CAMBRIDGE

John Butterworth and Geoff Thwaites

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Critical Thinking and Problem Solving

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1.1 Thinking as a skill

This book is about thinking. But it is not about any thinking. It is about those kinds of thinking that take conscious effort, and which can be done well or badly. Most of our thinking takes little or no conscious effort. We just do it. You could almost say that we think without thinking! If I am asked whether I would like coffee or tea, I don't have to exercise skill to reply appropriately. Similarly if I am asked a factual question, and I know the answer, it takes no skill to give it. Expressing a preference or stating a fact are not in themselves *thinking* skills. There are language and communication skills involved, of course, and these are very considerable skills in their own right. But they are contributory skills to the activities which we are calling 'thinking'.

This distinction is often made by assigning some skills a 'higher order' than others. Much work has been done by psychologists, educationalists, philosophers and others to classify and even *rank* different kinds of thinking. Most would agree that activities such as analysis, evaluation, problem solving and decision making present a higher order of challenge than simply knowing or recalling or understanding facts. What distinguishes higher orders of thinking is that they apply knowledge, and adapt it to different purposes. They require initiative and independence on the part of the thinker. It is skills of this order that form the content of this book.

Skills are acquired, improved, and judged by performance. In judging any skill, there are two key criteria: (1) the *expertise* with which a task is carried out; (2) the *difficulty* of the task. We are very familiar with this in the case of physical skills. There are basic skills like walking and running and jumping; and there are advanced skills like gymnastics or woodwork or piano playing. It doesn't make much sense to talk about jumping 'well' unless you mean jumping a significant distance, or clearing a high bar, or somersaulting in mid-air and landing on your feet. There has to be a degree of *challenge* in the task. But even when the challenge is met, there is still more to be said about the quality of the performance. One gymnast may look clumsy and untidy, another perfectly controlled and balanced. Both have performed the somersault, but one has done it *better* than the other: with more economy of effort, and more skilfully.

The first of these two criteria also applies to thinking. Once we have learned to count and add, tell the time, read and understand a text, recognise shapes, and so on, we do these things without further thought, and we don't really regard them as *skilled*. You don't have to think 'hard' unless there is a hard problem to solve, a decision to make, or a difficult concept to understand. So, as with physical performance, we judge thinking partly by the degree of challenge posed by the task. If a student can solve a difficult problem, within a set time, that is usually judged as a sign of greater skill than solving an easier one.

However, when it comes to assessing the *quality* of someone's thinking, matters are more complicated. Mental performance is largely hidden inside a person's head, unlike physical performance which is very visible. If two students give the same right answer to a question, there is no telling from the answer alone how it was reached. One of the two may simply have known the answer, or have learned a mechanical way to obtain it – or even just guessed it. The other may have worked it out independently, by reasoning and persistence and imagination. Although the difference may not show from the answer given, the second student scores over the first in the long term, because he or she has the ability to adapt to different challenges. The first is limited to what he or she knew and could recall, or simply guessed correctly.

Reasoning

Reasoning is the ability most closely associated with human advancement. It is often cited as the faculty which marks the difference between humans and other animals. The famous apes studied by the psychologist Wolfgang Köhler learned ways to overcome problems, such as using a stick to get at food that was beyond their reach; but they discovered the solution by trial and error, and then remembered it for the next time. This is evidence of animal intelligence, and certainly of skill; but it is not evidence that apes can 'reason'. As far as we can tell, no animal ever draws conclusions on the basis of observable facts. None of Köhler's apes thought anything like, 'That banana is further from the bars than the length of my arm. Therefore I need to find a stick'; or 'If this stick is too short, I will need a longer one.'

Reasoning is the process by which we advance from what we know already to new knowledge and understanding. Being rational is recognising that from some facts or beliefs others follow, and using that understanding to make decisions or form judgements with confidence. If there is one overriding aim of this book it is to improve students' confidence in reasoning.

Creative thinking

Reasoning is not the only higher thinking skill, nor the only kind of rationality. Imaginative and creative activities are no less important in the history of human development and achievement. But that is not to suggest that there are two distinct ways of thinking: cold hard reason on one hand and free-ranging creativity on the other. In fact, there is so much overlap and interdependence between the two that it is very difficult to say where one begins and the other ends. Clearly there are times when a seemingly insoluble problem has been cracked by an imaginative leap rather than a methodical process. Some of the greatest advances in science have been the result of creative thinking that appeared to conflict with reason when first put forward. Yet it is just as clear that many apparent flashes of genius, which seem to come 'out of the blue', actually come on the back of a lot of careful and methodical work. Likewise, new and creative ideas have to be understood and explained to be of any practical value. Reasoning is required both to enable and to apply creative thinking, just as creative thinking is needed to give a spark to reasoning.

Reflection

Another quality that is evidently exclusive to human thinking is reflection. Reflecting means giving deep or serious or concentrated thought to something, beyond the immediate response to stimuli. When we are engaged in reflection we don't just make up our minds on impulse, but carefully consider alternatives, think about consequences, weigh up available evidence, draw conclusions, test hypotheses and so on. Critical thinking, problem solving and decision making are all forms of reflective thinking.

Moreover, the reflective thinker does not focus only on the problem to be solved, the decision to be made, or the argument to be won, but also on the reasoning processes that go into those activities. Reflecting on the way we think – or *thinking about thinking* – helps us to evaluate how effective our thinking is, what its strengths are, where it sometimes goes wrong and, most importantly, how it can be improved.

Using this book

Throughout the book there are activities and discussion topics to prompt and encourage reflection on thinking and reasoning themselves. At regular intervals in the chapters you will find 'Activity' panels. You can use these as opportunities to close the book, or cover up the rest of the page, and think or talk – or both – about the question or task. Each activity is followed by a commentary offering an appropriate answer, or some guidance on the task, before returning to the chapter. By comparing the discussion or solution in the commentary with your own reflections and responses, you can judge whether to go back and look at a section again, or whether to move on to the next one.

Although it is not essential to do all of these activities, you are strongly urged to give some time to them, as they will help greatly with your understanding of the concepts and procedures that make up the Thinking Skills syllabus. The tasks also act as opportunities for self-assessment, both of your own personal responses, and of those of your colleagues if you are working in groups. Small-group discussion of the tasks is particularly valuable because it gives you insight into other ways to think and reason besides your own. You have the opportunity to compare your responses with those of others, as well as with the responses suggested in the commentary. The activities and commentaries are like a dialogue between you and the authors of the book.

The book can be used either for a school or college course in thinking skills, or by the student for individual study. It is divided into seven units with varying numbers of chapters within them. Although it is not a straight-line progression, there is an overall advance from basic skills to applied skills and to higher levels of challenge.

Preparing for examinations

The backbone of this book is the Cambridge syllabus for A and AS Level Thinking Skills. All of the assessment objectives for that examination are covered, though not necessarily in the same order as they appear in the specification. The book does not follow the syllabus step by step or confine itself to just one examination. If it did it would not help you either to think more effectively or to do well in your exam. Critical thinking and problem solving are very broad *skills*, not bodies of knowledge to be learned and repeated. A competent thinker is one who is able to deal with the unexpected as well as the expected. This book therefore takes you well beyond the content of one particular exam and equips you with a deeper understanding of the processes involved, as well as a flexible, adaptive approach to the tasks you are set.

Because thinking skills are general and transferable, the topics and concepts dealt with in the coming units will also prepare you for many other awards that involve critical thinking and/or problem solving. The table on pages 342–43 shows a range of public examinations and admissions tests whose content is covered by some or all of the chapters. These include A Level Critical Thinking (OCR and AQA); the BioMedical Admissions Test (BMAT); Cambridge Thinking Skills Assessment (TSA); Singapore H2 Knowledge and Inquiry; and Theory of Knowledge in the International Baccalaureate (IB).

Other subjects

Finally, the value of developing your thinking skills extends far beyond passing exams called 'Thinking Skills'! It has been shown, unsurprisingly, that confidence and aptitude in critical thinking and problem solving will assist students to achieve higher grades across all the subjects that they study. Accordingly you will find critical thinking, problem solving and presenting well-reasoned argument among the learning and assessment objectives of just about every senior-school or university course, whether in the sciences or the arts and humanities.

Beyond that, too, these are sought-after qualities in a great many professions and occupations. Hardly surprisingly, employers want staff who can think for themselves, solve problems, make decisions and construct arguments.

What to expect

To give a taste of the structure and style of the book, this chapter ends with an activity similar to those which appear at regular intervals in all of the coming units. You can think of it as a trial run. The task is to solve a puzzle entitled 'The Jailhouse Key'. It is a simple puzzle, but it introduces some of the reasoning skills you will encounter in future chapters, giving a foretaste of all of three disciplines: problem solving, critical thinking and decision making.

Activity

Two prisoners are held in a dungeon. One night a mysterious visitor appears in their cell and offers them a chance to escape. It is only a *chance* because they must first reason to a decision which will determine whether or not they actually do go free.

Their cell is at the bottom of a long flight of steps. At the top is the outer door. Three envelopes, marked X, Y and Z, are placed on the table in the prisoners' cell. One of them, they are told, contains the key to the outer door, but they may take only *one* envelope when they attempt to leave the cell. If they choose the wrong one, they will stay locked up forever, and the chance will not come again. It is an all-or-nothing decision.

There are six clues, A to F, to help them – or puzzle them, depending on how you look at it. Two are printed on each envelope. There is also a general instruction, on a separate card, which stipulates:

No more than one of the statements on each envelope is *false*.

On envelope X it says:

A The jailhouse key is solid brass.
B The jailhouse key is not in this envelope.

On envelope Y it says:

C The jailhouse key is not in this envelope either.
D The jailhouse key is in envelope Z.

On envelope Z it says:

E The jailhouse key is solid silver.
F The jailhouse key is not in envelope X.

The prisoners may look inside the envelopes if they wish, before deciding. They have five minutes to make up their minds.

Decide which envelope the prisoners should choose in order to escape from the cell.

The best way to do this activity is to discuss it with a partner, just as the two prisoners would do in the story. As well as deciding which envelope to choose, answer this further question:

Why is the envelope you have chosen the ***right*** one; and why can it not be either of the others?

Commentary

Throughout this book you will be given questions to answer, problems to solve, ideas to think about or discuss, followed, as we have said, by commentaries. The commentaries will vary: some will provide the correct answer, if there is one. Some will suggest various possible answers, or different directions you could have taken in your thinking. The purpose of the activities and commentaries is to allow you to assess your own progress and to give you useful advice for tackling future tasks.

Sometimes you may question or disagree with the commentary, especially later on when you have gained experience. On other occasions you will see from the commentary where you went wrong, or missed an important point, or reasoned ineffectively. Don't be disheartened if you do find you have taken the wrong tack. It is part of the learning process. Very often we learn more from making mistakes than we do from easy successes.

In the present example there is only one answer to the question: the key is in envelope Z. The clues, although they seem confusing and contradictory, do give you all the information you need to make the correct decision. Nonetheless, there are any number of different ways to get to the solution, and you may have found a quicker, clearer or more satisfying procedure than the one you are about to see. You may even have taken one look at the puzzle and 'seen' the solution straight away. Occasionally this happens. However, you still have to explain and/or justify your decision. That is the *reflective* part of the task.

Procedures and strategies

Procedures and strategies can help with puzzles and problems. These may be quite obvious; or you may find it hard even to know where to begin. One useful opening move is to look at the information and identify the parts that seem most relevant. At the same time you can write down other facts which emerge from them. Selecting *and* interpreting information in this way are two basic critical thinking and problem solving skills.

Start with the general claim, on the card, that:

[1] No more than one of the statements on each envelope is *false*.

This also tells you that:

[1a] At least one of the statements on each envelope must be *true*.

It also tells you that:

[1b] The statements on any one envelope *cannot both be false*.

Although [1a] says exactly the same as the card, it states it in a positive way rather than a negative one. Negative statements can be confusing to work with. A positive statement may express the information more practically. [1b] also says the same as the card, and although it is negative it restates it in a plainer way. Just rewording statements in this kind of way draws useful information from them, and helps you to organise your thoughts.

Now let's look at the envelopes and ask what more we can learn from the clues on them. Here are some suggestions:

[2] Statements B and F are both true or both false (because they say the same thing).

[3] A and E cannot both be true. (You only have to look at them to see why.)

Taking these two points together, we can apply a useful technique known as 'suppositional reasoning'. Don't be alarmed by the name. You do this all the time. It just means asking questions that begin: 'What if . . .?' For example: 'What if B and F were both false?' Well, it would mean A and E would both have to be true, because (as we know from [1a]) at least one statement on each envelope has to be true. *But*, as we know from [3], A and E cannot both be true (because no key can be solid silver *and* solid brass).

Therefore:

[4] B and F have to be true: the key is *not* in envelope X: it is in either Y or Z.

This is a breakthrough. Now all the clues we need are on envelope Y. Using suppositional reasoning again we ask: What if the key were in Y? Well, then C and D would both be false. But we know (from [1b]) that they can't both be false. Therefore the key must be in envelope Z.

Thinking about thinking

You may have approached the puzzle in a completely different way. For instance, you may not have started with the clues on X and Z, but gone for eliminating Y first. This is perfectly possible and perfectly sensible. If the key were in Y, both the clues on Y would be false. So it could not be there and must be in X or Z. Then you could eliminate X, as in the solution above.

You may not have used the 'What if . . .?' strategy at all. (Or you may have used it but without calling it that or thinking of it that way.) Different people have different ways of doing things and reasoning is no exception. The method used above is not the only way to get to the solution, but it is a powerful strategy, and it can be adapted to a wide variety of situations. The method, in general terms, is this:

> Take a statement – we'll call it S – and ask yourself: '*If* S is true, what else would *have* to be true too?' If the second statement can't be true, then nor can S. You can do the same thing asking: 'What if S is *false*?' If you find that that would lead to something that can't possibly be true, then you know that S can't be false but must be *true*. (If you do Sudoku puzzles you will be very familiar with this way of thinking, although you may not have a name for it.)

Whether you proceeded this way or not, study the solution carefully and remember how it works. Think of it as an addition to your logical toolbox. The more procedures and strategies that you have in the box, the better your chances of solving future problems or puzzles.

Summary

- When we talk of thinking as a *skill* we are referring to higher-order activities, such as analysing, evaluating and explaining; and to challenges such as problem solving and evaluating complex arguments.
- Three broad categories of higher-order thinking are reasoning, creativity and reflection. They all overlap.
- Reflection includes 'thinking about thinking'. In many ways the content of this book is thinking about thinking: thinking more confidently, more skilfully and more independently.

1.2 An introduction to critical thinking

What makes some thinking *critical*, others *uncritical*?

'Critical', 'criticism' and 'critic' all originate from the ancient Greek word *kritikos*, meaning able to judge, discern or decide. In modern English, a 'critic' is someone whose job it is to make evaluative judgements, for example about films, books, music or food. Being 'critical' in this sense does not merely mean finding fault or expressing dislike, although that is another meaning of the word. It means giving a fair and unbiased opinion of something. Being critical and *thinking* critically are not the same thing.

If critical thinking did just mean judging, wouldn't that mean that anyone could do it simply by giving an opinion? It takes no special training or practice to pass a judgement. If I watch a film and think that it is boring, even though it has had good reviews, no one can really say that my judgement is *wrong* and the professional critics are right. Someone can disagree with me, but that is just another judgement, no better or worse, you might say, than mine. In a limited sense, this is true. But a serious critical judgement is more than just a statement of preference or taste. A critical judgement must have some basis, which usually requires a measure of knowledge or expertise on the part of the person making the judgement. Just saying 'I like it' or 'I don't like it' is not enough. There have to be some *grounds* for a judgement before we can call it critical.

Critical Thinking (and critical thinking)

We should also be aware of the difference between 'critical thinking', as a general descriptive term, and Critical Thinking (with a large C and T), which is the name of an academic discipline with a broadly defined syllabus. This book addresses both. In Units 2, 4 and 7 it covers the Critical Thinking (CT) component of the Cambridge and other syllabuses. But it goes well beyond the confines of exam preparation. In fact, having mentioned the distinction, we can largely ignore it. To have maximum value, thinking skills have to be *transferable* from one task or context to others. The aim of this book is to instil in students a critical approach to reading, listening and reasoning *generally*; and to provide the conceptual tools and skills that enable them to respond critically to a wide range of texts. The CT syllabus gives the book its *structure* but not its whole purpose.

The objects of critical focus are referred to generically as 'texts'. The word is used in its broadest sense. In real life a 'text' can be spoken or written or visual: a television programme, for example, or Tweet or blog; or just a conversation. In a book, of course, the texts are restricted to objects which can be placed on a page, so that they are often referred to instead as *documents*. Most of the documents that are used in the coming chapters are in the form of printed texts. But some are graphical or numerical; or a mixture of these. Two other generic terms that are used are 'author' and 'audience'. The author of a text is the writer, artist or speaker who has produced it. The audience is the receiver: reader, watcher or listener.

Some CT textbooks give the impression that critical thinking is directed only at arguments. This can be quite misleading if it is taken too literally. Arguments are of particular interest in CT, but by no means exclusively so. Information, items of evidence, statements and assertions, explanations, dialogues, statistics, news stories, advertisements . . . all of these and more may require critical responses. What these various expressions have in common is that they all make *claims*: that is, utterances that are meant to be true. Since some claims are in fact *untrue*, they need to be assessed critically if we, the audience, are to avoid being misled. We cannot just accept the truth of a claim passively. Arguments are especially interesting because their primary purpose is to persuade or influence people in favour of some claim. The critical question therefore becomes whether the argument succeeds or fails: whether we should allow ourselves to be persuaded by it, or not.

Activities

The core *activities* of CT can be summarised under the following three headings:

- analysis
- evaluation
- further argument.

These recur throughout the book with different texts and different levels of challenge. As they are fully discussed in the coming chapters there is no need to flesh them out in detail here, but they do need a brief introduction:

Analysis means identifying the key parts of a text and reconstructing it in a way that fully and fairly captures its meaning. This is particularly relevant to arguments, especially complex ones.

Evaluation means judging how successful a text is: for example, how well an argument supports its conclusion; or how strong some piece of evidence is for a claim it is supposed to support.

Further argument is self-explanatory. It is the student's opportunity to give his or her own response to the text in question, by presenting a reasoned case for or against the claims it makes.

(In most CT examinations, including Cambridge, these three tasks are set and assessed in roughly equal measure. They are referred to as the three 'assessment objectives'.)

Attitude

As well as being an exercise of skill and method, critical thinking also relates to an attitude, or set of attitudes: a *way* of thinking and responding. Here is a fragment from a document. It is just a headline, no more. It belongs to an article exploring the history of aviation in the magazine section of a newspaper. It challenges the familiar story of the first manned, powered flight in a heavier-than-air machine, by Wilbur and Orville Wright in 1903. The headline reads:

WRIGHT BROS *NOT* FIRST TO FLY

Suppose you have just glanced at the headline, but not yet read the article. What would your immediate reaction be? Would you believe it on the grounds that the newspaper would not print it if it wasn't true? Would you disbelieve it because for so long it has been accepted as a historical fact that Wilbur and Orville Wright were the first? Might you even take the cynical view that journalists make claims like this, true or not, just to sell papers? (After all, it would hardly make 'news', over a century later, to announce that the Wright brothers *were* the first to fly!)

Such reactions are common enough among readers. What they are not is *critical*. They are either passively accepting, or too quickly dismissive. All suggest a closed mind to the question behind the headline.

Critical thinking, by contrast, should always be:

- fair and open-minded
- active and informed
- sceptical
- independent.

Most of these speak for themselves. Without an open mind we cannot judge fairly and objectively whether some statement or story is true or not. It is hard sometimes to set aside or discard an accepted or long-held belief; but we must be *willing* to do it. Nor can we judge any claim critically if we know nothing about it. We have to be ready to take an active interest in the subject matter, and be prepared to investigate and enquire. Hasty, uninformed judgements are never critical. At the very least we would need to read the article before an informed judgement is possible.

Some degree of scepticism is also needed: a willingness to question or to entertain doubt. Scepticism is not the same as cynicism. For example, it doesn't mean doubting everything that journalists write as a matter of course because you think that they are driven only by the wish to grab the reader's interest, with no regard for fact. Critical appraisal requires each claim or argument to be considered on its merits, not on blanket prejudgements of their authors – however justified those may sometimes seem.

Lastly, critical thinking requires independence. It is fine to listen to others, to respect their beliefs and opinions, to learn from teachers, to get information from books and/or from online sources. But in order to think critically you must also be prepared to take some initiative: to ask your own questions and reach your own conclusions. We get very used to being told or persuaded what to think, so that being faced with choices or decisions can be uncomfortable. The methodology of critical thinking can give you greater confidence in your own judgements, and more skill at defending them. But *exercising* the judgement – using it to form your own views – is ultimately up to you.

You cannot evaluate a bare assertion without considering the reasons its author has for making it. So the whole article is presented on the next page. Read the document and then have a go at the following question, a typical critical thinking task.

Activity

How strongly does the information in the article support the headline claim that the Wright brothers were *not* the first to fly?

You can answer this individually, or in a discussion group of two or more. Use your own words. It is an introductory activity, so you are not expected to use any special terms or methods.

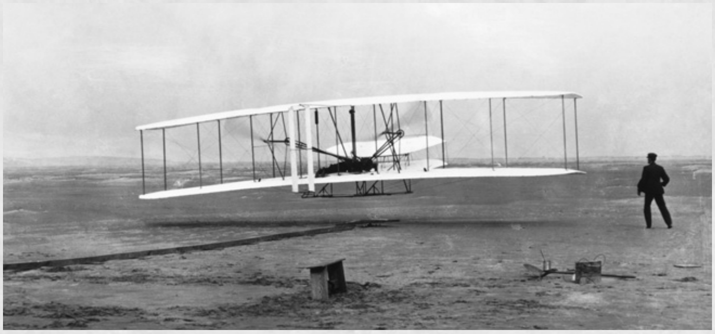
Commentary

This is a typical critical thinking question, and one you will be asked in one form or another many times on different topics. This commentary will give you an idea, in quite basic terms, of the kind of critical responses you should be making.

Firstly, with any document, you need to be clear what it is saying, and what it is doing. We know from this article's style that it is journalistic. But perhaps the most important point to make about it is that it is an *argument*. It is an attempt to persuade the reader that one of the most widely accepted stories of the 20th century is fundamentally wrong: the Wright brothers were not the first to fly a powered aeroplane. That claim is, as we have seen, made in the headline. It is echoed, though a bit more cautiously, in the caption beside the first photograph: 'Or did they (make history)?' The article then goes on to give, and briefly develop, four reasons to support the claim.

Two obvious questions need answering: (a) whether the claims in the article are

WRIGHT BROS *NOT* FIRST TO FLY



Wilbur and Orville Wright make history at Kitty Hawk, USA, December 1903. Or did they?

Many aviation experts and historians now believe that German-born Gustave Whitehead – seen here with his aeroplane 'No. 21' – beat the Wright brothers into the sky by as much as two or even three years.

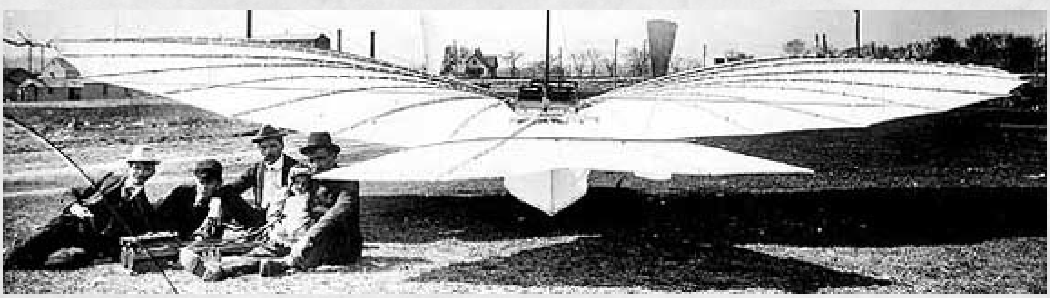
In a 1935 article in the magazine *Popular Aviation*, and a book published two years later, author and historian Stella Randolf tells of a steam-powered flight made by Whitehead in 1899, in Pittsburg, and of signed affidavits from 20 witnesses. One was Louis Daravich, stating that he was present and accompanied Whitehead on his flight. Randolf tells of two more flights, in 1901 in a plane that Whitehead named 'No. 21', and another in the following year in 'No. 22'.

A headline from the *New York Herald,* dated August 19, 1901 read: 'Gustave Whitehead travels half a mile in flying machine . . .', and quoted a witness who affirmed: 'The machine worked perfectly, and the operator had no problem handling it.'

Whitehead was a poor German immigrant to the United States, whose voice was easy to drown out in the debates that followed. The Wrights, by comparison, had influential friends and supporters. The prestigious Smithsonian Institute for Science, in return for ownership of the *Flyer*, agreed not to publish or exhibit anything referring to flights before 1903. The question we should be asking is: Why?

