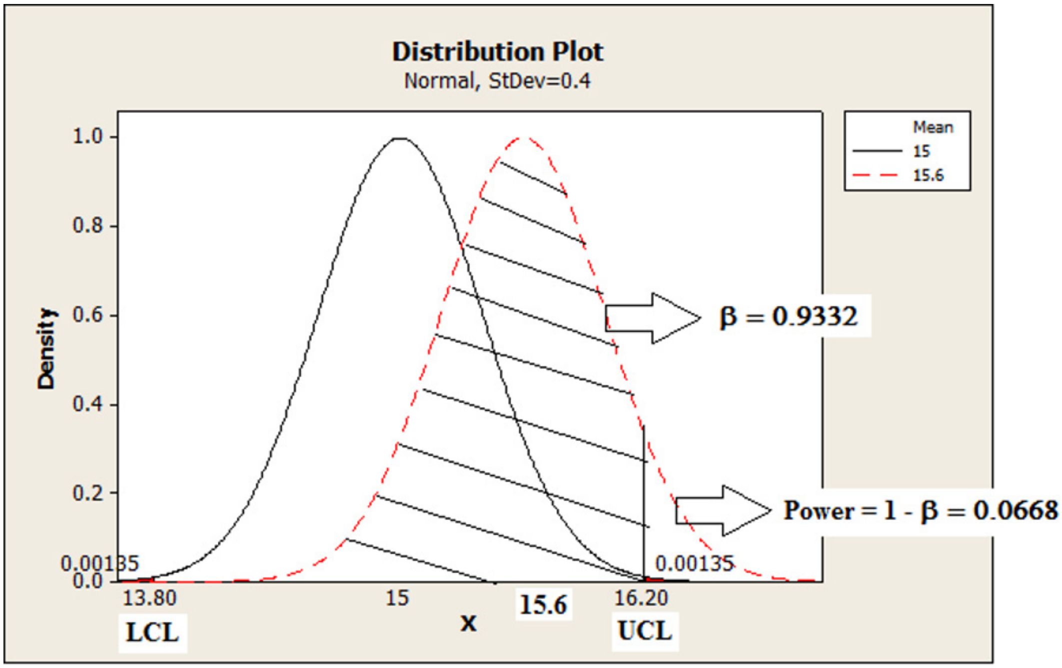


Computing Power

Example 1. (continued) The diameter of cotter pins: μ =15 mm, σ = 0.8 mm, n = 4

Sketch the graph of $\overline{X}$ assuming the true mean is 15 mm. Also, sketch the graph of $\overline{X}$ assuming the mean has shifted to U_1 = 15.6 mm. In your sketch, shade the Type II Error region given the shifted mean is U_1 = 15.6 mm. Use the LCL and UCL as “rejection regions” set at α = 0.0027.

Type II Error and Power for mean shift by 1.5 standard deviations with a sample size of n = 4. (same as slide 29 value)



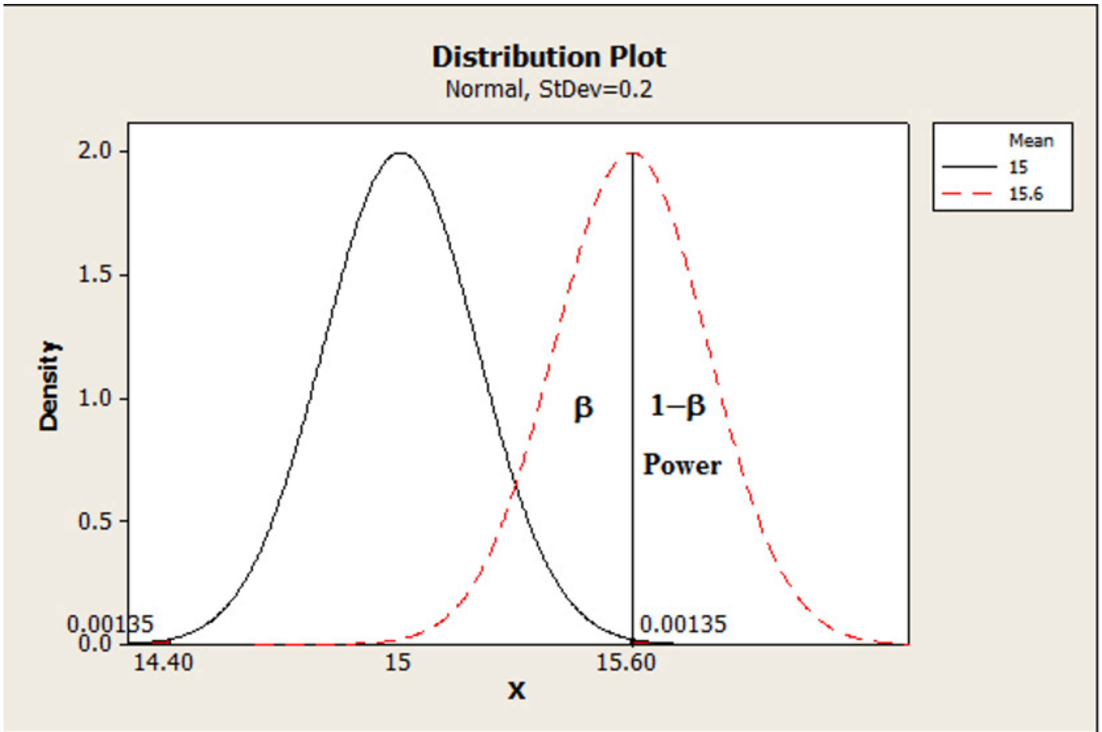
What sample size is necessary to detect a shift in the process mean by 0.6 mm with power 0.5 (or Type II Error = 0.5)?