The jury is not so much *out*. The jury has gone home, and the case is closed. History suggests it is time to reopen it.

Jacey Dare



Gustave Whitehead, pictured with his aeroplane 'No. 21', and his daughter and assistants believable; and (b) whether they support the headline claim. You cannot be expected to know whether or not the claims are true unless you have done some research. But it can be said with some confidence that they are believable. For one thing they could easily be checked.

As it happens, most if not all of the claims in the first four paragraphs are basically true. Firstly there are people who believe that Whitehead flew planes successfully before 1903. (You only need to look up Whitehead on the internet to see how many supporters he has. It is hard to say whether they count as 'aviation experts' or 'historians', but we can let that pass.) It is also true that Stella Randolf wrote books and articles in which she refers to numerous witnesses giving signed statements that they saw Whitehead flying. There really was a story in the *New York Herald* in 1901, reporting a half-mile flight by Whitehead, and quoting a witness as saying that the plane 'worked perfectly'. The photograph of Whitehead with his 'No. 21' is understood to be genuine; and no one disputes that Whitehead built aircraft. Lastly, it is a fact that Whitehead was a poor German immigrant, and it is thought that the Smithsonian had some sort of agreement with the Wrights in return for their donating the *Flyer*.

If all these claims are so believable, is the headline believable too? No single one of the claims would persuade anyone, but added together they do seem to carry some weight. That, however, is an illusion. Even collectively the evidence is inadequate. Not one of the claims is a first-hand record of a confirmed and dated Whitehead flight pre-1903. All the evidence consists of is a list of people who said that Whitehead flew. Author Jacey Dare *reports* that author Stella Randolf *wrote* that Louis Daravich *said* that he flew with Whitehead. Such evidence is inherently weak. It is what lawyers call 'hearsay' evidence, and in legal terms it counts for very little.

Here are three more negative points that you could have made, and quite probably did make. Firstly, the photograph of Whitehead's plane does not show it in the air. The Wrights' *Flyer*, by contrast, is doing exactly what its name implies: flying. 'No. 21' *might* have flown. (Apparently some 'experts' have concluded from its design that it was capable of flight.) But that is not the same as a photograph of it *in* flight; and had there been such a photograph, surely Jacey Dare would have used it in preference to one that shows the machine stationary and on the ground. The clear implication is that there is no photograph of a Whitehead machine airborne.

Secondly, the *New York Herald* report is not a first-hand account: it quotes a single unnamed 'witness', but the reporter himself clearly was not there, or he would have given his own account. Thirdly Stella Randolf's article and book were published 34 years after the alleged flight of 'No. 21', and the testimony of Louis Daravich was not made public until then either. Why? There are many possible reasons; but one, all-too-plausible reason is that it simply wasn't true.

An overstated conclusion

Another major weakness in Jacey Dare's argument is that she claims too much. The evidence she provides does not give sufficiently compelling grounds for rewriting the record books. What can be said, however, is that it raises a question mark over the Wright brothers' claim to fame. For even if the argument fails to show that they were *not* the first to fly, it doesn't follow that they *were*. Lack of evidence for something does not prove that it is false, or that the opposite is true.

There is a way, therefore, to be a little more positive about the document. We can interpret it as doing no more than opening up a debate. On that reading, the wording of the headline is just down to journalistic style. If we understand it as a provocative or 'punchy' title rather than a literal claim, and take the last sentence of the article as the real conclusion, then perhaps Jacey Dare has a more defensible point. Maybe it *is* time to reopen the debate. If that is all she is really saying, then she has a stronger case. Or you may feel that even that is going too far for the evidence available.

Whichever judgement you come to in the end, you have now had a taste of critical thinking, and in particular of two of its core components: *analysing* (or interpreting) an argument, and *evaluating* it. You have also seen how the activity sections of the book link up with the instructional part and the commentaries.

Looking ahead

There are three critical thinking units in the book, interspersed – and sometimes overlapping – with the problem-solving units. Unit 2 is entitled 'Critical thinking: the basics', which is self-explanatory. It covers the main concepts and methodologies of the discipline. Unit 4 is given over to 'Applied critical thinking', introducing longer and more complex documents and additional concepts such as evidence and credibility, inference, explanation. Unit 7 is entitled: 'Critical reasoning: Advanced Level'. As the name suggests, it moves into more challenging and sometimes more technical territory. It draws on some of the methodology of elementary logic and formal decision making, and concludes with two chapters on drawing together the different strands of critical thinking that have featured in the foregoing parts of the book.

Summary

- Critical thinking consists of making informed, evaluative judgements about claims and arguments.
- The main strands of critical thinking are: *analysis* (interpretation), *evaluation* and *further argument*.
- Critical thinking is characterised by being: fair and open-minded; active and informed; sceptical; independent.

1.3 Solutions not problems

Some people do not like the word 'problem'; they say, 'We don't have problems, we only have solutions.' The word 'problem' is used in different ways. It can mean something that is causing us a difficulty. The word 'problematical' implies a situation where we cannot see an easy solution to something. However, not all problems are like this. In some cases we may enjoy problems and solve them for fun: for example, when reading a puzzle book or doing a crossword. Most people have some sorts of problem in their lives and many of these may be solved with a little careful thought. The problem solving we are talking about here is based on logic; it is often related to mathematics, in the sense of shape or number, but does not require a high level of formal mathematics to solve. It is largely based upon the real world and is not abstract like much of mathematics. Many people, from carpenters to architects, from darts players to lawyers, use this type of problem solving in their everyday lives.

On the face of it, critical thinking and problem solving might appear as quite separate disciplines. Most critical thinking questions are primarily textual whilst many problem-solving questions contain numerical information. However, the skills used, especially in the application of logic, are quite similar and certainly complementary. Scientists, politicians and lawyers will frequently use both verbal and numerical data in proposing and advancing an argument and in drawing conclusions.

One of the reasons why the two disciplines may be thought of as separate is in the nature of thinking skills examination papers, which often present the tests with clear divisions between critical thinking (CT) and problem solving (PS). Some of this is due to the nature of short multiple-choice questions which mainly deal with testing sub-skills rather than looking at the full real-world application of thinking skills. However, there are areas where a more rounded evaluation is carried out, such as the Cambridge A2 papers, BMAT data analysis and inference, and in Unit 2 of the AQA syllabus. Some of the questions in both disciplines will be seen to be 'hybrid' where, for example, you may be asked to draw a conclusion or asked about further evidence when presented with a set of numerical data.

Although many of the skills used in problem solving in the real world are mathematical in nature, much of this mathematics is at a relatively elementary level, and needs little more than the basic arithmetical operations taught at elementary school. In fact, many problem-solving tasks do not need arithmetic at all. The origins of problem solving as part of a thinking skills examination lie in the processes used by scientists to investigate and analyse. These were originally defined by Robert J. Sternberg (*Beyond IQ: A Triarchic Theory of Human Intelligence,* Cambridge University Press, 1985) and can be summarised as:

- relevant selection: the ability to identify what is important in a mass of data, and thus to recognise what is important in solving the problem in hand
- finding procedures: the ability to put together pieces of information in an appropriate way and thus to discover the route to a solution of a problem
- identifying similarity: the ability to recognise when new information is similar to old information and thus to be able to understand it better and more quickly.

Problem solving in early thinking skills exams was firmly founded on these three basic processes. The BMAT and TSA syllabuses still refer to them explicitly. In the Cambridge examinations, the three basic processes have been expanded into a wider range of skills which are tested at AS Level using multiple-choice questions and at Advanced Level with longer, more open-ended questions which can draw on several of the basic skills. For example, the problem-solving category of 'searching for a solution' is one of the strands of 'finding procedures'.

Unit 3 of this book is entitled 'Problem solving: basic skills' and deals with these extended skills. The chapter structure is firmly based on the problem-solving skills defined in the Cambridge syllabus. Unit 5, 'Advanced problem solving', deals with the extension to Advanced Level and wider-ranging questions. Questions at this level will generally include the use of several of the basic skills. This covers the analysis of more complex data sets, and mathematical modelling and investigation. These questions have open, rather than multiple-choice, answers. Unit 6, 'Problem solving: further techniques', deals mainly with mathematical techniques which may be useful in examinations at all levels.

The end-of-chapter assignments have often been left open-ended rather than framed as multiple-choice questions. This is so you will have to solve the problem, rather than eliminating answers or guessing. Some of the activities and questions are marked as 'harder' and are intended to stretch candidates.

Here is a 'taster' problem to start with. It is certainly not trivial, but illustrates the essence of problem solving. The problem contains only three relevant numbers and the only mathematics required is the ability to add, subtract and divide some small two-digit numbers. Solving the problem requires no specialised knowledge, either of techniques or skills, just clear thinking.

Activity

Marina is selling tickets on the door for a university play. It costs $11 for most people to buy a ticket, but students only have to pay $9. Just after the play starts, she remembers that she was supposed to keep track of the number of students in the audience. When she counts the takings, there is a profit of $124.

How many people in the audience are students?

A 2 **B** 3 **C** 4 **D** 5 **E** 6

Commentary

The $124 is made up of a number of $11 tickets plus a number of $9 tickets. We need to find out what multiples of 11 and 9 will add to 124. We can do this systematically by subtracting multiples of 11 and dividing the remainder by 9. For example, if there were one audience member paying the full ticket price, there would have been $113 from students. This is not a multiple of 9, so cannot be correct. We can list the possibilities in a table:

Number of full-fee payers	Amount paid	Remainder from $124
1	$11	$113
2	$22	$102
3	$33	$91
4	$44	$80
5	$55	$69
6	$66	$58
7	$77	$47
8	$88	$36
9	$99	$25
10	$110	$14

We found the first multiple of 9 with 8 full-price payers: \$124 – \$88 = \$36, which means there were 4 students paying \$9. We carried on checking, just in case there were other solutions. There weren't any, so C (4) is the correct answer. In practice, most of the working could be done mentally as it is quite simple, so the problem could be solved quite quickly.

Problems you will meet later in the book will have similarities to this in that they are based on realistic scenarios and reflect the processes needed to function efficiently in much of employment.

The challenges of problem solving are, in principle, no different from doing a puzzle such as Sudoku in a magazine and many are the type of thing some people will do for fun. Solving such a challenge is a rewarding and enjoyable experience and one which can help you with many things in both your home and working life.

Summary

- Problem solving is about the use of logic, often including simple mathematics, to address real-life situations and aid decision making.
- The fundamental skills of problem solving are: *selecting relevant data*, *finding appropriate procedures to solve problems* and *comparing data in different forms*.
- Learning to solve problems successfully develops skills which are useful in everyday life: at home, in education and at work.

2.1 Claims, assertions, statements

A claim or assertion is an expression that is supposedly true. It may be spoken or written, or sometimes just thought.

We have to say '*supposedly* true' because obviously not all claims and assertions are true. Some are deliberate lies; some are based on mistaken belief. There are also some claims which, as we shall see, are not straightforwardly true *or* false, but can still be asserted, or denied. (A denial is a kind of assertion, an assertion that something is *not* so.)

Here are three illustrative examples:

[A] Angola shares a border with Namibia.
[B] The dinosaurs were cold-blooded.
[C] Top bankers earn too much money.

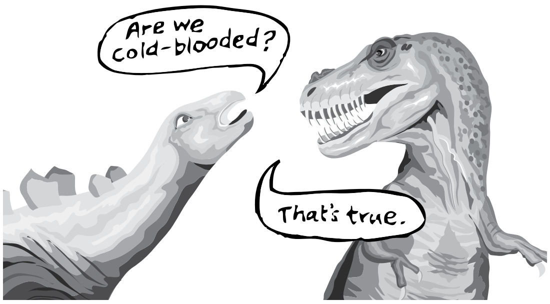
All three sentences are statements. 'Statement' here is used in the grammatical sense to distinguish between sentences that usually express claims and those which are used to ask questions or give commands. If you want a more formal grammatical term, the three sentences are all *declaratives* (or declarative sentences), as opposed to *interrogatives* (questions) or *imperatives* (commands).

It is important to keep in mind the distinction between an actual sentence – a string of words – and what is *expressed by* a sentence: the claim. A claim can usually be made in many different ways. For example, [A] could just as well have been expressed by the sentence:

[A_1] Angola and Namibia are immediate neighbours.

The wording is different but the claim is practically the same. Arguably the same claim or assertion could also be made by sketching and labelling a map showing the two countries next to one another.

Since [A], [B] and [C] are all claims, all three can be judged to be true or false. You may not know whether a particular claim is true, but at least it makes sense to say that it is; or that you agree or disagree with it. It makes no sense to say that a question or command is true.



Fact and opinion

Claims can be divided roughly into those that state facts and those that express opinions. This is a useful distinction, but it needs some clarification.

Activity

Look again at the three expressions above, [A], [B] and [C]. They are all grammatical statements. They all express claims. Discuss how, if at all, they differ from each other.

Commentary

A fact is a true statement. Of the three examples, the first, [A], is a fact. What is more, it is a *known* or an *established* fact. You can check it by looking in an atlas, or going there and crossing the border. Some people may not be aware of the fact, or even mistakenly think something different; but that doesn't in any way alter the fact. If someone says, 'No, these two countries do not share a border,' they are wrong, and that's all there is to it.

Note that *stating* a fact is not the same as *claiming* it – or making a factual claim. You can state a fact only if it really is a fact. But you can claim that something is a fact and be mistaken, or even be lying. Similarly, you can claim to *know* something and be mistaken. But you can't *actually* know something that isn't true. You can only *think* you know it.

Statement [B] that dinosaurs were cold-blooded is a *claim* to fact. But unlike [A], it is not a *known* fact, by the author or by anybody else. Scientific opinion on the subject is divided, with grounds for claiming either that the dinosaurs were cold-blooded (like modern reptiles), or that they were warm-blooded (like birds and mammals). The best we can therefore say of this claim is that it is a belief (or judgement or opinion); and unless or until there is more factual evidence available, it will remain so.

This does not mean, however, that this sentence is neither true nor false. For either the dinosaurs were cold-blooded or they weren't. Scientists may never know the truth, but the truth exists and is there to be discovered – even if it has to wait for the invention of a time machine!

The third claim, [C], is purely an opinion. Two people can disagree as to whether it is true or not, and neither of them is necessarily wrong. It comes down to what they think or believe to be a reasonable wage, and/or what they think of as 'too much'. To say that the sentence is true just means that you agree with it, or assent to it. And to say that it is false means you disagree. It can be 'true' in your opinion at the same time as being 'false' in someone else's.

Another way to distinguish this claim from the other two claims is to say that it is purely subjective. That means that its truth is decided by each individual person – or *subject* – who thinks about it. This is in contrast to the first two, which are *objective*. They are true or false regardless of what anyone thinks or knows. The fact that the truth is hidden does not mean that there is no fact to be discovered.

Value judgements

Claims like [C], that something or someone is good, bad, better, nice, nasty, greedy, too rich, underpaid, and so on, are also called *value judgements*, for the obvious reason that they are opinions about the perceived value or worth or rightness or wrongness of things. It is *not* a value judgement to claim that dinosaurs had cold blood. Nor would it be a value judgement to claim that some bank bosses earn more in a week than an average worker earns in a lifetime. For these are matters of fact which can be quantified and *verified* – or falsified, as the case may be – for example, by comparing the earnings of actual people.

It becomes a value judgement if you claim that there is something 'wrong' or 'excessive' or 'obscene' about a level of earnings; or if you say that, on the contrary, it is 'right' for such successful and talented individuals to get huge rewards. It might be difficult to justify a claim that such huge pay differentials are 'right'; but in the end it remains a matter of opinion or belief; and people may differ in their opinions.

When someone says, therefore, that a value judgement is true (or false), they are using the words in a broad sense to mean something like 'true (or false) *in my opinion*', or 'true (or false) *for me*'.

Predictions and probabilities

Another special kind of claim is a *prediction*. A prediction is a claim that something may or may not be true because it is still in the future, or is as yet unverified. For example, someone might claim, at a certain time and place:

[D] There's going to be a storm in the next 24 hours.

If there is a storm within one day of the sentence being spoken, then you can say, looking back, that the prediction (or forecast) was correct. But you cannot, even with hindsight, say that the prediction was a *fact* when it was made, because at the time of making it, it was not yet known to be true.

Even when a claim cannot be made with certainty, it can often be made with some degree of probability. If you are playing a game with five dice, and need five sixes with your next and final throw, it is a fairly safe prediction that you won't win, because the chances of throwing five sixes all at once are very low. But it is not impossible. *On average*, five sixes will come up once in every 7776 (6^5) throws. The claim that you will lose, therefore, has a high probability of being a correct prediction, but it is not a fact. Similarly, if someone said after you had thrown (and lost): 'I knew you wouldn't win,' you could correctly reply (as a critical thinker): 'You didn't *know* it. You predicted it correctly, that's all.'

Hypotheses

Strictly speaking, many of the claims that scientists treat as fact should be understood as probabilities of a very high order. These are often referred to as *hypotheses*, even when they are generally accepted as true.

Take the prediction that, if a dart and an empty drink can are dropped simultaneously from an equal height (under ordinary atmospheric conditions), the dart will land first. This claim is made on the grounds that, whenever two such objects are dropped, the result is always the same – or always *has been* the same – so that it is entirely reasonable to expect it to go on being the same in the future. The observed result is explained by the general principle that thin, arrow-shaped objects encounter less air resistance than bulkier ones, allowing the former to accelerate more rapidly under the same force (in this case gravity) than the latter.

The hypothesis has been so well tested that the probability of such a claim ever being wrong is practically non-existent. We call it a 'hypothesis', rather than an absolute certainty, because *conceivably* the laws of physics may not be the same in the far, unknowable future, or in all possible worlds.

Besides, there *have* been many scientific beliefs in the past that no one seriously doubted, but that have had to be revised because of later discoveries. One of the best-known examples is the belief that the Sun circled the Earth, or actually rose each morning from beneath the Earth and travelled across the sky. It was widely accepted by astronomers before the time of Copernicus. More recently, Albert Einstein's claim that nothing could exceed the speed of light seemed unchallengeable until, in 2011, a team of scientists at the Large Hadron Collider claimed to have measured a tiny subatomic particle – a neutrino – travelling fractionally faster. Their measurements have yet to be confirmed, and may have been proved wrong by the time you are reading this page. But whilst any uncertainty remains, Einstein's assertion is still just a hypothesis, and hence a claim, not a fact.

Recommendations

Recommendations or suggestions are claims of yet another sort. Here is one example:

> [E] The wages and bonuses of bankers should be capped.

This may seem quite similar to [C]: the claim that top bankers earn too much. Both express a similar sentiment, and both are opinions rather than hard facts. However, there is an important difference. [C] is an observation. It describes a situation as the author sees it: the way things are in his or her opinion. [E], in contrast, is a claim about how things *ought* to be, or what the author thinks should be done in response to the situation.

Recommendations, like value judgements, are not straightforwardly true or false. Two people – even two people who agree about [C] – may disagree about whether the recommendation to cap wages is the right way to deal with what they see as excessive earnings. Neither of the two will be factually wrong in their judgement. If one person says that it is 'true' that bankers' wages should be capped, it just means that he considers it to be a good idea. If another says it is 'false', she is claiming it is a bad idea.

Grammatical note

We saw earlier in the chapter that claims typically take the form of statements, or declarative sentences. In some cases, however, other grammatical forms can be used.
Take [C] again. A similar point could be made by 'asking':

> [C_1] How disgusting are bankers' wages?

'Asking' is in quotation marks because [C_1] is not a genuine question but a *rhetorical* one. (You could alternatively call it an exclamation, and punctuate it with an exclamation mark.) What defines a rhetorical question is that it is not really in need of an answer: it is making an assertion. In this case the assertion is:

> [C_2] Bankers' wages *are* disgusting.

Summary

- In this chapter we have discussed and analysed one of the most basic concepts in critical thinking: claims. These are also referred to as 'assertions' and 'statements'.
- Several important kinds of claim have been introduced. They include:
  - claims to fact
  - statements of opinion or belief
  - value judgements
  - predictions
  - hypotheses
  - recommendations.

  There will be more discussion of all of these kinds of claim in the coming chapters.

End-of-chapter assignments

1 Explain briefly, in your own words, the difference between a claim and a fact.
2 Is there any significant difference between a claim and an assertion? If so, how are they different? If not, what do they have in common?
3 For each of the five examples [A]–[E] in this chapter, suggest two other claims that have the same relation to the truth, but on different subject matter.
4 The word 'hypothesis' has several close relatives. Here are four:

- conjecture
- theory
- guess
- speculation

In what way is each of these different from the others? (You can use a dictionary to help you answer the question.)
5 How would you define the following special kinds of claim?

- allegation
- accusation
- insinuation
- confirmation
- denial
- verdict

6 The idea of *claims* is central to the discipline of critical thinking. Why is this so?

Answers and comments are on page 311.

2.2 Judging claims

When a claim is made, especially publicly, it is natural to think we are being told the truth. Most of the time we accept claims, especially claims to fact, at face value. For instance, if we read in the newspaper that there has been a plane crash, we are entitled to assume that such an event really has taken place. We don't jump to the conclusion that the statement is false just because we have not witnessed it ourselves. We hear the football results, or baseball scores, and assume they are correct, and not made up to please the fans of some clubs. We get a weather forecast telling us to expect heavy snow, and we plan accordingly: we don't ignore it just because it is a prediction, and predictions aren't facts.

Assuming that most of what we are told is true is entirely reasonable. Indeed, it is necessary for a normal life, and the functioning of a modern democratic society. If we questioned, or refused to believe, everything we read or heard, life as we know it would come to a standstill. That is why we all have a responsibility to tell the truth; and why people are understandably annoyed if they are told something that is not true.

Everyone knows the story of The Boy Who Cried 'Wolf!' or a story like it. The boy has a bad habit of raising false alarms, in particular frightening his community by shouting out that a pack of wolves is approaching the village. At first the villagers run to safety whenever he does this. But after a while they stop believing him, until the day comes when a real wolf appears. By then, of course, the boy has lost all credibility and his for-once genuine warning is ignored. (You can work out the ending yourself.)

The moral of the story is that truth and trust are both important. People need to be able to rely on what they are told most of the time; and people who speak the truth need others to believe them most of the time. But that does not mean we should respond with blind acceptance to everything that we read and hear. Obviously we cannot assume that just because something has been asserted – in spoken, printed or any other form – it is true, or we have to agree with it. People do make false assertions not only with intent to deceive, but also out of carelessness or ignorance. Even when there is a core of truth in what someone says, it may be exaggerated, or over-simplified, or a mere approximation, or a rough guess. There are many ways, besides being plainly false, in which a claim may be less than the whole truth.

None of this means that we should start routinely doubting everything. But it does mean we should keep an open and inquisitive mind.

Justification

As you saw in the previous chapter, it is not always possible to *know* whether a claim is straightforwardly true or false. Knowledge requires certainty and certainties are rare. In the absence of certainty, the best evaluation we can give of a claim or belief is to say whether it is *justified*, or *warranted*. These two words mean much the same as each other. A warrant is a right or entitlement. We are entitled to hold a belief, or to make a claim, if there are strong grounds – for example, evidence – to support it. Without such grounds a claim is unwarranted (unjustified).

At first sight it may seem that truth and justification amount to the same thing: a claim is justified if it is true, and unjustified (or unwarranted) if not. But neither of these is correct. A claim can be true but unjustified if the person making it does not have good grounds for believing it – or in extreme cases may not believe it at all. Suppose, for example, a crime has been committed. The victim (we'll call her Vera) claims that her neighbour (Nick) was the one who did it, perhaps because she doesn't like him, or perhaps because she wants to see someone convicted, and anyone will do. Other than this she has no reason for making the allegation, and certainly nothing that would count as evidence. But then suppose it is discovered that Nick, just as Vera has claimed, is guilty of the crime! Would the discovery of Nick's guilt justify Vera's accusation? No. It would just be *chance* that the claim she had made was true. Given her motives her claim would still be a lie.

Conversely, a false claim can be justified in some circumstances. Someone may make an assertion on the basis of all the information available at the time of making it. If that information gives convincing grounds for the claim, then it is fair to say that it is a justified claim to have made, even if it later turns out to be false on the basis of some new information.

In other words, truth and justification are different. Justification is provided by the reasons that can be found and given for a claim, but truth or falsity belong to the claim itself. We may never know for certain whether a particular claim is true, but we may be able to say that there is sufficient evidence or grounds or support to justify asserting it. Alternatively we may say that a claim is *unjustified*, because there are *not* sufficient grounds or support for it, or because there *are* sufficient grounds to cast doubt on it. This is different from saying that it is actually false.

Judging which of these is the right way to respond to a claim is at the heart of the discipline of critical thinking, and is part of what we mean by 'evaluation'.

Activity

Recall the example in the last chapter: the claim that the prehistoric dinosaurs were cold-blooded. Two facts are often cited in support of this:

[A] The dinosaurs were reptiles.
[B] Modern reptiles, e.g. snakes and lizards, are all cold-blooded.

Discuss whether these two facts between them justify the claim that the dinosaurs were cold-blooded.

Commentary

The two facts give some support to the claim, but only some. They are grounds for the *hypothesis* that the dinosaurs were cold-blooded inasmuch as they add some weight to that side of the debate. If you knew nothing else about dinosaurs, or reptiles, or evolution generally, you might be tempted to accept the grounds as sufficient. But it would be a big step to take. For one thing it would mean assuming that what is true of reptiles now must have been true of reptiles 70 million years ago, and earlier. It is not at all impossible that there were once warm-blooded reptiles running around, including some of the dinosaurs; but that these reptiles became extinct, leaving only the cold-blooded species surviving today. (Being cold-blooded may have given certain reptiles a survival advantage over the warm-blooded ones. Warm-blooded species use more energy than those with cold blood, and food sources may have become scarce.) This possibility alone means that the assumption is questionable, though not necessarily false.

So [A] and [B] on their own do not really justify taking the hypothesis as fact. It could be true, and many scientists consider it more probable than the counter-claim that the dinosaurs were warm-blooded. But there is no proof one way or the other.

Standards

It should be noted that 'justified' is not an all-or-nothing term like 'true' and 'certain'. A claim is either true or it is not. You may want to object that some claims are *partly* true (or partly false); or somewhere in between truth and falsity. But in strict terms 'true' means 'the whole truth and nothing but the truth', and does not allow degrees or approximations. A claim, on the other hand, can be more or less justified according to the strength of the supporting grounds and the context in which the claim is made.

Here is a simple example. (A '*marathon*', officially defined, is a running race over 42.195 km. There are various explanations and historical accounts for this rather peculiar distance. You may like to do some research and find out why. But for present purposes what matters is that it is a *fact*.)

Activity

Let us suppose that Katya has just returned from a training run of 42 km and announced to her friends:

[C] I have just run a marathon.

Discuss whether her claim is *justified* (or warranted), given that it is so close to the truth. Is it in any sense 'true'? Or is it altogether 'false'?

Commentary

The assertion is, strictly speaking, untrue. Even if we allow that by 'marathon' Katya means the marathon *distance* (rather than an organised race), her claim is short of the whole truth – by 195 metres. You may have thought it was fair to say that Katya's claim was *nearly* true, or *approximately* true; but this is really just a way of saying that Katya ran nearly a marathon or approximately a marathon. Indeed, it is *completely* true that Katya ran *nearly* a marathon, even though [C], as it stands, is not true.

Is [C] as it stands justified? That is a more difficult question. It depends on the circumstances or context in which it was asserted. If it is just a conversational context, which is what it sounds like, then it would be plainly silly to call Katya a liar. However, if she had to run at least one complete, officially recognised marathon – perhaps in a certain time – to pass some test, and she was counting the training run as her qualifying run, then we have to say that her claim is not justified. What makes the difference is the standard of accuracy or precision required.

The most familiar example of varying standards of this kind is in the law. Take a guilty verdict passed in a criminal trial. (A verdict is a special kind of claim. You were asked to define it in the assignment at the end of Chapter 2.1.) Under the justice systems of many countries, the UK included, a guilty verdict is justified only if it can be proven *beyond reasonable doubt*. That phrase sets the standard. So, even if the jury are pretty sure the defendant is guilty, but there is just a small, lingering uncertainty, they must give a verdict of not guilty – or in some countries an 'open verdict', or 'unproven'. Similarly, those who give evidence in a court are instructed to tell *the truth, the whole truth and nothing but the truth*. This, too, sets a very high standard on what counts as a justified or warranted assertion.

By contrast, the standard required for a 'not guilty' verdict is much lower: all that is required is that there is some room for doubt – at least in societies which hold the principle that a person is innocent until proven guilty. In a criminal case there is an imbalance between the standards that must be met by the prosecution and the defence respectively. The 'burden of proof', it is often said, 'lies with the prosecution'.

The balance of probability

Outside the criminal law we may find standards lower than proof being needed to justify a claim or decision. For instance, in a civil case, where both sides are treated equally, a verdict is justified 'on the balance of probability'. Obviously it is much harder to justify a claim beyond reasonable doubt than on the balance of probability.

What this means is that there are *degrees* of justification, depending on context. For critical thinking it means that when we judge a claim to be justified (warranted), or *un*justified (*un*warranted), we need to *qualify* the judgement by stating what standard we are applying. Expressions like 'wholly (completely, entirely) justified' are stronger than 'well supported' or 'highly likely'; and 'unwarranted' is stronger than 'open to question' or 'unlikely'. Choosing the right qualification for the judgements we make about claims and their justification is one of the most important critical skills to develop – arguably the most important.

Knowledge and certainty

With certainty, on the other hand, there are no degrees. It is true that people often talk about the degree of certainty that can be given to some claim or other; but what they really mean by this is the degree to which the claim falls short of certainty. The claim that you will never win the lottery is so highly probable that it can be stated as a near-certainty. But near-certainty is not certainty. Likewise, you don't *know* that you won't win the lottery. If everyone who bought a lottery ticket claimed to know that they would not win, sooner or later one of them would be wrong!

However, this does not mean we can never use the words 'know' or 'certain' appropriately. It is perfectly appropriate to say of some claims that they are certain. The truths of mathematics and logic are usually spoken of as certainties. No one doubts that $7 + 5 = 12$ or that a triangle has three sides, or that an object cannot be red and black all over at the same time. Claims like these are often said to be true *by definition*. For example, '12' just means the same as 'the sum of 7 and 5'.

Also there are claims which are *practically* certain even if they are not logically true. The old favourite is that the sun will rise tomorrow (as it has always done on previous days). It would be foolish to dispute this claim, despite the fact that some freak of nature could *conceivably* spell the end of the solar system in the next 24 hours. If you had to bet on winning the lottery or the sun not rising, you would bet on winning the lottery every time!

Complex claims

Sentences such as 'Katya just ran a marathon' or 'Dinosaurs were reptiles' express simple claims. The following, by contrast, are *complex* sentences, each expressing two or more connected claims:

[D] Katya just ran a marathon and completed the distance in under four hours.

[E] The dinosaurs were reptiles, yet they were warm-blooded.

[F] Sea levels are rising around the world because global warming is melting the polar ice caps.

[G] Many parts of the world will soon be submerged if nothing is done to reverse climate change.

Grammatical note

A simple sentence, when it becomes part of a complex sentence, is called a 'clause'. Words or phrases which express the relation between clauses are called 'connectives': for example, 'and', 'because', 'if'.

Activity

What difference does it make to the way we judge a claim if it is complex rather than simple?

For each of the examples [D]–[G] discuss the conditions that would have to be met to justify the whole claim.

Commentary

When assessing complex claims we also have to take note of the connective, and the relation it expresses between the parts.

In the case of [D] the job is quite straightforward. The connective is 'and'. This means that [D] as a whole is true if Katya did just run a marathon *and* that she ran it in under four hours. So, if either of these claims is at all questionable, [D] is not fully justified.

In [E] the connective is 'yet' which makes [E] a slightly more complex assertion than [D]. Again the two connected claims both have to be true: firstly that dinosaurs were reptiles, and secondly that they had warm blood. But the use of the connective 'yet' also suggests that there is something surprising or unusual in this: that the second claim is true *despite* the first being true. The implication is that reptiles are usually, or normally, cold-blooded; and if this is not the case then the use of 'yet' is not really justified, even if both the claims are true in themselves.

[F] also has more to it than just the two claims. [F] is an explanation, or more precisely a *causal* explanation, as indicated by the connective 'because'. Its author not only asserts that sea levels are rising and that global warming is melting the ice, but also that the first is caused by the second. If we are not satisfied that all three parts are true, then we are not justified in asserting [F]. (There is more about explanation later in the book.)

[G] is another complex claim, and one which is quite tricky to analyse accurately. First of all it is *not* claiming either that parts of the world will soon be underwater, or that nothing will be done about climate change. [G] is what we call a *conditional* claim, or a *hypothetical*. We will also be returning to these later in the book; but for now all you need to note is that a conditional is a claim that if one thing is true, then so is another. For instance, *if* nothing is done about climate change, *then* parts of the world will be underwater. If nothing is done and the prediction turns out to a false alarm, then [G] as a whole is untrue.

Strong and weak claims

Before concluding the chapter, there is one more important distinction that needs to be made. Some claims are *stronger* than others. The importance of this is that a strong claim is harder to justify than a weak claim. A 'strong' claim is one which says a lot, and/or says it very plainly or forcefully. A 'weak' claim in comparison is more *moderate*: it says less, and/or qualifies what it says.

Suppose for example that whoever asserted [G] had said instead:

[H] Whole regions of the world will soon be under water as a direct result of man-made climate change.

This is a very strong claim. It doesn't say 'may be . . .', or 'are at risk of being . . .', or anything else that softens the impact. It says, categorically, that whole regions will be flooded. The whole of [H] is stronger still, because it also claims, just as categorically, what the direct cause will be. [H] does not pull any punches. Moreover, it is clearly implying that climate change *is* taking place, and that it *is* man-made – a claim that some people deny or question. It would not make sense to add that this would be the cause if it were not also claimed to be a reality. All of these factors add up to make [H] a strong and far-reaching claim.

Because it says a lot, and says it so forcefully, it would take a lot to justify it in full.

One important point to add about this distinction is that if a claim is very strong it is easier to challenge, or to cast doubt on, because there is more, potentially, to find fault with. [H] could be made easier to justify if it were weakened, or modified, for example like this:

[H_1] Some parts of the world could one day be under water, and if so man-made climate change may be at least partly to blame.

Obviously [H_1] needs less to justify it than [H], and would be easier to defend if a denier of climate change wants to attack or disprove it. Words or phrases such as 'some', 'could', 'may' and 'one day' are weaker terms than 'whole', 'will' and 'soon'; and partial blame is easier to pin on something than direct cause. Whereas you need something approaching *proof* to justify [H], you need only danger signs to justify [H_1]. But then [H_1] does not have the impact that [H] has. It is not the same claim any more.

Generalisations

A generalisation is a claim that applies very widely – sometimes *universally*: that is, in every single case. For example:

[I] Women are better problem solvers than men.

This is a strong claim because it is about men and women generally. It is especially strong if it is taken literally to mean that *all* women are better at problem solving than *all* men. Clearly that would be unwarranted, since it would take just one or two counter-examples to prove it false. However, [I] could be understood to be the less sweeping claim that *on balance* women exhibit better problem-solving skills than men. Even so, it would be a generalisation, and a vague one too; and vague generalisations are hard to justify.

The opposite of the word 'general' is the word 'particular'. It would not be a generalisation to select a particular woman, or group of women, and talk about their thinking skills. Imagine that two teams – one all female, another all male – competed in a problem-solving competition, and the adjudicator concluded at the end that:

[J] The women (in the women's team) were more organised in their thinking than the men.

This would be a *particular* claim, not a general one, stating that these particular women, on this particular occasion, were superior to the men – at certain particular tasks. Claim [J] would be justified if the women won the competition. But no sort of general claim could be made on the strength of [J], especially not [I]. (You will meet up with this topic again in Chapter 2.10.)

Summary

- We have discussed what is meant by justifying a claim, and considered different standards of justification.
- We have looked at simple and complex claims.
- It has been shown that strong claims are harder to justify than weak claims.
- We have seen the distinction between general and particular claims.

End-of-chapter assignments

1 Invent a story or scenario in which a claim is made that is true but unwarranted.

2 Give an example of a claim that you consider to be:

a justified on the balance of probability
b justified beyond reasonable doubt
c completely justified; certain.

In each case say why your claim matches the description.

3 Compare these two claims:

[A] Polar bears will be extinct by the middle of the century.

[B] Polar bears are an endangered species.

One of these claims is stronger than the other. Which one is it, and why?

Answers and comments are on page 311.

2.3 Argument

An argument is a complex claim used to organise and express certain kinds of reasoning. It is composed of two or more claims, one of which is a *conclusion*; the others are *reasons* for the conclusion. A good argument is one in which the conclusion *follows from* the reasons, or is justified by the reasons.

This doesn't simply mean that the conclusion comes after the reasons. 'Following from', in the context of an argument, means that the conclusion is adequately *supported* by the reasons. If the reasons are true, and the argument is a good one, then the conclusion must be true as well. Obviously a false conclusion cannot follow, in this sense, from true reasons.

In practical terms arguments exist for the purpose of persuading others, or of satisfying oneself, that a particular claim is warranted.

An example

Until a few hundred years ago it was generally believed that the world was flat. This was a natural belief to have because the Earth's surface *looks* flat. But people had also observed (and been puzzled by the fact) that ships sailing away from land appeared to get lower and lower in the water, as if they were sinking, and appeared to rise up again as they approached land. Some argued – from this and other observations – that the Earth's surface could not be flat, but was curved. They drew this conclusion because if the Earth were flat, a ship would just appear to get smaller and smaller until it was too small to see. The argument went like this:

[1a] Ships appear to sink out of sight as they sail away. *So* the Earth cannot be flat.

This is a very simple argument. It consists of just one reason and a conclusion, and the connective 'so'. The words 'therefore' or 'so' are typically used before the conclusion of an argument, and are often called *argument indicators* (or inference indicators) for that reason.

However, this is not the only way to construct this argument. It could have been written:

[1b] The Earth cannot be flat *because* (*since / given that* / . . .) ships appear to sink out of sight as they sail away from land.

Note that the connective in [1b] reverses the order of the claims. Words like 'because' and 'since' are therefore sometimes referred to as *reason indicators* (or premise indicators). ('Premise' is a more formal word for a reason in an argument.)

Note also that it is not necessary to include an argument indicator at all: the reasoning may be just as clear without it. For example:

[1c] The Earth cannot be flat. Ships appear to sink out of sight as they sail away.

The form of an argument

In each of these examples the argument is expressed and/or arranged differently. But it is still the same argument, with the same reason and same conclusion. Because there are many ways in which an argument can be expressed, it is convenient to have one *standard form* for setting arguments out. The customary way to do this, both in logic and critical thinking, is to place the reasons in a list, and to separate them from the conclusion by a horizontal line. The line performs the same function as words such as ‘therefore’ or ‘so’ in natural language reasoning. We can set out this simple argument as follows:

[1] Ships appear to sink out of sight as they sail away.

The Earth cannot be flat.

In a formal argument like this, the reason or reasons are also known as ‘premises’. The word ‘premise’ is derived from Latin and means ‘put before’.

[1a], [1b] and [1c] are just three out of many ways of expressing [1] in ordinary language. [1] is the *standard* way. Reconstructing an argument in a standard form helps to make the reasoning clear and assists with its subsequent evaluation. It also helps with the identification of arguments. Obviously the exercise is unnecessary when an argument is as short and as plain as this one. But with more complex reasoning, which you will encounter as you progress through the chapters, formal reconstruction is a valuable tool.

Arguing back

Of course, not everyone has to accept an argument. Sometimes, even when you have given your reasons, people may still disagree with your conclusion. This certainly happened hundreds of years ago when the first ‘Round-Earthers’ began trying to persuade people that the world was spherical, not flat.

There may have been conversations like this.

[2] **Kris:** Did you know it’s been proven that the Earth is a huge ball hanging in space?

Bart: Don’t be ridiculous. Anyone can see the Earth is flat.

Kris: It can’t be flat. If you just let me explain . . .

Bart: There’s nothing to explain. All you have to do is use your eyes.

Kris: I am using my eyes, and they tell me the Earth is round.

Bart (*in a lowered voice*): Then I’ll tell *you* something. If you go around talking this kind of nonsense, someone is going to lock you up and throw away the key. Or tie you to a post and set you on fire.

Kris: But just listen –

Bart: No, you listen. The Earth is flat.

Kris: It’s round.

Bart: Flat. F-L-A-T, flat!

Kris: ROUND . . .

Activity

[1] and [2] are both called ‘arguments’. But do they have anything else in common besides answering to the same word?

Discuss how you would define an argument to include both the first kind and the second.

Commentary

The problem with the English word ‘argument’ is that it has several meanings. Two of them are given by the following dictionary entry:

argument (noun)
1 a reason or reasons supporting a conclusion; a case made for or against a point of view. **2** a debate or dispute, especially a heated one; quarrel; row.

As you can see, example [1] is an argument of the first sort whilst [2] is an example of the second. The main difference is that [2] is a *dialogue* engaging two or more people. It may involve some reasoning from one side or the other, or both, but it need not. In [2] there is very little *reasoned* argument. Kris tries to explain his position, but his opponent shouts him down. The two speakers are mostly just exchanging opinions, without giving any developed reasons to back them up.

However, it would be wrong to think that the two meanings of 'argument' are completely divorced from one another. As stated at the beginning of the chapter, arguments typically exist to persuade, and it is clear that in a dispute like [2] each of the participants is trying to change the mind of the other. In [1] there is no context given, but the argument being made is obviously aimed at *some* real or imagined opposition. Why else would its author feel any need to give reasons to support the claim? You don't hear people nowadays *arguing* that the Earth is spherical, because it is no longer disputed. Arguments of the first kind occur typically when some opposition to the conclusion has been expressed or is anticipated.

Conversely, most arguments of the second kind have some elements of reason-giving in them. Even in [2], which is predominantly a quarrel, both men are arguing on the grounds of what they claim to *see* – the evidence of their senses.

Bart: Anyone can see the Earth is flat.
Kris: . . . my eyes . . . tell me the Earth is round.

If we wanted to interpret Bart's words as an argument, we could write it as follows:

[3] The Earth looks flat (to me); therefore it *is* flat.

You may not think much of this argument now because you happen to know that, because of the size of the Earth, appearances are misleading. The Earth *does* look flat. Therefore the premise of [3] is true; but the conclusion is not. So the conclusion does not follow from the reason. [3] is an argument, but it is a bad one.

In some textbooks the impression is given that critical thinking is concerned only with arguments of type [1], and not with argument in the sense of dispute. But for reasons just given, we cannot understand the full meaning and purpose of arguments if we ignore their most obvious context. Much of our reasoning – perhaps all of it – arises in or from differences of belief or opinion. An *argument* that the Earth is *not* flat makes practical sense only if someone – past or present – thinks that it is flat, or needs proof that it is.

Evaluating argument

We have seen then that an argument is a complex claim, made up of simpler claims – the reasons (premises) and the conclusion. It is a *good* argument if the reason or reasons justify the conclusion. It is a poor argument if they do not. Evaluating argument means distinguishing good ones from bad ones. Much of the content of this book is about the critical evaluation of reasoned argument. But here is a taste of what it is like.

Activity

We have established that [3] is a weak argument; a bad one. Compare it with [1]: the argument that since ships appear to sink out of sight as they sail away, the Earth cannot be flat. Is [1] a good argument, or not? Would it persuade you that the Earth's surface was curved if you had previously believed it was flat?

Commentary

Argument [1] might seem like a strong argument now, because we already accept that the Earth is not flat. But, as we also know from history, arguments like [1] were not enough to convince the general public straight away. People needed more reasons if they were going to give up a belief that had persisted for centuries. Judged critically it becomes clear that [1] is no better than [3], because [1] also argues from appearances. If the flat appearance of the Earth does not mean that it *is* flat, then surely the *appearance* of ships sinking does not prove that they are dropping out of sight; nor that the curvature of the Earth is the cause of this appearance. It could be some kind of optical illusion; a kind of mirage perhaps. It isn't a mirage: it is perfectly true both that ships appear to sink and that the Earth's curvature is the reason. But we know that now *independently* of the argument. The single reason given in [1] does not, on its own, establish its conclusion.

More reasons

For an effective argument we usually need more than one reason. Imagine you were sent back in time several hundred years and had to convince people that the Earth was not flat. What would you take with you: pictures from space; stories of people who have sailed round the world? These would seem like a good start. Armed with such evidence, you could supplement [1] and thereby make it stronger, for example:

> [4] Ships appear to sink lower and lower the further they are from land. But they cannot actually be sinking, or they would not come back. Also, sailors have proved that if you set off in one general direction, for example east or west, and keep going, you eventually arrive back where you started from. These facts show that the Earth cannot be flat. Besides, photographs have been taken from space that show the Earth's curvature.

Here four reasons are given in support of the conclusion. The conclusion is introduced by the phrase: 'These facts show that', another way of saying 'so'. Three of the reasons are given first; then the conclusion; then a further, seemingly indisputable, reason. So the structure of the argument is as follows:

> Ships appear to sink as they sail away.
> They can't actually be sinking or they wouldn't come back.
> Ships sail in one direction but return to their starting point.
> Pictures from space show the curvature of the Earth.
>
> ---
>
> The Earth cannot be flat.

Obviously [4] is a much stronger argument than [1]. Whether it actually convinces its audience will still depend on their willingness to accept the evidence. But *if* they understand and believe the claims you are making, then it would be irrational of them not to accept the conclusion also.

Of course, the 'if' is a big one. In all probability the audience from that time would *not* accept your claims because they would not understand them. What could pictures from space mean to a 14th-century fisherman? They would lock you up – or worse – and carry on believing what they had always believed and could see with their own eyes: a flat Earth surrounded by flat sea.

This is why 'claim' is the right word for the statements that appear in arguments. Some of the claims made in an argument may be known facts, but others may be forecasts, suggestions, beliefs or opinions. Claims may also be *false*. It is perfectly possible to construct an argument from false claims, either out of ignorance, or out of deceit. (That is probably what people hundreds of years ago would have suspected you of doing, as they slammed the dungeon door.)

This point is important in understanding what argument is. An argument presents reasons and a conclusion. It does not guarantee that either the reasons or the conclusion are *true*. It is still an argument even if the claims in it turn out to be false.

Grammatical note

It was noted in Chapter 2.1 that claims can sometimes take the form of rhetorical questions, or other sentence types: imperatives, or exclamations. When reconstructing an argument in which one or more of the sentences is not a declarative sentence, but is making a claim nonetheless, it is good practice to transform it into a grammatical statement.

Summary

- An argument is a complex construction in which one sentence, the conclusion, is claimed to follow from another (or others) which are reasons.
- A more technical word for a reason, in the context of an argument, is 'premise'. In this book both terms are used, and have the same meaning unless otherwise stated.
- A good argument is one in which the conclusion follows from the premises, meaning that if the premises are true then the conclusion should be true too, because of the truth of the premises. (But there is a lot more to be said about this point in later chapters.)

End-of-chapter assignments

1 Think of a suitable conclusion that you could add to the following to make it into an argument:

> Police forces the world over face a dilemma. On top of dealing with murders and other major incidents, they have to divide their limited time and finite resources between tackling minor crimes such as shoplifting and street robbery, and traffic offences such as speeding or careless driving. Of course, the consequences of speeding can be as bad as or worse than the theft of a wallet or a mobile phone. They can be fatal. But there is a big difference of another sort. The thief intends to do harm and to deprive people of their rightful property, whereas any harm that is done by a car-driver, however serious, is usually accidental.

2 Think of one or two reasons that could be used to support the following viewpoints, and use them to construct arguments:

  a It is wrong to charge foreign students higher fees than other students.
  b Private cars with fewer than four occupants should be banned from city centres.
  c The stars of football, baseball and other popular sports deserve every cent of the millions that they are paid.

3 Find a short argument published in a newspaper or magazine or on the internet. Copy it down and underline its conclusion.

4 Write a short argument of your own consisting of two or three reasons and a conclusion that they support.

Answers and comments are on page 311.

2.4 Identifying arguments

Before an argument can be reconstructed and/or evaluated it must first be established that it *is* an argument. This can be harder than it sounds, especially if the argument is a poor one. In a good argument the conclusion follows from the reasons. In a bad argument it does not follow: the reasons do not justify the conclusion. It is this which makes it a bad argument. But how bad does an argument have to be before we decide that it is not an argument at all? Establishing that some piece of text is an argument may come down to deciding whether or not the author *meant* or *intended* one of the claims to be a conclusion, and the others to be reasons. Judging an author's intention, from a text alone, is not a very exact science!

Matters are made easier if the conclusion or reasons are marked by indicators such as 'therefore', 'so', 'since' and 'because'. However, these connectives have other functions in the language beside signalling argument. They occur frequently, for example, in explanations (see Chapter 4.2). Just finding two sentences joined by 'so' or 'since' does not automatically identify a reasoned argument. Think of the words of the rock ballad:

> But *since* you've been gone
> I can breathe for the first time . . .

There is no argument here. 'Since' in the song means '*ever* since', which is different from the meaning it has in front of a premise.

Besides, as stated in Chapter 2.3, there are plenty of examples of natural-language arguments which contain no connectives. An argument may just be conveyed by a pair or sequence of sentences. Obviously not every sequence of sentences is an argument. All too often it is left to the reader to interpret how a text is best understood.

For example, it is not an argument to say:

[1] Photographs from space show the Earth's surface as curved. The curvature does not show when a photograph is taken from ground level.

How we can establish that [1] is not an argument is by asking if either of the two claims supports the other, or states a reason for accepting the other. Despite what was said just now about indicators, a *partial* test can be applied by inserting 'therefore' or 'so' between the sentences and asking: Does it make sense? If it doesn't make sense, then there is no argument – although the converse does not necessarily apply. Here is the test applied to [1]:

[1a] Photographs from space show the Earth's surface as curved. *Therefore* the curvature does not show when a photograph is taken from ground level.