In Minitab:

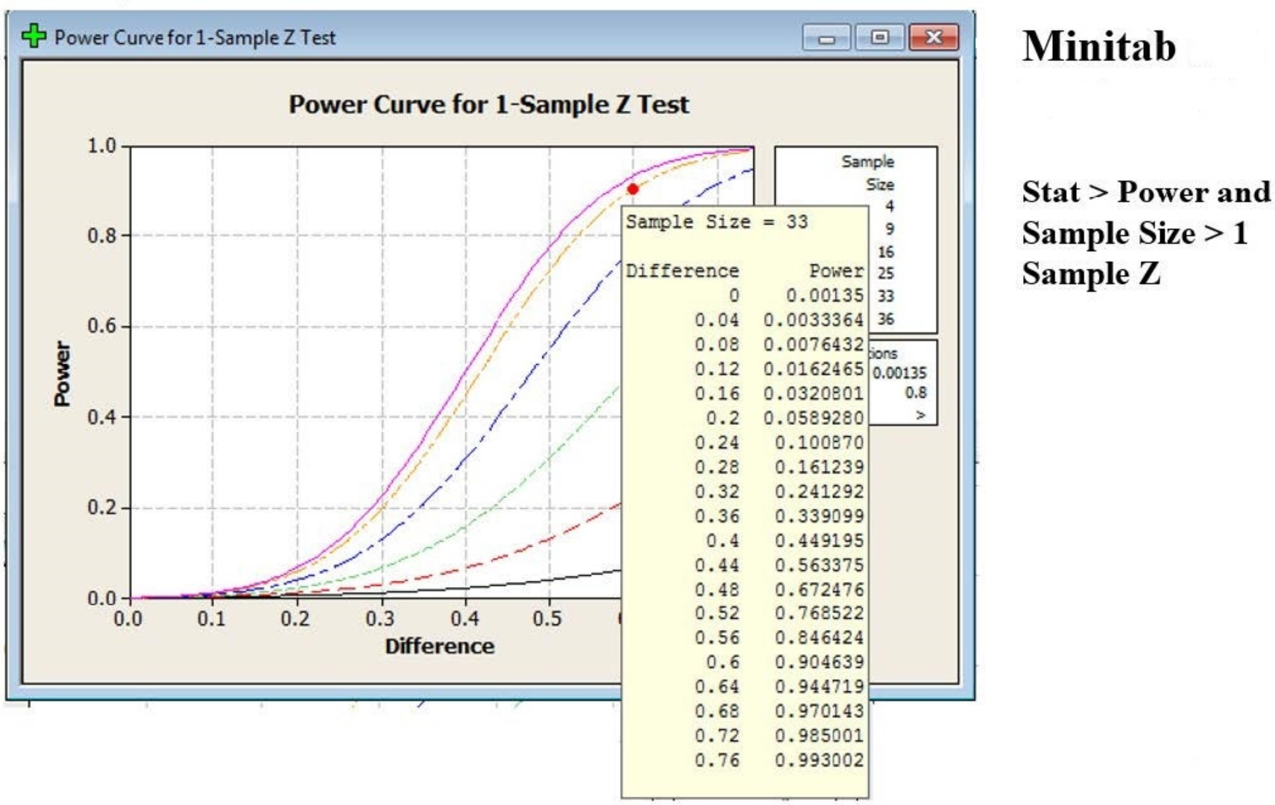
1. Choose **Stat** > **Power and Sample Size** > **1-Sample Z**.
2. In **Differences**, enter 0.6.
3. In **Power values**, enter 0.5.
4. In **Standard deviation**, enter 0.8.
5. Click **Options**.
6. Under **Alternative Hypothesis**, choose **Greater than**. In **Significance level**, enter 0.00135.
7. Click **OK** in each dialog box.