[1b] The curvature does not show when a photograph is taken from ground level, *so* photographs from space show the Earth's surface as curved.

Neither of these makes sense. So [1] is not an argument.

The same test can be applied to the next example, only as there are more claims there will be more rearrangements to try out.

[2] Completed tax forms and payments must be received by 31 July. Late payment may result in a fine not exceeding $100. Your payment did not reach the tax office until 12 August.

There are three possible candidates for the conclusion of [2], if there is one. So, applying the test, we have these possibilities:

[2a] Completed tax forms and payments must be received by 31 July. Late payment may result in a fine not exceeding $100. *Therefore* your payment did not reach the tax office until 12 August.

[2b] Late payment may result in a fine not exceeding $100. Your payment did not reach the tax office until 12 August. *So* completed tax forms and payments must be received by 31 July.

[2c] Completed tax forms and payments must be received by 31 July. Your payment did not reach the tax office until 12 August. *Therefore* late payment may result in a fine not exceeding $100.

In each rearrangement the attempt to use an argument indicator sounds unnatural, which indicates that none of the sentences is the *kind* of claim that could follow from the others in the way that a conclusion follows from reasons.

Activity

Using the 'therefore/so' test, and the definition of an argument as reasons and a conclusion, decide which of the following could be interpreted as arguments.

For those that are arguments, identify the conclusion and note what kind of claim it is.

Lastly, discuss how well supported the conclusion is, given the reasons.

[3] The Tokyo train leaves at 4.24. It can take up to 40 minutes to get to the station if the traffic is bad. We should leave for the station by 3.40.

[4] Raisa is the only person with a key to the safe. The police are bound to treat her as a suspect. The money went missing when she was in the building on her own.

[5] You are likely to get a fine. Completed tax forms and payments must be received by 31 July and people who miss the deadline are usually fined $100. Your payment did not reach the tax office until 12 August.

[6] From the 15th century European sailors reached the lands of the east by sailing west. Those who sailed on and survived eventually arrived back in Europe. When they claimed they had sailed around the world, few people believed them.

[7] There are only three possible causes of the leak in your system: the pump could be worn, a hose could be split or one of the connections could be loose. I've checked the hoses and tightened all the connections, but the machine still leaks.

Commentary

[3] is an argument. The conclusion, which is at the end, is a recommendation. This also is a useful clue: recommendations are often accompanied by reasons. Here there are two: the time of the train's departure and the possibility of a 40-minute journey to the station. If they are both true, then clearly they justify the conclusion.

[4] is also an argument. The conclusion is a prediction that the police will (definitely) suspect Raisa, firstly because she is the only key-holder, and secondly because she was alone in the building. The argument is perhaps not quite as solid as [3]. Do police *always* treat people as suspects in these circumstances? The words 'bound to . . .' make the conclusion a very strong claim. Even if both premises are true, there may be other

factors – CCTV footage for instance – that show Raisa was nowhere near the safe, and therefore make it less than definite that she will be treated as a suspect.

[5], too, is an argument. The conclusion is another prediction (of sorts). You could also have described it as a statement of probability: 'You are *likely* to get a fine.' The reasoning for the conclusion is that payment did not reach the tax office until 12 August, together with the second sentence which establishes that the payment was late and that late payment *usually* results in a fine. The argument is quite sound, mainly because the conclusion is a fairly weak claim. If fines are *usual* for lateness, then a fine is *likely*. If the claim had been that the person would get a fine, the reasons would not be adequate.

[6] is not an argument. None of the three sentences makes sense with 'therefore' in front of it, e.g. 'From the 15th century European sailors reached the lands of the east by sailing west. Those who sailed on and survived eventually arrived back in Europe. *Therefore* when they claimed they had sailed around the world, few people believed them.' The connective that makes most sense is 'but', not 'therefore'. None of the claims is a conclusion drawn from either or both of the other two; and it is the same whichever order the claims are placed in.

[7] is not an argument either – at least not an explicit one – because, like [6], none of its actual sentences is a natural conclusion. However, [7] does *point towards* a conclusion, even though it is not stated. In fact there is really only one conclusion that you could draw from [7] – that the pump must be worn – because both the other possibilities are ruled out. What we can say about [7] is that it is not complete. It is left to you (the reader or listener) to draw a conclusion – though in this case it leaves you in little doubt as to what the conclusion should be. We could say therefore that [7] is an implicit argument, or that it has an implicit conclusion.

Who wants an argument?

In the last unit we discussed arguments in dialogue form, as well as single arguments. Read the following passage – preferably aloud with a partner, taking a part each – and then answer the question that follows.

SCENE: **a table for two in a restaurant**

Anita: What are you going to have?
(*Sound of a mobile phone*)

Bara: Just a minute. I've got a message.

Anita: Not another!

Bara: I need to answer it.

Anita: Why don't you just switch it off? Restaurants are places for conversation. They're so antisocial, those things.

Bara (*texting at the same time*):
You wouldn't say that if you had one. You'd be on it all the time.

Anita: I wouldn't have one as a gift.

Bara: Yes, you would. I'll give you my old one.

Anita: Keep it. I'm better off without it. In fact the whole world would be better off if the wretched things had never been invented.

Bara: How do you work that out?

Anita: Well for a start, you can't sit anywhere quietly any more without having to listen to one end of someone else's shouted conversation. Secondly, they're a health risk because they pour out microwaves that cook your brain. Thirdly, they distract drivers and cause road accidents. So, like I said: they do more harm than good.

Bara: You just can't say that. No one thinks they are a health risk any more. They don't distract drivers unless the drivers are stupid enough to have them switched on in the car. Not everybody shouts into their phones, and not everyone finds them irritating. They help people to keep in touch. They save lives in emergencies. They access information when you need it. What more do you want?

Anita (*shouting*): I'm sorry, but people *do* shout into them. They don't even know they're doing it. And they do use them when they're driving, whatever the law does to stop them. If someone smashed into you because she was reading a text message, you would soon change your tune.

Bara: Hang on, you're blaming an inanimate object for what people do with it. Of course there are always some idiots who misuse stuff. It's like guns, isn't it? Guns don't kill, it's the people who fire the guns. You're making the same mistake.

Anita: I'm not making a mistake. The machines *are* to blame. I agree, a gun can't kill you until someone fires it, but you can't get shot either if there are no guns to do it with. And people couldn't be distracted by their phones when they're driving if there were no mobile phones. And you wouldn't still be sending that text and spoiling our lunch.

Bara: That's just silly. You've lost that one.

Anita: No I haven't.

Bara: You have. You're just old-fashioned, so you can't see the value of the new technology.

Anita: I'm not old-fash–

Bara: Be quiet, and let me finish this message. I'll be quicker if you just stop talking.

Activity

Is the conversation above just a quarrel, or is there reasoned argument going on here as well? If there is, identify some examples.

Commentary

Overall, this conversation is a quarrel, and parts of it are no more than exchanges of opinion, laced with mild insults. But in the course of the exchange there are examples of developed argument as well, coming from both sides.

The clearest example is Anita's first long paragraph. This is practically a *standard* argument, with three numbered reasons and a conclusion signalled by 'so'. Bara responds with a counter-argument. This gives three reasons which challenge or contradict Anita's claims, then two further reasons (the value of keeping people in touch, and of saving lives in emergencies) to support a position which is the complete opposite of Anita's. Bara's conclusion is expressed by the first sentence of the paragraph: 'You just can't say that.' In other words: 'It is not true that mobile phones do more harm than good,' (as Anita has just asserted). In natural-language arguments, conclusions may not always be spelled out in full, as they are in a standard argument. Expressions such as 'Yes', 'No', 'You're wrong!' can be understood as conclusions if it is clear what they refer to and they are supported by reasons.

In the three paragraphs that follow we see Anita and Bara each trying to reinforce their arguments with further reasons and objections. Then, as their tempers begin to fray, they go back to mere quarrelling and personal remarks.

Summary

- We have considered ways of identifying arguments using argument indicators.
- The difference between a reasoned argument and a mere quarrel has been established.
- We have seen examples of arguments in the context of a dialogue.

End-of-chapter assignments

1 Out of the following passages, only one is an argument. Which is it, and how can it be recognised as an argument? Why are the others not arguments?

A Since the last earthquake in California, engineers have been investigating what happens to man-made structures during a large seismic event. They were surprised that a section of the Bay Bridge, which connects Oakland to San Francisco, fell like a trapdoor. They also discovered that in some of the older double-decker freeways the joints that connect the lower column to the upper column may be suspect.

B The public should not expect the safety of drugs to be guaranteed by animal testing. Aspirin, which is a safe and effective painkiller for most humans, is fatal to the domestic cat. Penicillin poisons guinea pigs. These examples show that different species react to drugs differently.

C If more cash machines start making a fixed charge for each withdrawal, people who draw small amounts will pay more in the long run than those who make larger but fewer withdrawals. People with low incomes tend to make smaller withdrawals, but are more willing to look for machines that don't charge.

For questions 2 and 3 return to the dialogue between Anita and Bara.

2 Look back at the dialogue on pages 35–6 and find the paragraph that begins: 'I'm not making a mistake . . .' Is it an argument, and if so, what is its conclusion?

3 Who do you think 'wins' the argument: Anita or Bara? Give reasons for your judgement.
Note that this is an entirely open question: it is for you to choose which *criteria* to use in making your judgement, but you must say what they are.

Answers and comments are on pages 311–12.

2.5 Analysing arguments

In Chapter 2.3 you were introduced to the idea of a standard form of argument. In natural language an argument can be expressed in many different ways. Standard form shows what the underlying argument is. If a text cannot be reduced to a standard form of argument, we have to question whether it really is an argument.

In critical thinking we use the same basic way of formalising arguments as logicians have used for many centuries: we list the reasons (or premises), and then the conclusion. If we use R for 'reason' and C for 'conclusion' we can say that all arguments have the form:

$R_1, R_2, \ldots R_n / C$

The reasons and conclusion in a standard argument are all *claims*. In theory there is no limit to the number of reasons that can be given for a conclusion. In practice the number is usually between one and half-a-dozen.

The relation between the reasons and conclusion of standard argument is roughly equivalent to the phrase 'so', or '. . . and so . . .', which is why inserting 'so' or 'therefore' into the text is a clue – though not an infallible one. What the whole argument states is that R_1, R_2, etc. are true; *and* that C follows from them. Or that because R_1, R_2, etc. are true, C must be true as well.

Another way to say this is that C is true *as a consequence* of R_1, R_2, etc. being true.

Still another way is to say that C can be *inferred* from R_1, R_2, etc. (Note that it is not correct to say 'R_1, R_2, etc. infer C.' Inferences are always *from* one or more claims *to* another.)

Getting it right

Before you can respond critically to an argument, by evaluating it or by challenging it with a counter-argument, you need to have a clear and accurate interpretation, or *analysis*, of what the reasoning is. It is no good challenging an argument if you have misunderstood or misrepresented it. That is known as attacking a 'straw man' (from the stuffed sacks that soldiers and archers once used for target practice).

What analysis entails is identifying the parts of the argument and recognising how they relate to each other, especially how the reasons relate to the conclusion. One convenient way to do this is to reconstruct the argument in a standard form.

The simplest kinds of argument have one or two reasons followed by the conclusion, and no other content besides these. In practice such arguments don't really need analysing, as their structure is plain enough already. However, we will start with simple examples and build up to more complex, less obvious ones later.

Activity

Here is an example of everyday reasoning, which someone might use to persuade another to hurry.

[1] The train doesn't leave until 4.24, but it can take up to 40 minutes to get to the station, if the traffic's bad. It's 3.30 now. We need to leave for the station within ten minutes to be sure of catching the train.

How would this argument look in standard form?

Commentary

The prime purpose of analysis is to identify each of the claims that comprise the argument and to separate the reasons from the conclusion. Since there are three main reasons, we can label them R1 to R3, and the conclusion we can label C:

R1 The train leaves at 4.24.
R2 It can take 40 minutes to get to the station.
R3 It's 3.30 now.

C We need to set off within ten minutes to be sure of catching the train.

(You can use 'P' for premise to replace 'R' if you prefer.)

Notice that in [1] there is no argument indicator, such as 'therefore', 'so' or 'because'. That is because none is needed. It is obvious which of the claims is the conclusion: it is *because* of R1, R2 and R3 that the speaker claims C, not the other way round.

Also notice that there are more claims in [1] than there are sentences. The first two reasons are connected by 'but' to form a single compound sentence. Part of the job of analysis is to identify each of the individual claims. So, in standard form, these need to be listed separately. Logically 'but' means the same as 'and' in that both R1 and R2 have to be true for the whole compound sentence to be true. 'But' has a different meaning from 'and' in the natural-language version. But as far as the reasoning is concerned all that matters is that the train leaves at 4.24 and that the journey can take 40 minutes. Nor does it really matter to the argument *why* the journey to the station sometimes takes 40 minutes: it is sufficient that it sometimes does. So, when you are analysing an argument, it may not be necessary to include every detail.

On the other hand, not all detail is extraneous: some is essential. For example, the conclusion of [1] is incomplete without the phrase: '. . . to be sure of catching the train'. If the next train would do just as well, then there is no need to set off within ten minutes. Where possible, analysis abbreviates a text, but nothing essential can be left out. Sometimes for clarification purposes an analysis may even need to add explanatory detail.

How reasons relate to a conclusion

There is more to analysis, even of simple arguments like [1], than simply listing premises. We also need to know how the premises operate in supporting the conclusion.

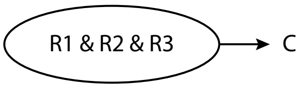
In some arguments the reasons function independently of one another, each giving support to the conclusion in its own right. If one premise is taken out, or found to be false, it doesn't fatally affect the argument because the other, or others, may still be sufficient. The argument may be a little weaker for the loss of a premise; but like a plane with two or more engines, the failure of one does not necessarily knock it out of the sky.

There are other structures, however, in which the reasons work together in support of the conclusion. They are *interdependent*. This is more than just an interesting detail. It is an important factor when we come to evaluation. In an argument with interdependent premises, both or all of them are necessary for the conclusion to follow. If one is omitted, or found to be false, the conclusion cannot be inferred from the other (or others) on their own.

In [1] the reasons are interdependent. It is the train time *together with* the time it can take to get to the station *and* the time it is now that justifies the conclusion. If any of these three reasons turned out to be unwarranted, then the argument would fail. For example, if the train were not due until 5.24, then the other two, on their own, would not establish the need for setting off at 3.40. Or if R2 was an exaggeration, and it *never* took 40 minutes to get to the station, leaving in ten minutes would not be necessary. The remaining premises would be true, but the conclusion would not follow from them. (If you want to check this, try crossing out each of the premises in turn and see the effect it has on the argument.)

Structure

The structure of argument [1] can be represented diagrammatically, for example like this:



The single arrow shows that it is the *combination* of all three premises that leads to the conclusion.

In comparison, look at the next argument.

[2] Short-range flights may have become cheap, but rail travel makes a lot more sense. Flying is responsible for ten times the carbon emissions of rail travel per passenger/km, and twice as much stress. What is more, trains take you to the heart of a city, not to some far-flung airport.

Activity

Rewrite [2] in standard form, and comment on the structure of the reasoning.

Commentary

The conclusion is the first sentence. It is followed by three supporting claims. So in standard form the reasoning is as follows:

R1 Flying is responsible for ten times the carbon emissions of rail travel.

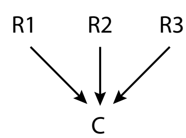
R2 Flying is twice as stressful (as rail travel).

R3 Trains take you to the heart of a city, not to some far-flung airport.

C Rail travel makes a lot more sense than short-range flights.

So far [1] and [2] look to have quite similar shapes: three premises, one conclusion. But there the similarity ends. In the case of [2] there is no interdependence between the premises. Each offers a separate line of reasoning to the conclusion. In the case of R3, for instance, the inference that rail travel makes more sense is made on the grounds that trains take passengers right into a city centre, unlike planes. (Actually, this is not always the case, but it is what the author claims.) True or not, R3 does not rely on the truth of either of the other two premises, nor they on it. So, even if you decide that R3 is not a justified reason, you can still argue that rail travel makes more sense on the basis of lower emissions (R1) and less stress (R2).

So, if you wanted to represent the structure of [2] in a diagram, you would need three separate arrows for the three independent reasons. For example:



Indeed, there are grounds for analysing [2] as three arguments, rather than just one. All three share the same conclusion, but each one is a separate line of reasoning.

Note that the first part-sentence, ‘Short-range flights may have become cheap’, is not a reason. In fact it is not part of the argument at all. The fact that flying may be cheaper would, if anything, be a reason for choosing to fly, so obviously it does not support the conclusion. What it does is show why an argument is needed. The author is saying: ‘OK, there may be a financial reason for going by air, but look at these other reasons for travelling by train.’ In other words, this opening clause puts the whole argument into the context of a potential debate: ‘Which is better: plane or train?’

Mixed arguments

In arguments with more than two premises there may be some that function independently, and others that combine forces.

Activity

Try rewriting the following argument in standard form, and explaining its structure in words or by means of a diagram:

[3] Rajinder cannot be trusted to keep a secret. He was the only person apart from me who knew about Jed and Jill getting engaged. I haven’t said a word to anyone, yet now the news is all round the college. And he spread another story about Jill that I told him in confidence.

Commentary

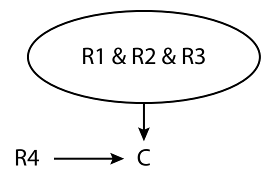
Once again the first sentence is the conclusion, but this time it is supported by four or five reasons (depending on how you choose to analyse them).

R1 Rajinder was the only person apart from me who knew about Jed and Jill getting engaged.
R2 I haven’t told anyone.
R3 The news is all round the college.
R4 Rajinder spread a story that I told him in confidence.

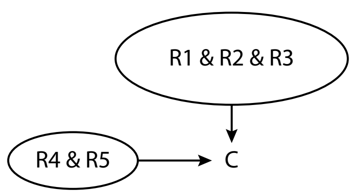
C Rajinder cannot be trusted to keep a secret.

The first three reasons depend on each other. Obviously, if I had told several people, or if others had known besides Rajinder, it might not have been Rajinder who was to blame; and if the news hadn’t spread there would be no reason to suggest Rajinder had told anyone the secret. R4, on the other hand, does not have to be true for the conclusion to follow from the other three. Therefore, although R4 adds strength to the argument, it is separate from the other reasons: an additional reason for inferring that Rajinder cannot be trusted.

As a diagram:



Don’t worry if you have structured the sentences a little differently. For example, some people might prefer to treat R4 as two reasons: Rajinder spread the story; and R5, I told it to him in confidence. These two reasons would of course be dependent on each other, so the alternative analysis would be:



You will find, as you work on more complex arguments, that there can be some differences in the way an argument is analysed. That is because analysis is a form of interpretation, and different interpretations can be found for the same text. The more complex the text, the more room there is for differing interpretations.

So, if your way of reconstructing an argument is not exactly the same as the one suggested in the book, this won't necessarily mean that yours is wrong. What is important is that you recognise the conclusion and the main reasons, and that you are satisfied that you understand the argument and can explain it clearly. Analysis helps you to be clear, but it should not be a straitjacket.

Summary

- We have looked at the workings of some relatively simple arguments, and suggested some ways of analysing and interpreting them, by identifying the reason(s) and the conclusion, and explaining the structure.
- Reasons (premises) can operate in combination with each other, or independently.
- Some parts of a text may not belong to the actual argument.

End-of-chapter assignment

Analyse the following arguments using the methods discussed in this unit.

a People shouldn't be fooled into buying bottled mineral water. It's meant to be safe but there have been several health alerts about chemicals found in some brands. It costs silly money, and anyway tap water, which is free, is just as good.

b It is inevitable that every year some athletes will give in to the temptation of taking performance-enhancing drugs. At the highest levels of sport, drugs can make the difference between winning gold and winning nothing. The rewards are so huge for those who reach the top that the risk will always seem worth taking.

c No sport should be allowed in which the prime object is to injure an opponent. Nor should any sport be allowed in which the spectators enjoy seeing competitors inflict physical harm on each other. On that score, boxing should be one of the first sports to be outlawed. What boxers have to do, in order to win matches, is to batter their opponents senseless in front of large, bloodthirsty crowds.

Answers and comments are on page 312.

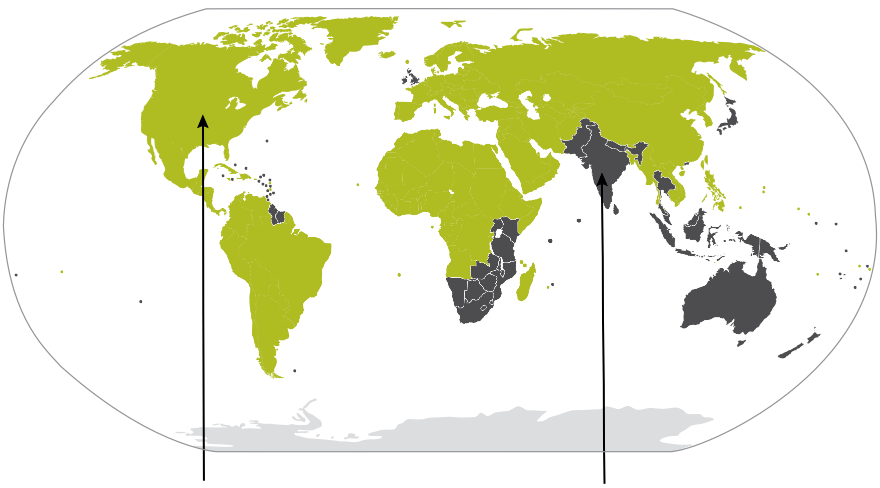
2.6 Complex arguments

In the last chapter we saw how reasons – independently or in combination – support a conclusion. In every case there was just one conclusion.

But in some arguments there may be more than one conclusion. One or more of the reasons may lead to an *intermediate* conclusion, which then leads on to a main or final conclusion. Intermediate conclusions together with their supporting reasons form *sub-arguments*. There may be two or more sub-arguments within the larger argument.

Here is an example:

[1] In some parts of the world, cars are still driven on the left side of the road. This can result in accidents involving drivers from other countries who are used to traffic being on the right. Pedestrians are also at risk from looking the wrong way before crossing the roads. Cities would be safer, therefore, if in all countries the rule were the same. Since countries where the drivers keep to the left are in a minority, those countries should change over to the right.



Activity

Identify the two argument indicators in [1], and use them to give an analysis of the argument.

Commentary

With the help of the two connectives, 'therefore' and 'since', you will have had no difficulty identifying two conclusions:

C1 Cities would be safer if in all countries the rule were the same.

C2 Countries where drivers keep to the left should change over to the right.

The first of these, C1, is drawn from two reasons (or premises):

R1 Driving on the left can cause accidents involving drivers from other countries.

R2 Pedestrians are also at risk from looking the wrong way.

The second conclusion then follows from the first, making a two-stage argument from R1 and R2 to C1; and from C1 to C2.

To put it another way, we have a sub-argument – (R1 & R2) → C1 – and a main argument, C1 → C2. This means that C1 functions as both a conclusion (of one argument) and a premise (of the other). Hence we call C1 the intermediate conclusion (IC), and C2 the main conclusion (MC – or just C).

However, you may have noticed that within the final sentence there is another reason that directly supports the main conclusion, namely that countries where drivers keep to the left are in the minority. As this is a premise we can call it R3.

What would you say if you were asked whether R1 and R2 count as reasons for the *main* conclusion? Strictly speaking they are not: they are reasons for the intermediate conclusion, and support the main conclusion only indirectly. C1 is a direct reason for the main conclusion. So is R3. This distinction between direct and indirect reasons – like the distinction between sub-arguments and main arguments – is very important, as you will see when we come to evaluating this argument and asking whether the reasoning does adequately support its conclusions.

Background information; context

You may also have wondered what to do with the first sentence: 'In some parts of the world, cars are still driven on the left . . .' You possibly listed it as a reason. This is not exactly wrong; in one sense it is because there are some drive-on-the-left countries that there are accidents. But there is another way to look at this which also makes good sense. The first sentence can be understood as the background information, or *context*, for the argument. It is because of the diversity of traffic rules that there is an argument to be had.

Neither interpretation would make your analysis wrong; nor would it make any difference to an assessment of the success or failure of the argument. In the interpretation that follows we have chosen to call the first sentence 'context'; but if you prefer to call it a reason, you can amend the analysis yourself. As stated in the previous chapter, there is often room for different interpretations. As long as you can *justify* your analysis, and it makes good sense of the text, you are entitled to give a different slant.

A full analysis

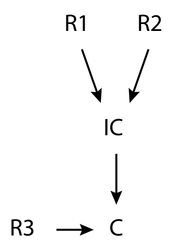
Context: In some parts of the world, cars are still driven on the left.

R1	Driving on the left can cause accidents involving drivers from other countries.
R2	Pedestrians are also at risk from looking the wrong way.
C1 (IC)	Cities would be safer if in all countries the rule were the same.
R3	Countries where drivers keep to the left are in a minority.

C2 (MC) Drive-on-the-left countries should change to the right.

Put into words, the fact that in some countries cars are driven on the left, *and* the claim that this can cause accidents, each leads (separately) to the conclusion that cities would be safer if all countries did the same. This, together with the fact that there are many more drive-on-the-right countries than left, then leads to a final, or main, conclusion that the drive-on-the-left countries should change to the right.

Complex arguments like this, where one argument links into another, are often called 'chains of reasoning'. The diagram shows clearly why this metaphor is used.



Study this argument carefully and make sure you follow the steps, or links, in it. It is important to understand how the conclusion of one argument can also be a reason given in support of a further argument. It is also very important to be able to distinguish between the main conclusion in an argument and any intermediate conclusions reached on the way, especially since this pattern of reasoning is very widely used.

Activity

Here is another argument that consists of a chain of reasoning. Analyse it using some of the techniques discussed in the last example. Then look at the suggested analysis that follows.

[2] We should not rush headlong into large-scale recycling projects without carefully weighing the gains and the losses. Recycling used materials may in the long run prove uneconomical. The cost of collecting up and sorting rubbish, plus the cost of the recycling process itself, often makes the end product more expensive than manufacturing the same product from raw materials. This extra cost has to be paid by someone: if it is not the consumer, then it is the taxpayer in the form of subsidies. Nor is recycling always the best solution environmentally. The high levels of energy required for processing waste can cause pollution. This can also add to global warming.

Commentary

This is a more complicated argument to unravel than the last one because the reasons and conclusions are in a different order, and there are no argument indicators to mark the conclusions.

The main conclusion is the first sentence: 'We should not rush headlong . . .' There are two direct reasons for reaching this conclusion. The first is that recycling may be uneconomical. The second is that it may harm the environment. Each of these has its own supporting premises, making each one an intermediate conclusion leading to the main conclusion.

The best way to list and label the reasons is for you to decide. But your analysis must identify the main conclusion, and recognise that there are *two* distinct sub-arguments leading to the main conclusion. For example:

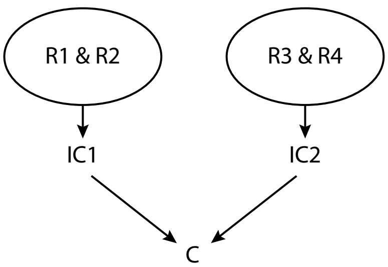
R1 The cost of recycling often makes the end product more expensive than manufacturing the same product from raw materials.

R2 This extra cost has to be paid by someone: if it is not the consumer, then it is the taxpayer in the form of subsidies.

IC1 *(from R1 & R2)*
Recycling used materials may in the long run prove uneconomical.
R3 The high levels of energy required for processing waste can cause pollution.
R4 This can also add to global warming.

IC2 *(from R3 & R4)*
Recycling is not always the best solution environmentally.

C *(from IC1 & IC2)*
We should not rush headlong into large-scale recycling projects.



In this example the diagram really helps to show the complex argument structure. There are two separate lines of reasoning and therefore two arrows leading to the conclusion. If you took away one of the lines, say R3 & R4 → IC2, you would still have an argument for C. It would not be as strong, because it would present only the economic reasons for not rushing into recycling, not the economic *and* environmental reasons. Similarly, if you took away or refuted the sub-argument leading to IC1, you would still have an environmental argument to fall back on.

A useful strategy

You saw in both [1] and [2] that there were direct and indirect reasons. A good strategy for analysing difficult arguments is this: first select what you think is the main conclusion, then look for the direct reasons that support it. Then look for reasons (if any) that support the direct reasons. In other words, work backwards from what you think is the main conclusion. Find:

> *(first)* conclusion ← *(then)* direct reasons ← *(then)* reasons for the reasons.

Put them together to see if they make sense as an argument. If not, try again.

Reported or 'embedded' arguments

Very often, in the media, or in magazines and journals, arguments are *reported*, rather than being expressed directly. Another way of saying this is that an argument may be *embedded* in a report or article or piece of research, and so on. Argument [2] is a *direct* argument. But originally it appeared in the following way:

> [2a] An environmental consortium has advised against rushing headlong into large-scale recycling projects without carefully weighing the gains and the losses, pointing out that recycling used materials may in the long run prove uneconomical. 'The cost of collecting up and sorting rubbish,' said their representative, 'plus the cost of the recycling process itself, often makes the end product more expensive than manufacturing the same product from raw materials.' This extra cost, she went on, has to be paid by someone: . . . [etc.]

Strictly speaking this is not an argument: it is a report of an argument, made by someone other than the author of the report. The author of the report is not arguing for or against large-scale recycling projects; and we have no idea from the report alone whether he or she agreed or disagreed with its premises or conclusion, at the time of writing.

Nonetheless, there is an argument embedded in [2a], and it can be analysed and evaluated like any other argument, once it has been extracted from the report. Instead of being asked to respond to the author's argument, you would be asked to respond to the consortium's argument, as it is represented in the report. To extract the argument, all you have to do is transpose the reported speech back into direct speech, at which point it will have the same standard form as [2].

More about context: targets and opposing views

As already noted in argument [1], interpreting an argument can leave you with parts of the text which don't seem to be reasons or conclusions. In fact they don't seem to belong to the argument at all. In some cases there are parts that even appear to oppose it.

Here is an example:

[3] Top women tennis players used to grumble that their prize money was less substantial than that paid to top male players in the same competition. They argued that they were being unequally treated. But the disparity was entirely justified and should never have been abolished. Male players just have more prowess than women. They need to win three sets out of five to take the match; the women only two. They have to play harder and faster, and expend far more energy on court than the women. But most of all, if the best woman in the tournament played any of the men, there would be no contest: the man would win.

Activity

What do you make of the first two sentences of [3]? Discuss where you think they fit in.

Commentary

The short answer is that the first two sentences *don't* fit in – not to the actual argument. They are a necessary part of the text, of course, because without them the argument would not make much sense. (Try reading the passage without them and you will see this for yourself.) But they are neither reasons nor conclusions of the author's argument. In fact they really belong to an opposing argument, because they are about the women's case for equal prizes, not the author's case for keeping the men's prize money higher.

We can think of these opening sentences – everything preceding the word 'But . . . ', as the *target* for the author's argument. The whole point and purpose of that argument is to *respond* to the women's alleged claim of unfairness and inequality. Another way to put this is that the first two sentences place the argument into a context. Or you could say that they introduce it, or provide background information. Any of these labels would do.

Some textbooks refer to parts of a text which function as the target for an argument as a *counter-argument*, but this is misleading. If anything is to be called a counter-argument here it is the author's argument, because the author is the one responding, not the women. What the first two sentences are doing is explaining the context; setting the scene.

So, in standard form we have:

Context (or target): Top women tennis players used to complain about the inequalities of prize money.

But . . .

R1 Men have to win three out of five sets; the women only two.

R2 The men play harder and faster and use more energy.

R3 Any of the men would beat the best woman.

IC The men have more prowess.

C The disparity was justified and should not have been abolished.

The value of analysis

Thoroughly analysing an argument is the surest way to get a clear understanding of its meaning and structure. It also gives you the best chance of responding to it appropriately. When you see its parts laid out for inspection, and the links between them, you can quickly spot strengths, weaknesses, gaps, and so on which may not be at all obvious when the argument is wrapped up in ordinary, everyday language.

The kind of detailed analysis you have practised in the last few pages will not always be necessary. Once you become more skilled at it, you will be able to recognise the main conclusion of an argument and see the lines of reasoning more instinctively, without having to list and label all the parts. Listing and labelling is the way to acquire and embed the skills.

Summary

- Some arguments have intermediate conclusions that lead on to a main conclusion.
- An intermediate conclusion has its own supporting reason(s). It is both a conclusion and a reason for a further conclusion.
- Some sections of a text may not be reasons or conclusions: they may just introduce or provide a context – sometimes in the form of a 'target' – for the argument itself.
- Often an argument will be embedded in a report, and needs to be extracted from the text by converting it into direct speech.

End-of-chapter assignments

1 Draw a diagram which shows the structure of argument [3]. (You may follow the model used in the commentaries, or invent your own method of representation.)

2 Look again at argument [1] and the accompanying visual material. How might this material be understood as part of the argument? What part would it play?

3 Analyse the following arguments to show their reasons and conclusions, including any intermediate conclusions. Also, separate and label any background information or opposing views which are there as a target for the argument.

   a Recently the operators of a cruise liner were fined $18m for dumping oil and other hazardous waste at sea. This may seem substantial, but in the same year the ship earned profits of $340m. The company could well afford the fine, and dumping saved them the considerable expense of storing and legally disposing of the waste. So emptying their tanks into the ocean was probably a risk worth taking. Nor was it much of a risk. In the last decade only a handful of companies have been fined and every year there are unsuccessful attempts to prosecute. We must give the authorities greater powers and demand that they use them. Otherwise the oceans of the world are in danger of becoming open sewers.

   b The South Pole must once have been much warmer than it is today. Scientists have recently discovered some three-million-year-old leaves preserved there in the ice. Despite their age, they are so undamaged, and preserved in such fine detail, that they could not have been carried there by wind or sea. Therefore, they can only be from trees that once grew there. The leaves belong to a species of beech tree that grows only in warm or temperate regions; and beeches do not evolve quickly enough to adapt to changes in climate.

OCR (adapted)

4 Extract the argument from the following report, and identify its conclusion and supporting reasons.

   A top tennis coach, Annabel Aftar, has reacted angrily to calls for a ban on grunting. Players who emit a loud explosive sound each time they hit the ball have been accused by some of putting opponents off their game. Ms Aftar opposed a ban by saying that grunting is a natural and unstoppable accompaniment to sudden effort, and that making women play in near-silence would reduce the power of their shots, placing an unfair handicap on some but not on others. Some women can control grunting, others can't, she said, adding that it is not just a female thing. Some men grunt almost as much as the women.

Answers and comments are on pages 312–13.

2.7 Conclusions

The most important function of argument analysis is identifying the conclusion. Once it is clear what the author is seeking to establish or justify, the rest of the argument usually falls into place.

The kind of detailed analysis you have been studying in the last two chapters is not always necessary. If an argument is quite short and straightforward, the conclusion often stares you in the face. But with longer and more complex arguments, it can be very easy – as the saying goes – 'to get the wrong end of the stick': to mistake a reason for a conclusion, or an intermediate conclusion for the main conclusion; or to misunderstand the direction of the argument altogether. It is in order to avoid this kind of misinterpretation that you need skill and confidence in argument analysis generally, and the recognition of conclusions in particular.

As already noted in previous chapters, the conclusion of an argument is often marked by the word 'so' or its equivalent. Alternatively the conclusion may be followed by 'because' (or some equivalent), to indicate that a reason or reasons are being given to support the preceding claim. In the absence of such linguistic clues – and they often are absent – we have to look to the claims themselves to decide if there is an argument present, and if so which part or parts of it express the conclusion.

Here is a very simple example:

> [1] The government won't raise taxes this close to the election. Tax rises are not vote-winners.

In [1] there are two claims: the first is a prediction, the second a claim to fact. It is quite obvious here that the factual claim is being given as a reason for the prediction; not the reverse. It is *because* raising taxes is not a vote-winner that the author is predicting that the government will not do it. If instead we try to say that tax rises are not vote-winners because the government will not raise them, we end up with something that barely makes sense.

However, this does not mean that the second sentence couldn't be a conclusion, in a different argument. Suppose I were to reason as follows:

> [2] People say they want good public services, but they don't like it when any more of their hard-earned money is taken to pay for them. Tax rises are simply not vote-winners.

Here it is perfectly reasonable to interpret the first sentence as a reason to assert, and believe, the second. In standard and abbreviated form:

People don't like paying more (for public services).

Tax rises are not vote-winners.

Activity

In [1] and [2] there was a single premise and a single conclusion. In the next passage there is more work to do.

> [3] Most spoken languages come in many different accents and dialects. They also contain colloquial, even slang, expressions that vary from region to region, or class to class.

The only way to learn a foreign language properly is to go and live in the country where it is spoken. Classroom teaching, books or DVDs cannot give students the necessary exposure to the variations and subtleties of everyday speech.

Which sentence is the conclusion of argument [3] – and why?

Commentary

Your discussion should have led you to see that the conclusion is the last-but-one sentence: the claim that the only way to learn a language properly is to go and live in the country where it is spoken. The author is claiming this on the grounds that spoken languages have many 'variations and subtleties' – such as dialects and colloquialisms – *and* that school language lessons cannot give students the requisite exposure to these features.

Remember that what we are primarily concerned with here is identifying the conclusion. We are not yet evaluating the argument or responding to it. But although analysis and evaluation are separate activities, there is inevitably some overlap between them. For a claim to be recognisable as a conclusion we have to be able to say that there is *some* level of support given by the claims we identify as the reasons, even if it is not entirely convincing support.

The difficulty comes when there is more than one possible way to interpret a text as an argument. How can we be confident that in [3] the penultimate sentence really is the conclusion for which the author is arguing, rather than, say, the last sentence? Might the author not be saying that *because* of all the dialects and colloquialisms that are found in spoken languages, school lessons cannot give students the exposure they need to learn a language properly?

Well, the author might be saying this. Critical thinking is not mind-reading. But nor is it guesswork. What we should be asking, when we analyse a piece of text as an argument, is not what the author might have been thinking, but which interpretations gives us the best or most persuasive argument. Another way to ask this is: Which interpretation makes the best sense *as an argument*? It is for this purpose that the 'therefore/so' test becomes a useful tool.

Compare:

[3a] The only way to learn a foreign language properly is to go and live in the country where it is spoken. *Therefore* classroom teaching, books or DVDs cannot give students the necessary exposure to the variations and subtleties of everyday speech (dialects, slang, etc.).

with:

[3b] Classroom teaching, books or DVDs cannot give students the necessary exposure to the variations and subtleties of everyday speech (dialects, slang, etc.). *Therefore* the only way to learn a foreign language properly is to go and live in the country where it is spoken.

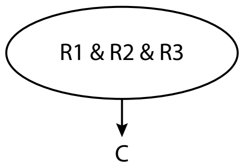
The difference is quite clear. [3b] not only makes better sense than [3a]; it is a better *argument* than [3a]. In fact it makes better sense because it is a better argument. The best interpretation that we can place on [3] is that the first, second and fourth sentences are being presented as grounds for the third. Abbreviated, and in standard form, we have:

R1 Spoken language has different accents and dialects.
R2 There are also colloquialisms and slang.
R3 Classroom teaching, books and DVDs cannot give requisite exposure (to these).

C The only way to learn is to go and live in the country.

You may have wanted to say that R3 was an intermediate conclusion from R1 and R2.

However, R3 does not so much follow from the previous two claims as *join with them* to support C. The structure then would be:



The principle of charity

The rule that says we should interpret a supposed argument in a favourable way – that is, as a good argument rather than a poor one – is known as the principle of charity. Note that despite the name, this doesn't mean being kind or generous to the author. All it means is that we should assume that the author is a rational individual who understands the difference between good and bad reasoning at least as well as we do ourselves. So, if we have in front of us a text that could be understood as 'X therefore Y', or as 'Y therefore X', and we can see that X is a good reason for believing Y, but Y is not a good reason for believing X, then on the principle of charity we should accept the first interpretation and not the second.

This explains why there is often a slight overlap between analysis and evaluation. We are not just looking for lists of sentences that can be called an argument (however bad), but one which goes some way towards being a *good* argument. By the same token, if a piece of text makes much better sense as a *non*-argument than as a argument, we should not just assume it is bad argument.

We shall return to this important principle when we discuss evaluation and counter-arguments in Chapters 4.9 and 7.7.

Complex arguments and multiple conclusions

The procedure is the same for longer and/or more complex arguments, except that you may have to repeat it for each of the sub-arguments, or steps. To distinguish main conclusions from intermediate conclusions, you still just ask yourself: Which follows from which? or: Which makes better sense as a reason for the other?

Activity

Look at the next example and answer the multiple-choice question that follows it.

> [4] Parents naturally tend to think that, because they are older and more experienced, they know better than their children. They consequently assume that their judgements and decisions are the right ones. But in many ways children are much cleverer than their parents give them credit for. They frequently display problem-solving skills that their parents do not possess; and they are more adventurous in their thinking, if only because they are less afraid of making mistakes. Parents should pay closer attention to what their children have to say, and allow them to make more decisions for themselves. Apart from anything else, this would help to relieve many unnecessary family tensions.

Which one of the following best expresses the main conclusion of the argument? As well as making your selection, give a brief reason why you think it is right, and why you thought the others were wrong.

A Children are much cleverer than their parents give them credit for, and frequently display problem-solving skills that their parents do not possess.

B Parents naturally assume that their judgements and decisions are the right ones.

C Children don't mind making mistakes to the extent that their parents generally do.

D Parents should attend more to what their children say, and allow them to make more decisions.

E A reduction in family tensions would result if parents listened more to what their children think.

Commentary

There are multiple-choice questions like this in some but not all critical thinking syllabuses and examination papers, and in some admissions tests to universities or professions. It is good practice to try some from time to time, and you can find plenty of sample papers with sets of such questions on various examination websites.

Unless you are told otherwise, only one of the options is correct. That is the case here. The other options either correspond to one of the reasons, or to an intermediate conclusion, or to a piece of background information; or they misrepresent the conclusion altogether. Usually in such tests, you are not required to give any explanation or justification for your choice, but because this is a learning activity, you were asked to say why you made the choice you did, and why you rejected the others. (You should always do this when you are using multiple-choice questions to improve your skills.)

So how did you go about the task? Did you read the passage, then immediately look through A–E to find the most promising response? If so, you were asking for trouble. This is not a good strategy. Although the incorrect responses are not designed to trick you, they are designed to make you think. They are called *distracters*, and with good reason, for it is very easy to be tempted by an answer because it echoes something in the passage, or simply because it 'sounds right'.

A much safer approach is to ignore the responses A–E completely while you analyse the argument and identify its conclusion yourself; *then* to look for the response that best matches your analysis. That way you are not so much looking for an answer as looking for confirmation of your own answer. If you find a response that matches yours, you will have two good reasons for choosing it, not one.

So, what's the argument here? The passage starts by claiming that parents tend to think they know best and consequently assume their decisions and judgements are the right ones. This has the look of an argument already, but it is clearly not making the author's own point. For, like the tennis argument in Chapter 2.6, the opening sentences are followed by the word 'But', signalling an opposing view. What parents think is therefore just the introduction or target for the real argument.

The author's own argument stems from the claim that children are often wiser than parents think, supported by observations about their problem-solving skills, and so on. Then comes the recommendation that parents should pay children more attention and allow them to make more decisions. This also looks a likely conclusion, but does it *follow from* the claim that children are wiser than their parents think, or *support* it?

Clearly it does follow: the passage is not saying (nor would it make much sense to say) that parents should pay closer attention to their children, and therefore children are wiser than their parents think. So, a full and fair analysis would be:

Context: Parents naturally tend to think that . . . they know better than their children, etc.
But . . .

R1 Children frequently display problem-solving skills that their parents do not possess.

R2 They are more adventurous in their thinking.

IC In many ways children are much cleverer than their parents give them credit for.

R3 Paying closer attention etc. would help to relieve family tensions.

C Parents *should* pay closer attention to what their children have to say, and allow them to make more decisions for themselves.

Now look at the responses A–E. Which of them, if any, matches the meaning of the main conclusion of the passage? Obviously it is D: 'Parents should attend more to what their children say, and allow them to make more decisions.' We can safely select that as a close paraphrase of the actual conclusion.

What about the other options, the 'distracters'? Even though you may feel confident in your choice, it is sound practice to reassure yourself that none of the others is as good or better – and why. It is easy to do this once you have carefully analysed the argument. Here are responses A–E again:

A Children are much cleverer than their parents give them credit for, and frequently display problem-solving skills that their parents do not possess.

B Parents naturally assume that their judgements and decisions are the right ones.

C Children don't mind making mistakes to the extent that their parents generally do.

D Parents should attend more to what their children say, and allow them to make more decisions.

E A reduction in family tensions would result if parents listened more to what their children think.

A is not the main conclusion: it is a combination of R1 and IC. B looks like a conclusion partly because in the original text this claim begins with the word 'consequently'. However, on a proper reading of the whole passage it becomes clear that it is only a target for the main argument, once again showing that indicator words do not tell the whole story but must be understood in the context of the text as a whole. C is an explanatory detail, suggesting why children may be more adventurous. It is not supported by any other claims and is not therefore a conclusion. E comes at the end of the argument, which is a natural place for a conclusion. However, it should have been clear that it is there to give extra support to the argument, and is not its conclusion.

Eliminating A, B, C and E in this way is a worthwhile exercise to reassure yourself that you have made the right choice. But beware of using it as the *only* way of selecting the correct response. You need to have positive reasons for making your selection as well as negative reasons for rejecting the others.

Diffuse conclusions

The conclusion in each of the foregoing examples has been a self-contained sentence in the text of the argument. We come now to a rather different situation, and one which requires even more perceptive, interpretative skill.

Sometimes a conclusion is not expressed in one go, but is broken up, or repeated, or stated in more than one way, at different points in the text. (A useful word for this is 'diffuse', or 'diffused'. A diffuse conclusion is one that is *spread through* the argument, rather than being one component.) Identifying a conclusion, in these circumstances, means gathering or summarising it.

For example, look at the next argument:

[5] We are taught from an early age that we should tell the truth, the whole truth and nothing but the truth at all times and without question. But it is simplistic to pretend that truth-telling is always right and falsifying always wrong. Some people may tell the truth just to cause trouble; others may decide *not* to tell the truth just to save someone else from distress or to protect them from danger. The morality or immorality of a deed depends on its consequences and the motives for doing

it. On its own the simple act of saying what is so, or what is not so, can be judged neither right nor wrong.

Activity

Which would you say was the main conclusion here? Try to summarise it in your own words.

Commentary

What makes this a tricky argument to analyse cleanly is that the conclusion is spread out, rather than stated in a single sentence or phrase. It is clear enough that the first sentence is the target, setting up the standard principle that we should *always* tell the truth. It is also clear that the rest of the passage is contesting the principle, by giving two counter-examples as reasons:

R1 Some people tell the truth to cause trouble.

R2 Some people do not tell the truth to save others from distress, etc.

Between them these reasons support three closely related claims, out of which it would be difficult to decide which was *the* conclusion. Instead of forming a chain of reasoning, they all seem to be making roughly the same point, only in slightly different ways:

- It is simplistic to pretend that truth-telling is always right and falsifying always wrong.
- The morality or immorality of a deed depends on its consequences and the motives for doing it.
- On its own the simple act of saying what is so, or what is not so, can be judged neither right nor wrong.

In such circumstances you can do one of two things. You can either choose the sentence which you think is the clearest expression of the conclusion. Or you can *summarise* the conclusion to which all three appear to be contributing. For example:

Without considering motives and consequences, lying and truth-telling cannot be judged right or wrong.

You could be excused for thinking that [5] is a badly written argument, because its conclusion is not clearly stated once and for all. However, writers – good ones and bad ones – do this all the time, as a way of emphasising or reinforcing or clarifying the point they are making. In analysing such arguments you must be ready to summarise the conclusion and premises in your own words. The main purpose of analysing and standardising arguments is to simplify their meaning. When dealing with real texts by real authors you cannot always expect the job to be done for you!

Summary

- The primary purpose of argument analysis is to identify or summarise the conclusion.
- When identifying a conclusion, we should apply the principle of charity, by interpreting the text in the way which makes the best sense as an argument.

End-of-chapter assignments

Consider each of the following arguments, then answer the multiple-choice question which follows. There is only one correct answer to each question.

As well as answering the question, justify your selection by saying why you think it is the right one, and why the others are wrong. This will help you to improve your scores in multiple-choice tests, and your analysis skills generally.

1 When cities become congested with traffic, the usual solution is to make a charge for bringing a car into the centre. This works, but it is wrong to do it, because it discriminates in favour of those who can easily afford to pay. The less well-off in society are penalised so that the rich can enjoy the luxury of clear streets. Therefore congestion charges everywhere should be abolished. A system of rationing car use should be introduced instead, allowing each driver into the city just once or twice per week. Then everyone benefits equally.

Which of the following expresses the main conclusion of the argument?

A The usual solution to congestion is charging to drive cars into the city centre.
B It is wrong to charge drivers because it discriminates in favour of the rich.
C Rationing car use should be brought in to replace congestion charges.
D Everyone would benefit from an abolition of congestion charges.

2 Train fares differ enormously, with the most expensive always applying when people have to commute to and from work, and when the trains are most crowded. Some call this a cynical and unfair policy because it exploits the fact that commuters have to travel then, and will pay whatever is charged and put up with the over-crowding because there is no alternative. But it is perfectly fair, as well as necessary, to do this. For one thing it is simply market forces at work. For another it is the only way the system can function at a profit. During off-peak periods people are travelling from choice and would not travel at all if there were no cheap fares. But the cheap fares would not be economical for the transport companies unless they can be subsidised by high fares at peak times.