Minitab returns sample size n = 16.

Type II Error and Power for shift of mean to 15.6 mm with a sample size of n = 16.

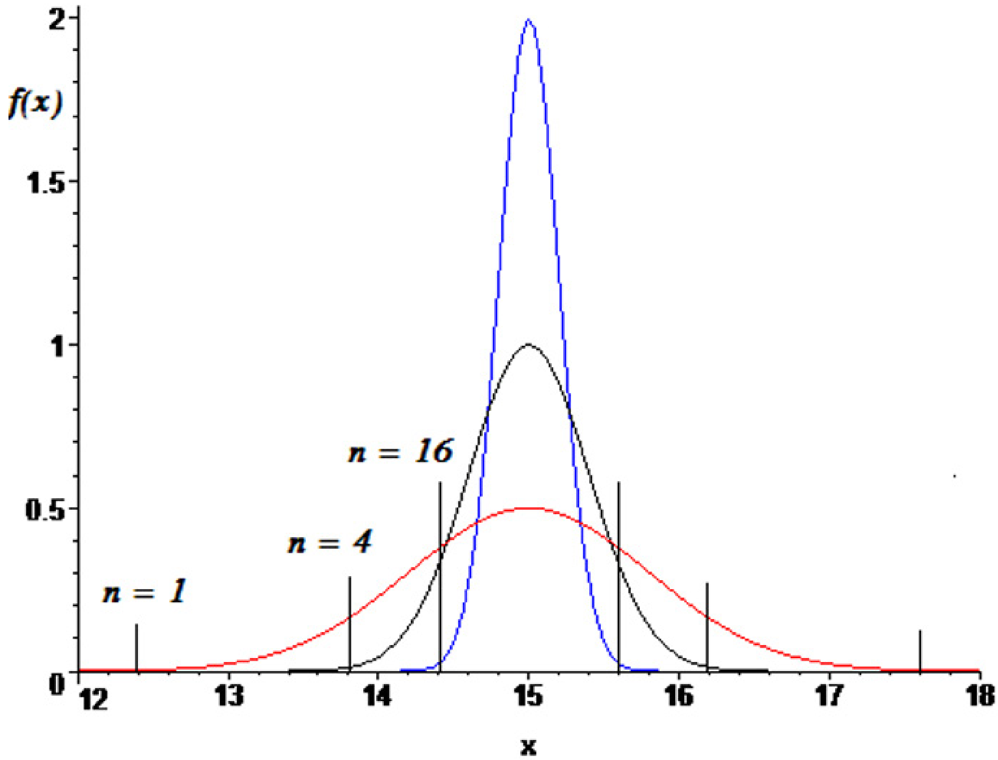


What sample size is necessary to detect a shift in the process mean to 15.6 mm with power 0.9 (or Type II Error = 0.1)?



Sample Size and Type I Errors

What happens to Type I errors as n is increased?



Summary of Control Limits and Type I & II Errors

	Effect on Type I Errors	Effect on Type II Errors
Increasing Control Limits		
Increasing Sample Size		

Problem?

38

Average Run Length: ARL

An alternative measure of the performance of a control chart with respect to Type II Error (in addition to the OC curve).

Definition: The ARL is the number of subgroups, on average, required to detect an out-of-control process.

Let p denote the probability of an observation plotting outside the control limits. Let X be the trial number at which the first out of control point occurs or run length of the process.

Then

- P(X = 1) =
- P(X = 2) =
- P(X = 3) =
- P(X = x) =

for positive integer x

The average or **mean run length** before observing an out of control point is E(X), where E(X) =

Average Run Length (ARL) – Computing for Out of Control Process

For an **Out of Control** process, the ARL is:
ARL = 1/(1-b) = 1/(Power of Test), where b is probability of Type II error

- (1 – b) is the probability of an observation being outside the control limits
- We want the ARL to be small for a process that is out of control for fast detection of an out of control process

Average Run Length (ARL) – Computing ARL for In Control Process

For an In Control process, the ARL is:
ARL = 1/a, where a is probability of Type I error.

- For 3 sigma chart, ARL = 1/0.0026 = 385 …
- Even if a process is in control, a point will plot outside control limits every 385 samples or so
- We want ARL to be big for a process that is **in control**; otherwise, false alarms

Returning to Example 1:

Shifted Mean	β	Power: 1 – β	ARL
15.1	0.9964	0.0036	281
15.2	0.9936	0.0064	155
15.3	0.9877	0.0123	81
15.4	0.9772	0.0228	43
15.5	0.9599	0.0401	24
15.6	0.9332	0.0668	14
15.7	0.8943	0.1057	9
15.8	0.8413	0.1587	6
15.9	0.7734	0.2266	4