Which of the following best expresses the conclusion of the argument?

A It is fair and necessary to charge commuters the highest fares.
B Charging commuters peak rates is the only system that will work.
C It is cynical and unfair to charge commuters more than other travellers.
D Train companies exploit commuters because they have to travel at peak times.
E Cheap fares would not be economical without the subsidy of peak-time fares.

3 Meat eaters, in defence of their eating habits, often give the excuse that they (and we) do not have the teeth or the stomachs of natural herbivores, and therefore we must be carnivores. This is nonsense. We may not have the digestive equipment to eat raw grasses, but nor do we have the teeth and digestion systems of predators: we are as far removed from the wolf as we are from the horse. Seeds, nuts, berries, leaves and roots are the natural diet of our closest relatives in the animal kingdom.

Which of the following best expresses the conclusion of the argument?

A It is nonsense to say that we must be carnivores.

B Seeds, nuts, berries, leaves and roots are our natural diet.

C We do not have the teeth or stomachs of predatory animals.

D We are no more like wolves than we are like horses.

E Eating meat is a disgusting habit.

Answers and comments are on pages 313–14.

2.8 Reasons

Reasons are expressions which tell us why something is as it is. Their primary function is to explain. Recall the example you first considered in Chapter 2.2:

[1] Sea levels are rising around the world *because* global warming is melting the polar ice caps.

This complex claim offers an explanation for rising sea levels. As you have seen, it consists of two simple sentences joined by the connective 'because'. Grammatically, therefore, [1] looks very much like an argument, with the second sentence being given as a reason for the first. It could even be rephrased with 'so' or 'therefore' as the connective:

[1a] Global warming is melting the polar ice caps and therefore sea levels are rising . . .

But the claim that global warming is melting the ice is not a reason in the sense of a premise. [1] and [1a] do not make the argument *that* sea levels are rising: they assert *why* sea levels are rising. This is an important difference. The claim that the seas are rising is not a conclusion in need of support, but a claim to fact in need of an explanation.

Compare:

[2] Global warming must be happening because the polar ice is melting and sea levels are rising.

Superficially there is not a lot of difference between [1] and [2]. Again in [2] we see two claims connected by the word 'because', indicating that the second is being given as a reason for the first. But this time global warming is not being *explained* by rising sea levels: rising sea levels are being offered as grounds (or evidence) for *arguing* that global warming is taking place. The phrase 'must be' helps us to see that the author is urging the reader to accept the claim. But even without this clue it is quite obvious that rising seas could not be the *cause* of global warming, whereas it makes good sense to offer rising seas as *evidence* of global warming. It may not be conclusive evidence, but it is supportive.

What we learn from this is that the word 'reason' is ambiguous, depending upon whether it is a reason *why* (as in an explanation), or a reason *for* (as in an argument). This can make it quite hard on occasions to be sure whether a set of sentences is expressing an argument or giving an explanation, especially if there are no indicator words (such as 'because', 'therefore', 'for this reason') to label the sentences.

Reasons as premises

Premises are claims from which a conclusion is said to *follow*. But 'follows' in this sense means more than just coming after. When we say a conclusion follows from certain premises, we mean that it follows *logically*. In natural-language arguments the premises can appear before or after the conclusion: it is only in standard form that the conclusion is always at the end.

What we mean by 'follows from' is that if the premises are true, the conclusion must be true too. If the conclusion does not follow from the premises, then even if the premises are true, the conclusion might be false. So a really good argument is one in which the premises *are* true and the conclusion *does* follow. That is why, in a good argument, the premises are *reasons* for believing, or agreeing with, the conclusion.

In logic the term ‘premises’ is preferred over ‘reasons’. In critical thinking it tends to be the other way around, although there are exceptions. This is because critical thinking is a less formal subject than logic. In this book we have used both words, and up until now treated them as having roughly the same meaning when used in connection with arguments. However, there are differences which sometimes make one term more appropriate to use than the other. ‘Premise’, being the more formal word, is defined by its position in an argument – literally meaning ‘placed before’ – whereas a reason is identifiable more by its meaning: what it *claims*. Logicians often work with symbols rather than sentences. In an argument such as:

P & Q therefore R

‘P’ and ‘Q’ are *premises*. But nothing about these letters makes them recognisable as *reasons* for ‘R’. You would have to know what ‘P’ and ‘Q’ stand for – and ‘R’ too – before you could recognise them as grounds for believing R.

Relevance

For one thing, a premise cannot be understood as a reason for a conclusion unless it is *relevant* to the conclusion. Suppose someone tried to argue that:

[3] Seawater is salty, so Mars is a planet!

The premise of this ‘argument’ is true, and so is the conclusion. But knowing that seawater is salty gives no reason to believe that Mars is a planet, since the two claims are completely unrelated. In [3] the second claim is known as a ‘non sequitur’, because it does *not* follow from the premise in any logical sense of the word, even though both claims are true. Nor, for that matter, does the saltiness of water *explain* why Mars is a planet.

Compare with the following argument:

[4] Mars is a planet since it can be seen to orbit the Sun.

Its orbiting of the Sun justifies the claim that Mars is a planet. If I did not already know that Mars was a planet, [4] would give me a reason to believe it (provided I knew that planets are objects that revolve around suns).

To summarise so far, there are two ways in which a claim can be understood as a reason: as grounds for drawing a conclusion, or as an explanation. Usually you can tell from the meanings of sentences what their functions are, or from the context surrounding them. Sometimes, however, it is quite difficult to tell, especially if a short passage is taken out of context. But there is another complication, too: sometimes argument and explanation are both recognisable in a text at the same time. Indeed sometimes an argument consists of an explanation. Some or all of these complications are reflected in the following examples.

Activity

Discuss the following pairs of sentences. Can either of the sentences in each case be understood as a reason for the other? If so, what kind of reason?

[5] Tax rises are not vote-winners. In the last four decades, every time a government has raised taxes, their poll-ratings have fallen significantly.

[6] The government will not raise taxes this close to a general election. The result could be very close and tax rises are not vote-winners.

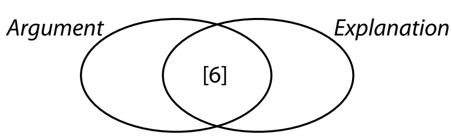
[7] The accused was at her desk in the office at 3 p.m. but no one reported seeing her again until after 4. That was plenty of time to get to the scene of the crime and back.

Commentary

We’ll take these examples in turn, starting with [5]. This is not exactly a trick question, but it is a tricky one. The straight answer is that either of the sentences could be understood as a reason for the other, depending on whether you interpret [5] as an argument or as an explanation. You might wonder how anyone can decide whether [5] is an argument or an explanation without knowing which of the sentences is the reason. This is a very good question. Without some context – which we do not have – the clues are insufficient for us to work out what point the author is making. It might be that the first sentence is meant to explain *why* tax-raising governments have experienced a slide in the polls; or the slide in the polls may be meant as evidence that tax rises are not vote-winners. Both make reasonably good sense, so even the principle of charity is little help. The right answer with regard to [5] is that it is *ambiguous*.

The next example, [6], is an interesting one. It is plainly an argument. The first sentence is a prediction. The second supplies two reasons (joined by 'and') which can be taken as support for the prediction. This is a perfectly acceptable interpretation of [6]. But would it not be just as accurate to say that the two reasons in the second sentence are explaining why the government will not raise taxes close to an election? If so, then it would seem that [6] is both an argument and an explanation; or that the explanation *is* an argument (and vice versa).

And that is the right answer. What [6] illustrates is that one way of supporting a conclusion is to offer an explanation for it. By explaining it, successfully, the author also makes it more believable. The boundary between argument and explanation is not always a clean line. If the relationship between the two concepts were represented in a Venn diagram (see Chapter 3.5), it would look like this, with [6] in the intersection:



Example [7] is another very interesting case. If either of the sentences is a reason for the other it looks like it is the first. You might have decided that the time lapse between sightings of the accused is being stated to explain *how* she managed to get to the crime scene and back. Or you might have thought that the sightings were evidence that she had plenty of time, making [7] an argument of sorts, with the second sentence as the conclusion.

But there is a third, more plausible reading, namely that *neither* of the sentences explains *or* supports the other. They are related by being part of the same story, but aside from that they are really independent claims. The first is that no one reported seeing the accused for an hour; the second that an hour was time to get to the crime scene and back. But if the second claim is true then it is true *whether or not* anyone saw the accused between 3 and 4 p.m. (If there is an explanation it would be about the distance of the crime scene from the office, or how long it would take to get there.) And the claim that no one saw the accused between 3 and 4 p.m. has nothing to do with the accessibility of the crime scene. Any attempt to make an argument out of [7] would result in a *non sequitur* – where the supposed conclusion does *not* follow from the premises. A non sequitur, as we know, is a bad argument. So, on the principle of charity, we have little justification for calling [7] an argument or an explanation.

An implied conclusion

What could be said about [7] is that it is leading towards some form of accusation. If some conclusion (or inference) were drawn from [7], that would make it an argument. For example, [7] could lead to the inference that the accused had had the time, or the opportunity, to commit the crime. However, this is such an obvious inference to draw that it does not need to be stated explicitly. We could think of it as something a prosecuting counsel might leave unsaid, and let the jury members make the inference themselves.

On that interpretation [7] consists of two reasons and an *implied* (implicit) conclusion. In standard form:

[7a] R1 The accused was at her desk in the office at 3 p.m. but no one reported seeing her again until after 4.

R2 That was plenty of time to get to the scene of the crime and back.

C (*implied*) The accused had the opportunity to commit the crime.

In practice many arguments are left unfinished in this way. Sometimes it makes for a more persuasive case if the audience, rather than the author, is left to draw the conclusion. Two questions that are frequently set in critical thinking assignments are:

What conclusion can be drawn, reliably, from such-and-such a claim, or claims?

Or:

How reliable (or *safe*) would it be to draw such-and-such a conclusion?

You might like to discuss the second question with reference to the implicit conclusion in [7a].

Are reasons always claims?

In a word, yes. This does not mean that reasons are always grammatical statements (declarative sentences). As we saw in Chapter 2.1, a claim can be made using a rhetorical question or even an imperative sentence. For instance, the prosecutor could have asked the jury:

'Did anyone see the accused at her desk between 3 and 4 p.m.?'

and mean it as a claim. It would be very hard, if not impossible, to think of a reason or premise, however it is expressed grammatically, that does not have the meaning of a claim. It is also difficult to see how a *genuine* question – with no obvious answer – could be grounds for a conclusion.

What about conclusions? Could the conclusion of an argument be a *genuine* question or command? This is a more debatable point. Obviously there are plenty of examples where people give reasons for demanding something. Take the following well-known example:

[8] Shoot her! She's a spy.

It makes perfectly good sense to call this an argument to justify an order. On the other hand, it also makes sense to interpret the conclusion of [8] as a claim: for example, 'She should be shot' or 'You must shoot her'. The question is whether you want to call 'Shoot her!' a genuine command, or just a way of asserting something. Since both interpretations are equally defensible, you must make up your own mind.

Summary

- There are two senses of the word 'reason', depending on whether it is found in an argument in support of the conclusion, or in an explanation.
- However, the boundary between argument and explanation is often blurred, making interpretation quite difficult at times.

Note: it should be becoming more and more evident as you progress through this book that not all critical thinking questions have plainly right or wrong answers. Being critical takes judgement. What matters in many cases is being able to back up your judgements with reasons of your own. In a critical thinking assignment, the same credit may be given for two quite different answers, if both are equally well argued.

End-of-chapter assignments

1 No team has come back from being three games down in the World Series, so can the Red Sox still win?

Is this an argument? Explain why this is a problem question, and write a short paragraph justifying your answer.

2 Some students in a San Francisco art school were told they were about to see an example of prize-winning modern art and were then shown a photograph of a pile of discarded drinks containers. It was nothing more than garbage, but the students took it quite seriously and agreed that it was worthy of an award.

Suggest a conclusion which could be drawn from the above claims.

3 Just look at the statistics and see for yourself how crime has been rising over the past few years. Could there be any clearer signal that the current soft approach to offenders isn't working? Either the courts get back to zero-tolerance and harsher sentencing, or we face defeat in the war on crime.

Identify the reasons and conclusion in the above argument, and comment on the grammar of the sentences used to express them. Then translate the argument into standard form.

4 Try to find – or invent – an argument in which the conclusion is supported by an explanatory reason (or reasons).

5 No one has anything to fear from giving the police random stop-and-search powers so long as they have nothing to hide. If you are carrying a knife or gun or stolen goods, then of course it's a different story. Opponents of the bill to grant the police more wide-ranging powers can only be helping to protect the guilty.

How would you interpret the above passage? Is it an argument? If so, what is its conclusion? If it isn't an argument, why isn't it?

Answers and comments are on page 314.

2.9 Assumptions

An assumption is a claim or belief that is accepted as true, even if it hasn't been proven or justified. Another similar word is 'presumption'.

We often assume (presume) something just because there is no reason *not* to believe it, even though we cannot be certain that it is actually true. Suppose, for example, I have five banknotes in my wallet, each for 20 euros. I have come by them in a normal way, so I *assume* they are genuine – as anyone would unless there were some particular reason to think otherwise. It is perfectly rational to make this assumption because the vast majority of banknotes we receive are genuine. Yet I know, as well as anyone else, that some banknotes in circulation are forgeries. Therefore, although my assumption is a reasonable one, it is not entirely justified; nor entirely *safe*. Under most circumstances it will be true; but in others it may be false.

This is the ordinary meaning of 'assumption', deriving from the verb 'assume'. An assumption differs from an *assertion* in that an assumption doesn't have to be stated – although it can be. In order to make an assertion I have to say something explicitly. But I can make an assumption without saying anything, or even consciously thinking it. In fact, in the above case, I would probably give no conscious thought whatever to the genuineness of the notes in my wallet, unless or until someone questioned it. My assumption that they were genuine would be evident in my behaviour: for example, taking the money out to pay for something – without a second's thought. You could say that the assumption I was making was *implicit* in my behaviour.

Two kinds of 'assumption'

We can see therefore that an assumption can be *explicit* (stated) or *implicit* (unstated). This raises an important distinction, because in critical thinking, both kinds of assumption play major roles. Unfortunately, in some critical thinking textbooks, the impression is given that 'assumption' always means something unstated, and therefore implied, whereas it is quite clear that in many if not most arguments the premises themselves are no more than assumptions. Unless a premise is a known fact, the best that can be said of it is that it is an assumption.

Take the following argument:

[1] The technology for detecting forgeries has improved in recent years. Unfortunately, the skills and techniques of the forger are more than keeping pace. So we are going to see ever-increasing amounts of counterfeit money in circulation.

The conclusion (C) is the last sentence; and the single premise (P) is the sentence before. (The first sentence is just context.) So, it is argued, C follows from the explicit claim that forgery is improving faster than detection. But what are the grounds for that claim? We are given none. It *may* be true, of course. But equally it may be false or exaggerated. Ultimately we have to take P on trust if we want to accept the conclusion. It is in that sense that we treat P as an assumption, not a fact.

But there is more to be said about [1]. For even if we assume that P is true, it is insufficient to establish the conclusion fully. C is a strong claim predicting that we will see increasing amounts of forged money. That follows from P only if the skills and techniques of the forger *continue* to advance ahead of detection; or, alternatively, if the technology for detection does not catch up in the foreseeable future. The fact that forgery is outstripping detection *at present* does not mean the balance won't change. By drawing the conclusion that we are 'going to see ever-increasing amounts of counterfeit money', the author is assuming more than he or she is saying. And because these assumptions about the future are extremely questionable, [1] is not a reliable argument.

Hidden premises

Another way to think of implicit assumptions is as missing, or hidden, premises. They are premises because they are necessary for the success and soundness of the argument. They are hidden because they are unstated. In the example above, there was at least one hidden premise that was unwarranted, making the argument as a whole unacceptable. But implicit assumptions need not always be detrimental to an argument.

Here is another case to consider, on the same topic:

[2] These banknotes all have the same serial number, so they can't be genuine.

Is this a sound argument? As it stands, no: it is incomplete. Without an additional premise, to the effect that genuine banknotes have *unique* numbers, the conclusion does not follow from the single, stated premise. (If all, or even some, genuine banknotes had duplicate numbers, then obviously the first clause of [2] would not be a good reason for claiming that the notes weren't genuine.) In [2], therefore, it is implicitly assumed that if the banknotes were all genuine they would have different numbers. And because it is necessary to assume this for the argument to make sense, we treat it like an unspoken premise.

In standard form:

[2a] R (*stated*)
These banknotes all have the same serial number.
A (*unstated*)
Genuine banknotes all have unique serial numbers.

C These banknotes can't all be genuine.

By contrast with the claims in [1], R and A are both *well* justified: R by the photographic

evidence; A by common knowledge. Moreover, A is arguably true by definition, since a 'serial' number means one number in a series. Therefore, although [2] in its original form is incomplete, when we add in the obvious assumption, we see that what is intended is a good and plausible argument.

Interestingly, the same conclusion could have been reached by *stating* A and *assuming* R:

[3] Genuine banknotes would have different serial numbers, so these notes can't be genuine.

Again the single premise makes sense as a reason for the conclusion only if it assumes that some of the banknotes in question have the same number. In [3] this is not stated, but only because it does not need to be. We can understand the argument perfectly well without it.

Remember that, under the principle of charity, we start from the presumption that the author of an argument is as rational as we are, and would not have left out a crucial premise through carelessness or stupidity, but would have meant it to be taken as read.

Identifying implicit assumptions

In the examples we have examined so far it would be very difficult not to recognise the implicit assumptions. But with longer and more complex arguments it can require careful and thorough analysis. Consider, for example, the following passage:

[4] In the days before the arrival of the internet, publishers and booksellers effectively controlled what people read, since very few would-be authors could afford the high financial risks of publishing themselves. The internet has changed all that, with Facebook and Twitter leading the charge. Now anyone can express their views publicly, or distribute information, at little or no cost, and without the tyranny of censorship.

Those who are fearful of the internet should therefore stop worrying about its dangers and acknowledge that, on balance, its growth is in the public interest, not against it. For, almost at a stroke, it has given us freedom of information on a scale that could never previously have been imagined.

Activity

Analyse the above argument so that you are clear about its reasons and conclusion. Then decide which of the following is a key underlying but unstated assumption. (There is only one correct answer.)

A There are some reasons to be worried about the internet.

B Freedom of information is in the public interest.

C The internet is here to stay.

D Everyone has the right to publish their opinions.

Commentary

In simplified form the argument runs as follows:

R Now anyone can express views publicly or distribute information at little cost.

IC The internet has given us freedom of information on an unimaginable scale.

C Those who are fearful of the internet should . . . acknowledge that its growth is in the public interest.

The first two sentences of the passage can be interpreted either as background information or as additional reasons to supplement the sub-argument, from R to IC. Either way the main argument is from IC to C. This step works only if we assume that freedom of information is itself in the public interest, since that is the reason given for saying that the internet benefits the public. If it could be shown that on balance freedom of information is *not* in the public interest – i.e. that it did more harm than good – then the argument would be considerably weakened. Option B plainly expresses this assumption; so, out of the four, it is the correct answer.

None of the other claims is required by the argument, even if it is suggested or indirectly implied. A, for example, is something that the author apparently acknowledges, given that he says that we should *stop* worrying. But A is not essential to the conclusion for which the author is arguing. It is just a passing remark. His argument would be no less sound if there were *no* reasons to worry: in other words if A were false. If anything, it would be stronger. So clearly A is not an assumption required for, or helpful to, the argument.

C is not implied at all. According to the author, the internet has brought with it freedom of information and expression. But that does not mean that it will continue to do so, or that other technology will not replace it.

You might have been tempted by D. It may seem reasonable to assume that freedom of expression etc. is an entitlement, and so it may be. But the argument here is that the freedom of expression afforded by the internet has benefits that are in the public interest, not against it; and that therefore it should not be feared. To draw that conclusion, it is not necessary to assume that such freedoms are a *right*. D claims more than is required for the argument; it goes too far.

Missing pieces

Sometimes a key premise is omitted from an argument, not because it goes without saying, but because it suits the author to leave it out, perhaps because it is a questionable assumption and the author may prefer not to draw attention to it by making it explicit. To see an example in which this might be the case, we return to the argumentative text you first encountered in Chapter 2.6 about prize money in tennis. Here it is again:

> [5] Top women tennis players used to grumble that their prize money was less substantial than that paid to top male players in the same competition. They argued that they were being unequally treated. But the disparity was entirely justified and should never have been abolished. Male players just have more prowess than women. They need to win three sets out of five to take the match; the women only two. They have to play harder and faster, and expend far more energy on court than the women. But most of all, if the best woman in the tournament played any of the men, there would be no contest: the man would win.

This argument has two steps. (There is a full analysis of it on pages 47–8 in Chapter 2.6.) The first step, or sub-argument, is clearly intended to establish that the men have more physical prowess than the women. It gives three reasons for this claim, including the explicit assumption that any of the men in a major tournament would beat even the best woman. Let's assume firstly that these claims are true and that they do show that the men have greater tennis-playing prowess. The next step – the main argument – is that therefore the differences in prize money were just, and should not have been abolished.

It is here, in the main argument, that a crucial premise has been left out. For it raises the question: *why* should this difference in physical strength and so on determine the prize money? And that question in turn shows us what is being smuggled into the argument without being stated. For the argument only succeeds if it is justified to say that prize money should depend on prowess, and so, in turn, on factors such as power and speed. Suppose the women were to object that these factors are irrelevant, and to argue that *their* game is actually more entertaining than the men's and attracts as many, if not more, spectators and television viewers. If the women bring no fewer fans, and no less money into the sport, they should have no less reward than the men get for their brute force!

Superficially [5] looks like a fairly strong case, until you look below the surface and see what is being assumed. The fact is there are many criteria which could be used to determine prize money. The author of [5] relies on just one: one which, of course, favours the men, and therefore suits his own argument. This might also explain why the author has omitted to add, in so many words, that 'muscle' should be the decider. Since he has no grounds to support that assumption perhaps it seemed better not to state it openly, and thereby invite an obvious challenge.

Whether or not the omission was intentional makes no difference. It is a seriously inadequate argument, either way, simply because the unstated assumption is unwarranted.

Deep-rooted assumptions

In some arguments, such as [4] or [5], what is assumed is a matter of opinion. You could easily imagine someone who initially thought freedom of information was a good thing changing her mind after seeing websites that encourage violence, racism or gross indecency. You could also imagine someone moving the other way and deciding that freedom of information is a good thing, and that it should be encouraged even if some minority groups abuse it.

But in other cases the assumptions we make are more deeply rooted or unshakable. Many arguments make assumptions based on strong beliefs, strict laws, political leanings, or shared cultural attitudes and loyalties that we grow up with and keep for a lifetime. Realising when an argument rests on assumptions which we take more or less for granted, and rarely question, is an important part of critical thinking and intelligent debate.

Activity

Read the following passage and discuss one or more major assumptions underlying the argument. Consider too how someone might oppose this argument.

[6] After a much-publicised legal battle, Harvey and Hanah Steinberg watched with satisfaction as a family of travellers was forcibly escorted off the corner of their 12,000-hectare estate where the group had been living in a mobile home for 18 months. Not before time. It had taken four appeals and cost the Steinbergs a small fortune in legal fees, but justice had prevailed in the end. The travellers claimed they were following a nomadic way of life going back thousands of years, but their ways show no respect for private property or the rule of law. They did not have the landowners' permission, and they did not pay rent. The Steinbergs therefore have nothing to be ashamed of in prosecuting the trespassers, and the court did the right thing in ordering their eviction.

Commentary

The deep assumptions in this passage are about property rights. The author clearly presumes that property owners, like the Steinbergs, have right completely on their side to choose who can and cannot stay on their land; and just as clearly assume that travellers have no comparable rights to live the life they choose if it means infringing property laws. There is also an assumption that trespassing is not only illegal (which in this case is a fact), but wrong (which is a value judgement). Without this assumption it would not follow that the Steinbergs had 'nothing to be ashamed of', or that the court did 'right' – as well as enforcing the law – to order the eviction.

The fact that the author assumes all this rather than stating it, or offering any argument for it, indicates that he or she simply takes it for granted, and no doubt expects that many if not all readers will do the same. In the culture to which the author belongs there are laws that protect property and punish trespass, and the majority accept such laws because it is in their interests to do so. Laws that prevent travellers from setting up home wherever they like also prevent them from moving into *your* house or setting up camp in your front garden. Consequently, people who own or rent homes of their own tend to accept such laws, and assume they have some moral backing, even if at times they seem harsh. The author does not see any need to spell all this out or argue for it. It 'goes without saying'.

But that doesn't mean the argument or its assumptions cannot be challenged. Not every social group adopts the same attitudes to private property as the author. There are people who choose to live, or would prefer to live, nomadic lifestyles without permanent homes, who might start from the entirely opposite assumption that *no one* has the right to own a piece of land and keep others from using it, especially a large estate like the Steinbergs'. Many people seriously question the assumption that trespass is morally (and not just legally) wrong, or that trespass laws are just laws, or that anyone needs 'permission' to set up a home where they choose. One might argue that the Steinbergs showed a complete lack of compassion in prosecuting the family: that they used their money and power to evict underprivileged people, of minority ethnic status, for no obviously good reason other than exercising their legal right. Some might say that the Steinbergs have *everything* to be ashamed of, and certainly much more to be ashamed of than the travellers.

How you evaluate and respond to an argument like this depends very much on your own political and cultural assumptions. But whichever side you take on the issues, you will not have dealt critically with the argument unless you have recognised and given thought to these assumptions as well as the explicit premises.

Summary

- An assumption, under the ordinary meaning of the word, is a claim or belief that is presumed true, without necessarily being warranted or justified.
- The premises of many arguments are assumptions. In other words the conclusion of an argument often rests on one or more assumptions. If the assumption can be shown to be false or unwarranted, then the argument must be judged unsound.
- Some assumptions that are made in the course of an argument are implicit rather than openly stated.
- Calling a claim or belief an assumption means that it is questionable, open to challenge, or in need of justification. It does not mean that it is necessarily false or unacceptable.
- Some assumptions reveal deep-rooted beliefs or attitudes.

End-of-chapter assignments

1 Study each of the following arguments and say which of the multiple-choice options below it are implicit assumptions on which the argument depends. To make it more interesting, there may be more than one right answer.

a Raisa will hate this book. For a start it's non-fiction, not a novel. But worse still it's all about mountain-climbing.

A Raisa hates non-fiction.
B Raisa hates mountain-climbing.
C Raisa likes novels.

b Nashida is claiming compensation from her former employers on the grounds that she was forced to leave her job. The employers are saying that they did not actually dismiss Nashida. However, they do admit that they altered the terms and conditions of her job. The law allows that, if employees are forced to accept changes in their working conditions that mean they would suffer as a result, and for that reason only they choose to leave, then their entitlement to compensation is the same as if they had been dismissed. Therefore Nashida's claim should be upheld.

A Nashida would have suffered as a result of the changes to her job.
B Nashida had done nothing to deserve dismissal.
C Nashida would not have left if the job changes had been favourable.
D Nashida had no choice about the changes that were made to her job.

OCR (adapted)

c 'Alcopop' is the name given to a range of drinks that contain alcohol but taste like fruit drinks. Their sale in the shops has been blamed for a recorded rise in alcohol consumption by children and young people, and with good reason. It is common sense that if you make alcohol sweet and fruit-flavoured you are encouraging children to drink it. Therefore its sale should be banned.

A Alcopops were manufactured specially to appeal to children.
B Children of an early age do not like the taste of alcohol.
C Children like the taste of sweet, fruit-flavoured drinks.
D Sweet drinks do not appeal as much to adults as to children.

OCR

2 Read the following argument and suggest one or more hidden assumptions that it relies on:

The internet has brought many advantages. It is a wonderful source of knowledge and, used intelligently, it provides for a healthy exchange of views. But history will prove that the internet is a far greater force for harm than for good. Its great flaw is that the information on it is not, and indeed *cannot be*, regulated. Anyone can access it and anything can be published on it, freely and at little or no cost.

3 Do you agree or disagree with the following statement – and why?

Every argument must make at least one unstated assumption.

4 With reference to argument [6]:

Either have a class or group discussion and debate the motion:

The Steinbergs have nothing to be ashamed of in evicting the travellers from their land.

Or write a short argument for or against the above motion.

Answers and comments are on pages 314–15.

2.10 Flaws and fallacies

A good argument is one that satisfies two rules.

Rule 1 is that the reasons should be true. We cannot trust an argument that is based on false premises. If we know that one or more of the premises are false, we must reject the argument.

Rule 2 is that the conclusion must follow recognisably from the reasons, meaning that if the reasons are all true, the conclusion cannot be false.

An argument that passes both these tests is said to be *sound*. An argument that fails one or both of them is *unsound*. Interestingly we use the same words to talk about structures like boats or buildings, and more abstract objects such as ideas, advice or plans. When you describe something as sound, what you are saying about it is that it is safe, reliable, free of faults. You would not call a boat sound if it had a hole in it and sank ten minutes after setting off from the shore. You would not call a plan sound if it led to a disaster. And you don't call an argument sound if it leads to a false or dubious conclusion. (A bad argument is often said to have a hole in it – something missing from the reasoning.) Nor do you call an argument sound if you know, or have reason to believe, that one or more of its premises are false.

Another word for an unsound argument is 'flawed'. A flaw is a fault. There are two main ways in which you can find fault with an argument. You can disagree with one or more of the reasons; and/or you can show that, whether the reasons are true or not, the conclusion doesn't follow from them. Arguments that are unsound for this second reason are said to contain 'reasoning errors', or 'flaws in the reasoning'. They are also called 'fallacies'. A fallacy is a flawed line of reasoning. Because it is very often not possible to know the truth or otherwise of the premises, most of the critical evaluation of arguments focuses on the reasoning, and whether it is sound or fallacious. (If you *know* that either the reasons or the conclusion is false, there is no further critical thinking to do on the argument!)

Note: the word 'fallacy' is often used casually to mean a false or mistaken *claim*. For example, after 1912 a person might have said, 'It was a complete fallacy that the *Titanic* was unsinkable.' In critical thinking, or any formal context, 'fallacy' is never used that way. A fallacy is always a defective argument.

Activity

Read the following argument and decide whether or not the reasoning is flawed. If it is flawed, explain what you think the flaw is.

[1] The outstanding success of Amulk's company, which was launched against the advice and without the support of bankers, business consultants and financiers, just goes to show that one person's vision can prove all the experts in the world wrong. Anyone thinking of setting up in business should therefore trust their own judgement, and not be influenced by the advice of others.

Commentary

First we need to analyse the argument so as to identify the conclusion and the reasons. Then we need to ask whether or not the conclusion follows from the reasons, according to Rule 2.

The conclusion is the second sentence. The first, longer sentence is the reasoning given in support of it. On inspection we can see that this long sentence really contains three claims rolled into one. So a full analysis of it would be:

R1 Amulk's company is/was an outstanding success.

R2 It was launched against the advice of bankers . . . etc.

IC One person's vision can prove all the experts in the world wrong.

C Anyone thinking of setting up in business should trust their own judgement, and not be influenced by the advice of others.

We don't know whether or not the two initial reasons, R1 and R2, are true, but we'll assume that they are. There is no reason to believe they are untrue. If they are true then it does seem that IC is also true; for if Amulk's company really *was* such a success, and the bankers and others all advised against it, then it seems fair to say one man's success (Amulk's) can prove the experts wrong. It means assuming that the bankers and others are 'experts', but we can let that pass. So we can accept that the first stage of the argument is sound.

The big question is whether the main conclusion follows from the intermediate one (IC). This time the answer is 'No'. Even if everything we are told is true, we cannot conclude from this one single example of success, or from this one misjudgement by the 'experts', that anyone setting up in business should ignore expert advice. It would be a crazy conclusion to draw, a reckless thing to do. It would be like arguing as follows:

[2] Beth passed all her exams without doing any work. So anyone taking an exam should stop studying!

Not studying may have worked for Beth, just as ignoring advice worked for Amulk, but that doesn't mean it will work for anyone else – let alone everyone.

Generalising from the particular

It is easy enough to see that [1] and [2] contain a serious flaw in the reasoning, one that makes the conclusion unreliable. It is also easy to see that it is the same kind of flaw in each case, even though the contexts are different. But what exactly is the flaw? How do we identify it?

[1] and [2] are both examples of a very common flaw. It is known as generalising from the particular. We call something a particular if it is just one instance, or one of a limited number of instances. The *particular* in [1] is the success of one company. In [2] it is a single person's exam results. Neither of these is a strong enough reason to support a sweeping generalisation. (See also Chapter 2.2.)

Arguing from anecdotal evidence

Another way in which you could describe the flaw in both of these arguments is to say that they rely on anecdotal evidence. An anecdote is a story, usually just one among many, often different, stories. So a piece of anecdotal evidence is a kind of particular; and arguing from anecdotal evidence can be a reasoning error if the conclusion is an unwarranted generalisation.

However, anecdotal evidence can support some conclusions. Look, for example, at this next argument:

[3] Three people fell through the ice last winter when they were walking across the lake. Seriously, you should think twice before you try to cross it.

If the anecdote – the first sentence – is true, then it is a sound argument, and its conclusion is sound advice. There is nothing wrong with the evidence in [3], even though it is still purely anecdotal. The fact of three people falling through the ice last year is a very good reason for thinking twice about walking on it now, and it would be irrational *not* to think twice about it, if you value your safety and you believe the story. But compare [3] with the following case, which uses exactly the same evidence:

> [4] Three people fell through the ice last winter when they were walking across the lake. You should never walk on frozen lakes.

Activity

Discuss the difference between [3] and [4].

Commentary

[3] is a sound argument and [4] is not. [4] is flawed, like [1] and [2], and in the same way: its conclusion is too general to draw from one, or even three, particular pieces of (anecdotal) evidence. In the right conditions it is perfectly safe to walk on frozen lakes, and people do it regularly. What happened to the three unfortunate people who fell through the ice was no doubt caused by the conditions being unsafe at that time. But it doesn't mean, as [4] concludes, that frozen lakes are never safe.

Insufficient reason

Another way to say what is wrong with [1], [2] and [4] is that in each argument the reason is insufficient or inadequate – i.e. *not strong enough* – to support the conclusion. In all three cases the argument goes too far, or claims too much. In [3], by contrast, the conclusion is much more limited in what it claims: it just suggests a bit of caution.

Here we see again why the distinction between strong and weak claims (Chapter 2.2, page 25) is so important in evaluating some arguments. Flaws occur when weak claims are expected to provide support for strong claims. Not surprisingly, strong claims need equally strong, or stronger, claims to support them adequately. 'You should *never* walk on frozen lakes' is not just strong: it is indefensible. It would need to be assumed that no freshwater ice, however thick, could bear a person's weight – which is obviously unwarranted.

In the next example the story is a bit different, and so is the conclusion.

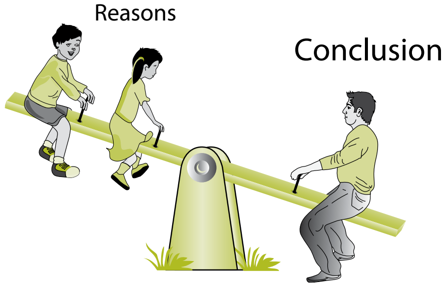
> [5] People cross this lake every year from November through to March. The ice can be anything up to a metre thick. People drive cars across it. I've even seen bonfires on the ice at New Year and folk sitting round having a party. So there is no risk of anyone ever falling through in the middle of February.

Activity

Assuming the reasons are true, is this argument sound, or does it have a flaw?

Commentary

This is a classic example of anecdotal evidence being used carelessly. The reasons are insufficient for the conclusion they are being used to support, even if you add all four of the reasons together. The fact that people have done various things on the ice in the past, and come to no harm, does not mean there is never going to be a risk in the future. In fact, if some scientists are right about global warming, what has been observed about frozen lakes up until now will not be very reliable evidence in years to come. On many lakes the ice in February may become thinner and less safe – just like the reasoning in [5]!



A useful metaphor for an argument is a see-saw, or balance arm, with reasons on one side and the conclusion on the other. If the conclusion is too strong, or asserts too much, the reasons may not have sufficient 'weight' to support it. For an argument to be sound the reasons must outweigh the conclusion. In [5] they don't even counter-balance it. They are insufficient.

Identifying flaws

It is one thing being able to see that an argument is flawed. It is another being able to say what the flaw is. It is not enough just to say that the reasons are insufficient or inadequate, or that the conclusion doesn't follow from the reasons, because that is the same as saying the argument is fallacious. We need a deeper explanation.

In this unit you have seen two very common reasoning errors. One was taking a particular point (e.g. about one person's business experience) and drawing a general conclusion from it (e.g. about how to start up *any* business), as in argument [1]. Another, illustrated in argument [5], involved using past experience to draw an unwarranted conclusion about the future.

Thus, if you were asked to describe the kind of flaw that weakens [5] you could answer:

> It assumes that what has been true in the past remains true now, or in the future.

Or, with more specific reference to [5]:

> It assumes that because people have walked on the ice safely in February in the past, it is always safe to do so.

Either of these would be a correct answer.

There is another way of identifying an error of reasoning which does not describe the flaw directly, but reveals or exposes it – shows it up. It may be a counter-argument, an example or explanation, or even a question.

Activity

Recall the argument at the start of the chapter:

> The outstanding success of Amulk's company, which was launched against the advice and without the support of bankers, business consultants and financiers, just goes to show that one person's vision can prove all the experts in the world wrong. Anyone thinking of setting up in business should therefore trust their own judgement, and not be influenced by the advice of others.

Discuss each of the following responses to this argument. Do any of them put a finger on the flaw in the reasoning?

A Many people may have been put off starting their own businesses because they paid too much attention to the advice of so-called experts.

B Business consultants and financiers know far more about setting up in business than the man in the street knows.

C Might Amulk just have been lucky, or the 'experts' to whom he spoke not so expert?

Commentary

We will take the options one at a time. A does not expose any flaw in the argument because if it does anything at all it supports the argument. It appears to sympathise with the conclusion that people should trust their own judgement.

B looks much more of a challenge than A did. But challenging an argument is not the same as showing up its internal flaws. Even if we accept that B is true, you could still argue that Amulk's experience proved them wrong on this occasion. The flaw is not that Amulk *knew less* than the experts, because nowhere in the argument is it claimed that he knew anything at all – only that he was successful. The mistake is in drawing a conclusion about other people's chances of success from Amulk's success alone. So B does not point to the flaw.

That leaves C. C effectively raises a doubt about the conclusion by suggesting that the real explanation for Amulk's success may simply have been that he was lucky on this one occasion. That way it would still be better as a general rule to heed expert advice, contrary to the conclusion of the argument. Alternatively, in Amulk's case, the particular individuals who advised him may not have been the best. Again, that does not mean that going against advice is more likely to succeed than following it. By identifying other equally likely explanations for Amulk's success, C exposes a serious flaw in the reasoning.

Drawing inferences

To infer something means to draw it as a conclusion, usually from some evidence or information. A sound or 'safe' inference is one that is adequately supported by the information. Otherwise it is *un*safe. (Other words you could use are 'unreliable', 'unjustified' or 'unwarranted', all of which can be applied to claims generally.)

Consider the following report in a local newspaper:

> [6] Doctors investigating an outbreak of suspected food poisoning discovered that four of the people who had reported sick had eaten at the Bayside fish restaurant the day before; and all had eaten fish. Any establishment that is found to be responsible for food-related sickness will be closed by the authorities and not permitted to reopen until it has been given a certificate of fitness from hygiene inspectors. Today the Bayside is closed.

Activity

Can any of the following claims *safely* or *reliably* be inferred from the passage above?

- **A** The source of the outbreak of food poisoning was the Bayside fish restaurant.
- **B** Fish was the cause of the outbreak.
- **C** The Bayside has been closed down by the inspectors.

Commentary

According to the passage we have three facts:

- Four people who reported sick had recently eaten at the Bayside.
- Any establishment responsible for food-related sickness is closed by the authorities.
- The Bayside is closed (today).

So, between them, do they justify any of the three claims? We'll take the claims in order, starting with A. Although there is a suspected link between the restaurant and the people reporting symptoms, it cannot be inferred that the restaurant was responsible for the outbreak. If still in doubt, read [6] again. Note, for example, that we are told nothing about the four people other than that they ate at the Bayside and then reported sick. It is possible that there were other connections between them: that they were all friends or family and had shared other food and drink besides the meal at the restaurant. Nor are we told if there were others who were sick besides the four who were mentioned in the report. There may have been others who did not report their illness. If there were others, we do not know whether they had eaten at the Bayside or somewhere else. The facts that we have, even if true, do not support the inference.

The same goes for inference B, that fish caused the sickness. We are told that all four of those who ate at the Bayside and reported sick had eaten fish. But we are not told what else they may have eaten or drunk. Even if the Bayside was the source of the illness, which is not certain, it need not have been the fish that caused it. The cause may have been a side dish, or a sauce, or contaminated water, or a general lack of hygiene in the kitchen.

Nor is it safe to infer C, that the inspectors closed the restaurant. The statement in the newspaper that restaurants found responsible for food-related sickness have to close is actually irrelevant: it does not mean that because a restaurant closes it *is* responsible for the sickness. Many restaurants close on one or more days of the week. Today may be the chef's day off. Many explanations for the closure are possible besides the *seemingly* obvious one, that it was closed because of food poisoning.

Jumping to conclusions

Often when people read of incidents like this they infer too much, given what they know – or rather, despite what they *don't* know. Without more than the information in the report, it would be *jumping to a conclusion* to draw any of the three proposed inferences about the restaurant, its food, or the reasons for its closure.

It is particularly tempting to jump to a conclusion if you carry some prejudice in the matter. Suppose, for example, you had eaten a couple of times at the Bayside and had not enjoyed the experience. Perhaps one of the waiters had been rude, or the service had been slow; or you just don't like fish. In other words, you had reasons to be critical of the restaurant, but 'reasons' in the sense of motives rather than reasons for a sound argument. With that motivation, you argue as follows:

[7] I knew the Bayside was bad news. I've never liked the food there, and certainly never eaten the fish. Now we hear that four people who went there have all reported sick, and the next day the restaurant is closed. So, it's pretty clear that their food is to blame. My suspicions were correct all along.

A classic fallacy

A fallacy, you will recall, is a flawed argument. It is also the word we use for the flaw itself. We can say that [7] *is* a fallacy, because it is a flawed argument. But we can also say that it *commits* a fallacy, or has a fallacy in it. Some fallacies appear over and over again in different arguments. The best-known examples were discovered and classified centuries ago, and many have Latin names. They are often referred to as the *classic* fallacies, for that reason.

There is a classic fallacy lurking in [7], and in the three inferences from [6] that we discussed. It is known as the *post hoc* fallacy, or in full: *post hoc ergo propter hoc*, meaning literally: 'after this, therefore *because* of this'. The fallacy is in assuming that when one thing happens and then another, that the first must be the cause of, or reason for, the second. The absurdity of this assumption can be illustrated if we imagine someone opening an umbrella just before it starts to rain, and arguing that opening the umbrella made it rain! Of course, there are many situations in which one act or event does cause another. If a tree falls into the road and a driver swerves to miss it, it is perfectly reasonable to infer that the falling tree caused the driver to swerve. The fallacy is not that there is never a causal connection between two events, but that a causal explanation cannot and should not be *assumed*, even when it looks quite plausible. Indeed, it is when a causal explanation looks quite plausible that the fallacy is most dangerous, because it is then that people are most likely to jump to a conclusion that may be false.

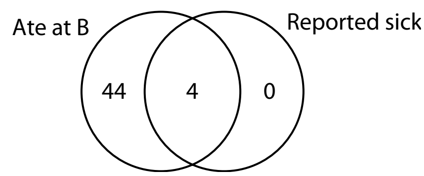
[6] is a good example. We are told that a number of people ate at a certain restaurant and reported sick the next day, with suspected food poisoning; then that the restaurant closed. It is natural enough to assume that eating in the restaurant caused the people to be ill. People often justify such assumptions by saying that there is no other explanation; or that it is all too unlikely to be a coincidence. But on reflection there often are other possible explanations; and coincidences *do* happen.

Cause and correlation

The *post hoc* fallacy is itself an example of a more general reasoning error known variously as the 'false cause' or 'mistaken cause' or 'cause–correlation fallacy'; or more descriptively as confusing correlation with cause. A correlation is any observed connection between two claims or two facts, particularly between two sets of data or trends. For instance, if there were an observed upward trend in violent crime in a city, at a time when sales of violent computer games were on the increase, it would be right to say there was some correlation between the two trends.

It would also be tempting to conclude that the games were at least a factor in causing the actual violence to increase. Many people make this inference, and not unreasonably, since a significant number of computer games have violent content. It is perfectly justified to claim that if such games did turn out to be a cause of violent crime it would be no surprise, and it would help to explain the trend in a convincing manner. But the plausibility of an explanation does not make it true. It can be posited as a reasonable hypothesis (see Chapter 2.1), but not safely inferred.

The inferences from [6], and the reasoning in [7], also exhibit the cause–correlation fallacy. There are not two different fallacies there: just two different ways of describing the same fallacy, one more general than the other. One could say that there is a correlation between the people dining at the restaurant and the people reporting sick. Let's suppose the figures for people dining at the Bayside (B) and reporting symptoms were as shown in the following diagram:



There is a correlation – 4 out of the 4 who reported sick had all eaten at the restaurant. But it is a *weak* correlation: 44 ate there without reporting sick, and although none who did not eat there reported sick, we have no information about those who may have been sick but did not report it. We have a plausible hypothesis. But to infer the Bayside's guilt from the data alone would be fallacious. Arguments or inferences that assume causal connections from correlations alone are generally flawed.

Recognising and avoiding flaws

There are many other classic fallacies and common reasoning errors besides those you have seen in this chapter. Some have names such as 'slippery slope' or 'restricting the options' or *argumentum ad hominem*. Many of these will feature in Unit 4, and you will learn to recognise them, so that you can reject unsound arguments and avoid making similar errors in your own reasoning.

It is a good idea to keep a diary or notebook of common flaws that you come across. (There is a suggestion in the end-of-chapter assignments on how to organise this.)

Summary

- An argument is flawed if the reason or reasons given are untrue, or give inadequate support to the conclusion.
- Some common flaws are:
  - arguing from a particular case to a general conclusion
  - relying too heavily on anecdotal evidence, or past experience
  - mistaking a correlation for a cause.

There are many more flaws and fallacies than these. In many flawed arguments you will find that there is more than one way to name or describe the fault.

End-of-chapter assignments

1 Recent research suggests that, contrary to popular belief, the firms that are making the most money tend to have the least happy workers. Therefore firms which impose conditions that make workers less happy can expect a rise in profits.

a Which of the following, if true, identifies the flaw in the argument above?

A It assumes workers are unhappy because of their work.
B It assumes that worker-unhappiness is the cause of higher profits.
C It assumes that workers do not get a share of the high profits.
D It assumes that successful managers have to be hard on their staff.

b Which of the following, if true, would weaken the argument above? (There may be more than one.)

A It has been found that workers in rich and successful companies become resentful and disgruntled.
B It has been found that the owners and managers of highly profitable companies stop caring about the welfare of employees.
C It has been found that companies that try to make their employees happy are not always financially rewarded for their efforts.

2 The famous author Farrah Lavallier died at the age of 98, just before finishing the 35th book of her distinguished literary career. Critics were in almost unanimous agreement that it was as sharp and witty as any she had written. Clearly she had all her faculties right up to her last days. She also left a diary that revealed, amongst other things, that she had never done a stroke of physical exercise in her entire life. She was fond of joking that if she walked once round her study, she needed to sit down for a rest. So, if a long and productive life is what you want, you should forget about jogging or joining a gym. Save your energy.

a How would you name or describe the fallacy in the above argument?

b Which of the following, if true, helps to expose the flaw in the reasoning, and so challenges the argument?

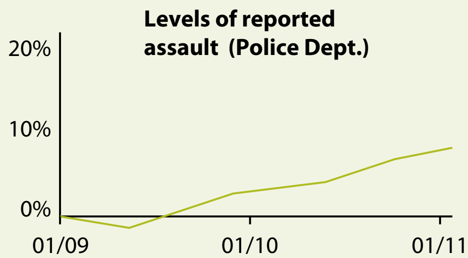
A Women didn't go to gyms when Farrah was young.
B Farrah's grandfather lived to 104, and her mother to 106.

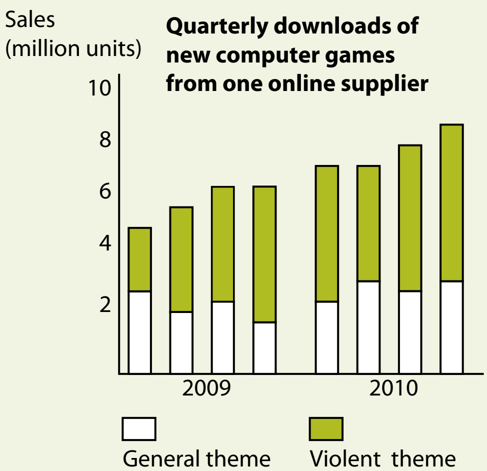
C According to her diaries Farrah had never been seriously ill.

D Few people are still working in their late nineties.

E Many writers live physically inactive lives.

3 Would the data in the two graphs below support the conclusion that computer games contribute to violence? Give reasons for your answer.





4 Start a file, or database, of common reasoning errors by listing the ones you have met in this. You could use three headings, or fields:

> ***Name*** *(or brief description), e.g. Relying on anecdotal evidence*
> ***Explanation****, e.g. Using a single occurrence of something and drawing a general conclusion from it.*
> ***Example****, e.g. I know someone who fell through the ice at this spot. Therefore it is never safe to cross this lake.*

Whenever you encounter flawed or suspect arguments, add them to the file.

Answers and comments are on page 315.

3.1 What do we mean by a 'problem'?

Consider the action of making a cup of instant coffee. If you analyse the processes you need to go through, they are quite complicated. Just the list of items you need is quite long: a cup, a teaspoon, a jar of coffee, a kettle, water, and milk and sugar if you take them. Having found all these items, you fill the kettle and boil it; use the teaspoon to put coffee into the cup; pour the boiling water into the cup, just to the right level; stir; add milk and sugar; then put all the things you used away again. In fact one could break this down even more: we didn't really go into very great detail on, for example, how you boil the kettle.

Although this is complicated, it is an everyday task that you do without thinking. However, if you encounter something new, which may be no more complicated, the processes required to achieve the task may need considerable thought and planning. Most of such planning is a matter of proceeding in a logical manner, but it can also require mathematical tasks, often very simple, such as choosing which stamps to put on a letter. This thought and planning is what constitutes problem solving.

Solving most problems requires some sort of strategy – a method of proceeding from the beginning which may be systematic or may involve trial and error. This development of strategies is the heart of problem solving.

Imagine, for example, trying to fit a number of rectangular packages into a large box. There are two ways of starting. You can measure the large box and the small packages, and calculate the best way of fitting them in. You may make some initial assumptions about the best orientation for the packages, which may turn out later to be wrong. Alternatively, you may do it by trial and error. If you have some left over at the end that are the wrong shape to fit into the spaces left, you may have to start again with a different arrangement. Either way, you will have to be systematic and need some sort of strategy.

With some problems the method of finding an answer might be quite clear. With others there may be no systematic method and you might have to use trial and error from the start. Some will require a combination of both methods or can be solved in more than one way.

The words 'problem solving' are also used in a mathematical sense, where the solution sought is the proof of a proposition. 'Problem-solving' as tested in thinking skills examinations does not ask for formal proofs, but rather asks for a solution, which may be a calculated value or a way of doing something. Although many of the problems we shall look at here use numbers and require numerical solutions, the mathematics is usually very simple – much of it is normally learned in elementary education. Many problems do not use numbers at all.

As we saw in Chapter 1.3, there are three clearly defined processes that we may use when solving problems:

- identifying which pieces of data are relevant when faced with a mass of data, most of which is irrelevant
- combining pieces of information that may not appear to be related to give new information

- relating one set of information to another in a different form – this involves using experience: relating new problems to ones we have previously solved.

When solving problems, either in the real world or in examinations, you are given, or have, or can find, information in various forms – text, numbers, graphs or pictures – and need to use these to come up with a further piece of information which will be the solution to the problem.

The processes described above are the fundamental building blocks of problem-solving and can be expanded into areas of skill that may be brought together to solve more complex problems. The chapters in this unit divide these into smaller identifiable skill areas which can be tested using multiple-choice questions. Examples of such sub-skills are searching for solutions and spatial reasoning (dealing with shapes and patterns). Later units deal with more complex problems, which can only be solved using several of these sub-skills in combination, and are closer to the sort of problem solving encountered in the real world.

The activity below gives an example of a simple problem; you can give either a simple answer or a more complicated one, depending on the degree of detail you consider necessary.

Activity

Luke has a meeting in a town 50 miles away at 3 p.m. tomorrow. He is planning to travel from the town where he lives to the town where the meeting is by train, walking to and from the station at both ends.

List the pieces of information Luke needs in order to decide what time he must leave home. Then work out how you would proceed to plan his journey from these pieces of information.

Commentary

The chances are that you missed some vital things. You may have thought that all he needed was a railway timetable. Unless you approached the problem systematically, you may not have thought of everything.

Let us start by thinking of everything he does from leaving his house to arriving at the meeting.

1. He leaves his house.
2. He walks to the station.
3. He buys a train ticket.
4. He goes to the platform.
5. He boards the train when it arrives.
6. He sits on the train until it reaches the destination.
7. He leaves the train.
8. He walks to where his meeting is being held.

You can construct the pieces of information he needs from this list. They are:

1. The time taken to walk from his house to the station.
2. The time needed to buy a ticket. (Remember to allow for queues!)
3. The time to walk to the platform.
4. The train timetable.
5. The time taken to walk from the station to where the meeting is being held.

Did you find them all? Perhaps you thought of some that I missed. For example, I didn't think of allowing for the train being late. You could estimate this by experience and allow some extra time.

Now, to find out when he should leave home we need to work backwards. If his meeting is at 3 p.m., you can work out when he must leave the destination station to walk to the meeting. You can then look at the timetable to see what is the latest train he can catch (allowing extra for the train to be late if appropriate). Then see from the timetable when this train leaves his home town. Continuing, you can determine when he should have bought his ticket, and when he should leave home.

Of course, you could do the whole thing by guesswork, but you might get it all wrong and, more to the point, you cannot be confident that you will have got it right.

In the sense we are using the word in this book, a 'problem' means a situation where we need to find a solution from a set of initial conditions. In the following chapters we shall look at different sorts of problem, different kinds of information, and how we can put them together to find solutions to the problems. These chapters will lead you through the types of problem-solving exercises you will encounter in thinking skills examinations and give some indications about how you might approach such problems. However, learning to solve problems is a generally useful life skill and also, we hope, fun!

Summary

- In this chapter we have looked at what a problem is and how the word can be used in different ways.
- We have seen how information is used to contribute to the solution of a problem.
- We have looked at how various methods of using information can lead to effective solutions.

End-of-chapter assignments

1. Imagine you are going to book tickets for a concert. List the pieces of information you need and the processes you need to go through in order to book the tickets and get to the concert. In what order should you do them? First list the main things, then try to break each down into smaller parts.
2. Consider something you might want to buy, such as a car, mobile phone or computer. Make a list of the pieces of information you would need in order to make a decision on which make or model to buy.
3. Find a mileage chart that gives the distances between various towns (these can be found in most road atlases or on the internet). Pick a base town and four other towns. Consider making a journey that starts at the base town, takes in the other four and ends at the base town. In what order should you visit the towns to minimise the journey?
4. The following questions are based on a very simple situation, but require clear thinking to solve. Some are easier than others.

   A drawer contains eight blue socks and eight black socks. It is dark and you cannot tell the difference between the two colours.

   a. What is the smallest number you will have to take out to ensure that you have a matching pair?
   b. What is the largest number you can take out and still not have a matching pair?
   c. What is the smallest number you can take out to be sure that you have one of each colour?
   d. What is the largest number you can take out and still have all of one colour?
   e. What is the smallest number you can take out to be sure you have a blue pair?

Answers and comments are on pages 315–16.

3.2 How do we solve problems?

We have seen that a problem consists of a set of information and a question to answer. In order to solve the problem we must use the information in a certain way. The way in which we use it may be quite straightforward – it may for example be simply a matter of searching a table for a piece of data that matches given conditions. In other cases, instead of searching for a piece of data, we may have to search for a method of solution. The important thing in either case will be to have a strategy that will lead to the solution.

Many publications give (in various forms) the procedure:

Data ➔ Process ➔ Solution

This is all well and good, and indeed represents a way problems can be solved. It says nothing about what the words and, in particular, the arrows mean. It is in this detail that the key to problem solving is found. In simple terms, we are concerned with identifying the necessary pieces of data and finding a suitable process. There are no hard and fast rules; different problems must be approached in different ways. This is why problem solving appears in thinking skills examinations; it tests the ability of candidates to look at situations in different ways and to be able to use many different strategies to find one that works. Whilst a knowledge of the different categories of problem, as identified by the syllabuses and the various chapters of this unit, will help, you will always need to have an open mind and be prepared to try different approaches.

There are several ways problems may be approached. A term that is used a lot is 'heuristic' (see for example *How to Solve It* by G. Polya [Penguin, 1990] – a book on mathematical problem solving). This word comes from the Greek 'to find' and refers to what we might call 'trial and error' methods. Alternative methods depend on being systematic: for example, an exhaustive search may lead to an answer. Previous experience of solving similar types of questions will always be a help.

Imagine you are going out and can't find your house keys. Finding them is a problem in the sense meant by this section of the book. The heuristic method (and sometimes the quickest) is to run around all the likely places to see if they are there. After the likely places, you start looking at the less likely places, and so on until they turn up or you have to resort to more systematic methods. There are two systematic ways of searching. The first (using experience) involves thinking carefully about when you last came into the house and what you did; this can be the quickest method. The other (which in mathematical terms is often known as the 'brute force' method) involves searching every room of the house thoroughly until they are found. This is often the most reliable method but can take a very long time and most people will use it as a last resort.

When people are solving problems, they may use all of these methods, often in the order given above. This is quite logical, as the heuristic method can lead to a very rapid solution whilst the systematic search is slowest. One of the prime skills you need in tackling problem-solving questions in examinations is to make a good judgement of which method is the most appropriate one to use in any set of circumstances.

In any problem you will be presented with some initial pieces of information – these may be in the form of words, a table of numbers, a graph or a picture. You will also know what you need to produce as a solution (the answer to a question). The first thing to do is to identify which pieces of information are most likely to be useful in proceeding to the solution and to try to work out how these pieces of information may be used. Problem-solving questions often contain redundant information, i.e. that which is not necessary to solve the problem. This echoes real life, where the potential information is infinite.

The activity below is a relatively easy example. It is not difficult to find a way of approaching the problem, and the necessary calculations are clear and simple. See if you can do it (or at least work out how you would tackle it) before looking at the commentary which follows.

Activity

Julia has been staying in a hotel on a business trip. When she checks out, the hotel's computer isn't working, so the receptionist makes a bill by hand from the receipts, totalling $471. Julia thinks she has been overcharged, so she checks the itemised bill carefully.

Room:	4 nights at $76.00 per night
Breakfast:	4 at $10.00 each
Dinners:	3 at $18.00 each
Telephone:	10 units at $1.70 per unit
Bar:	various drinks totalling $23.00
Laundry:	3 blouses at $5.00 each

It appears that the receptionist miscounted one of the items when adding up the total. Which item has Julia been charged too much for?

Commentary

The sum of the charges on the itemised bill is $453. This is $18 less than her bill, so she has been overcharged for one dinner. None of the other items could come to exactly $18, either singly or severally.

Although this example is simple, it illustrates many of the methods used in solving problems:

- Identify clearly and unambiguously the solution that is required. Reading the question carefully and understanding it are very important.
- Look at the data provided. Identify which pieces are relevant and which are irrelevant.
- Do you need to make one or more intermediate calculations before you can reach the answer? This can define a strategy for solving the problem.
- You may need to search the given data for a piece of information that solves (or helps to solve) the problem.
- Past experience of similar problems helps. If you had never seen this type of problem before, you would have had to spend more time understanding it.
- The above problem was solved using a systematic procedure (in this case calculating the correct bill, a value not given in the original problem).

The activity below, whilst still being relatively simple, involves a slightly different type of problem where the method of solution is less obvious.

Activity

The SuperSave supermarket sells Sudsy washing up liquid for $1.20 a bottle. At this price they are charging 50% more than the price at which they buy the item from the manufacturers. Next week SuperSave is having a 'Buy two get a third free' offer on this item. The supermarket does not want to lose money on this offer, so it expects the manufacturers to reduce their prices so SuperSave will make the same actual profit on every three bottles sold.

By how much will the manufacturers have to reduce their prices?

A $\frac{1}{6}$ **B** $\frac{1}{4}$ **C** $\frac{1}{3}$ **D** $\frac{1}{2}$ **E** $\frac{2}{3}$

Commentary

This could be solved in a variety of ways. We could just guess. As we are letting $\frac{1}{3}$ of the bottles go for free, option C, $\frac{1}{3}$, is tempting. This is wrong.

It could be done by trial and error: for example, start with the manufacturers charging 60¢ (this would be option B) and see what that leads to. For three bottles they will charge $1.80 and the supermarket sells for $2.40, so their mark-up is 60¢ for three bottles or 20¢ each. This is not enough, so the manufacturers' price must be lower.

In fact there is a straightforward, systematic way of solving this which is made clear by writing down all the relevant values which can be calculated:

Normally SuperSave sell at $1.20, so they buy at 80¢ (selling at 50% more than they buy), so each bottle is sold for 40¢ more than the price at which it is bought.

Under the offer, they will sell three bottles for the price of two, i.e. three for $2.40, or 80c each. If they are still selling for 40¢ more than the price at which they have to buy, they will be buying from the manufacturer at 40¢. So, the manufacturers will have to halve their price. Option D is correct.

This method was quite quick, and certainly quicker than the trial and error method. It is the sort of solution that you are more likely to come up with if you have seen a lot of similar problems before and you think carefully about the information given.

Finally, to be sure that you have found the correct solution, check the answer. The profit on one bottle was $1.20 – 80¢ = 40¢; the profit on three bottles under the offer is $2.40 – $1.20 = $1.20, or 40¢ per bottle. That's correct!

You should have learned a little about finding a method of solution from this example. The guesswork method can only work by luck. This may be called the 'pirate's gold' approach – we know the treasure is on the island somewhere so we dig a hole. If it's not there, we dig another one somewhere else. Sometimes this method may seem to work, but it is usually because a little previous experience has been used, even unknowingly. The trial and error method, sometimes using a common sense strategy which turns it into a partial search, can be effective for solving some problems. Other problems may need an exhaustive search to solve; these are discussed in Chapter 3.6.

In the case above – and in many others – the method of finding a clear strategy was the most efficient. Strategies are not always found by rigorous methods; the discovery of an appropriate strategy usually depends on past experience of similar problems.

Summary

- We have looked at some methods of solving problems, investigating how different methods may be used in different circumstances.
- We have recognised the value of experience in identifying problem types and appropriate methods of solution.
- We have seen how important it is to read and understand the information and the question.
- We have looked at the relative merits of guesswork, searching and strategic methods of solution.

End-of-chapter assignments

1 The petrol usage of a number of cars has been measured. Each car started with a full tank, then made a journey (all journeys were over similar roads). After the journey the tank was filled to the top, the amount of petrol needed to fill it being recorded. The results are shown below. Put the cars in order of their petrol efficiency (km/litre), from lowest to highest.

Car	Length of journey (km)	Petrol used (litres)
Montevideo	120	10
Stella	150	16
Riviera	200	25
Roamer	185	21
Carousel	230	16

2 The votes have recently been cast at the local elections. Voting is carried out using the alternative vote system. This means that each voter ranks the candidates in order of preference. Votes are counted initially on the basis of all voters' number one ranking. The candidate with the least votes is excluded and the votes of those people who placed him or her number one are reallocated using their second preferences. The process then continues until a winner is established. The results of the first count are shown below. How many candidates still have a chance of winning?

Patel	323
Brown	211
Walsh	157
Ndelo	83
Macpherson	54
Gonzalez	21

3 Rajesh is cooking a meal for some friends. This will involve roasting a chicken, which takes 2 hours' cooking time plus 15 minutes resting on removal from the oven. The oven takes 15 minutes to warm up. He will also cook some rice (30 minutes' soaking plus 15 minutes' cooking), broccoli (5 minutes to prepare and 5 minutes to cook) and a sauce (10 minutes to prepare and 15 minutes to cook).
What should be the timing of events if the friends are to eat at 7 p.m.?

4 Joseph is making a bookcase. This requires two vertical side-pieces of wood 1.2 m high and three shelves 1.6 m long. All are 20 cm wide. He will cut these from a sheet of wood 2.4 m × 1.2 m.
Draw a diagram showing how the pieces may be cut to leave the largest possible uncut rectangle. Are there other ways to cut it?

Answers and comments are on pages 316–17.

3.3 Selecting and using information

In one very simple form, problem solving involves understanding and making use of information. In the examples considered in this chapter, the problem to solve is to select the correct pieces of information and to use them in an appropriate manner.

Information can come in a great variety of forms and, if you want to be good at using it, you will need to practise extracting data from a range of sources.

Here are some forms that sets of information can take:

- Tables: these could include summaries of surveys, specification sheets or transport timetables.
- Graphs: these are used in science and business to provide information in such a way that it can be absorbed quickly and easily. For example, a graph may show variables such as temperature over time; financial data may be shown in bar charts.
- Words: numerical, spatial, logical and many other types of information can be summarised or described in words.
- Pictorial: pictures, for example in the form of engineers' or architects' drawings, can be used not only to show what something looks like, but also to give information about relative sizes and positions.
- Diagrammatic: diagrams come in a wide range of forms: flow charts, maps, schedules, decision trees and many other types can summarise numerical and spatial information.

The following series of activities is based on various different forms of information. Try to work them out by yourself before looking at the answers and comments. These activities also introduce some problem-solving methods that are discussed further in later chapters.

Activity

Tabular information

The table shows the results of a survey into participation in three types of regular exercise taken by people from three age groups.

Although the row and column totals are correct, one of the individual figures in the table has been typed incorrectly. Which is it?

Age	Type of exercise			
	Gym	Swimming	Jogging	Total
10–15	14	57	32	103
16–20	86	92	45	232
21–25	67	58	44	169
Total	167	207	130	504

Commentary

This table has a lot of figures, and finding the incorrect one might seem quite daunting. However, we must look at what we are trying to do and what information we have.

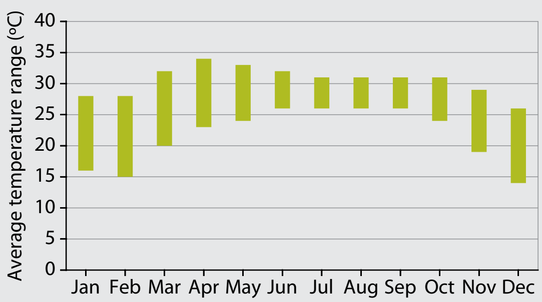
In this case we know that only one individual entry is incorrect and that the totals are correct. All we have to do is check each total in turn. Looking at the column totals, we find that the first two (167 and 207) agree with the entries by adding the three numbers above them. However, the third one does not agree with the entries above it, which add up to 121.

We are not quite there yet; the wrong number could be any of those in the 'Jogging' column. Repeating the procedure for the row totals, we find that the second row (age 16–20) adds up to 223. The error must be where the wrong totals cross over. The incorrect figure is the '45' in the age 16–20 Jogging entry. It should be 54; we know it should be 9 higher because both the row and column totals were 9 too low. A transposition of the digits is a common error when entering data.

Activity

Graphical information

The graph shows average monthly temperatures for Bangladesh. The lower end of the bar shows the average of the lowest daily temperatures during the month and the top end of the bar shows the average of the highest daily temperatures during the month.



What is the difference between the lowest average temperature and the highest average temperature during the year?

Commentary

There are two skills involved here. Firstly one must understand the verbal description of what the graph means. Then, based on the question, one must interpret the graph in the required way.

The solution is quite simple and involves subtracting the lowest point on any of the bars from the highest point on any of the bars. These values are (reading as accurately as possible) 14° and 34°, so the total range is 20°.

Activity

Verbal information

In an inter-school hockey knockout competition, there are initially 32 teams. Teams are drawn by lots to play each other and the winner of each match goes through to the next round. This is repeated until there are only two teams left, who play each other in the final, and the winner gets a cup. Matches have two halves of 20 minutes each. If the teams are level at the end of normal play, two extra 10-minute periods are played. If it is still a draw, teams take penalty shots at goal to decide the winner.

Chorlton High were eventually knocked out in the semi-final (without extra time). In one of the earlier rounds they had to play the two extra periods before they won.

For how long in total had Chorlton High played when they were knocked out?

Commentary

There is a considerable mass of information here, all presented as words. It must be read carefully. The method of solution is not difficult; the skill lies in choosing the correct pieces of information and using them appropriately. First we need to know how many matches Chorlton High played. The first round had 32 teams; subsequent rounds had 16, 8 and 4, which is when they were knocked out – so they played 4 matches.

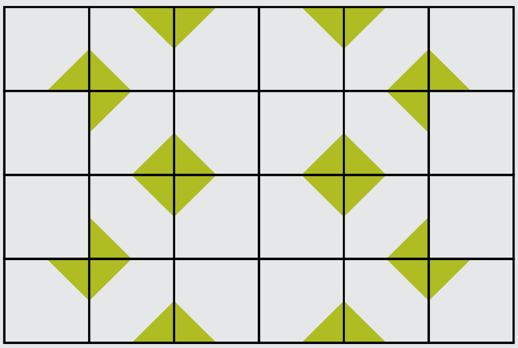
Next we need to know how long each match lasted. This is 2 × 20 minutes = 40 minutes. We must also note that Chorlton High played the two extra periods in one match – a total of 20 minutes. So their total playing time was 4 × 40 minutes + 20 minutes = 180 minutes or 3 hours in total.

Activity

Pictorial information

The picture shows a tiled floor where 24 individual tiles with different printing on them are used to make up the overall pattern.

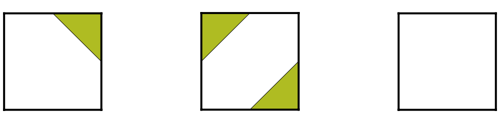
How many different patterns of tile are needed to make up the overall pattern?



Commentary

Solving this requires a systematic evaluation of the picture. We not only need to identify the apparently different tiles, but also to look at how tiles can be used in different orientations. The procedure is to eliminate tiles one by one, noting each time whether a new tile is needed or whether one we have already seen can be used in a different orientation.

In fact, only three tiles are needed:

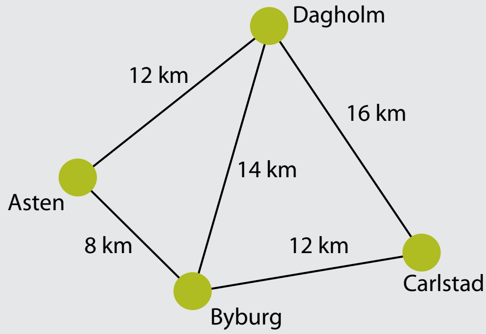


If you did not get the right answer, can you now convince yourself that three tiles as shown is correct?

Activity

Diagrammatic information

The map is a simple representation of the only roads joining four towns.



I live in Asten and wish to visit a friend in Carlstad. I normally go via Byburg but have discovered (before setting off) that the road between Byburg and Carlstad is blocked by an accident. How much will this add to my journey?

Commentary

To solve this you need to look at the length of the normal route and then consider the alternatives. The journey is normally 8 km + 12 km = 20 km. If I cannot use the road between Byburg and Carlstad, the only alternative (shown on the map) is via Dagholm. The distance will be 12 km + 16 km = 28 km. This is 8 km more than my normal route.

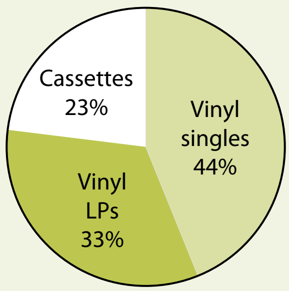
Summary

- In this chapter we have seen how data can be presented in several different forms.
- We have also seen the importance of reading the question carefully to ensure that the correct pieces of data are extracted from the information given and used correctly.

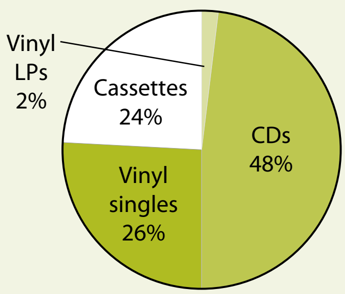
End-of-chapter assignments

1 Using the data in the first example above (tabular information, page 86), draw a graph of an appropriate type showing the proportion of types of exercise regularly taken by the 16–20-year-olds in the sample.

2 The pie charts illustrate the change that the introduction of the CD in 1985 had on the recorded-music market. Total annual sales of all types of recording in 1984 were 170 million and in 1994 they were 234 million.



1984



1994

What, approximately, happened to the actual annual sales of vinyl singles between 1984 and 1994?

A They fell by 14 million.
B They fell by 5 million.
C They were unchanged.
D They rose by 17 million.
E They rose by 64 million.

3 The table below shows the finishing positions in the Contrey handball league. The five teams play each other once each. Three points are awarded for a win and one for each team in a drawn match.

Team	Points
Alency	8
Bresville	7
Argest	5
Euroland	4
Saint Croix	2

How many of the games were drawn?

4 A carpenter is fitting some bookcases to an alcove, using as much of the space as possible from floor to ceiling, a height of 2.5 m. The books to be fitted into the shelves are 210 mm high and a gap of at least 30 mm is necessary above each book so they can be removed. The shelves are 20 mm thick. The alcove is 1.2 m wide. The bottom shelf should not be less than 300 mm from the ground, as the house-owner cannot bend down easily. How many shelves can be fitted into the alcove?

Answers and comments are on page 317.

3.4 Processing data

In the previous chapter we looked at solving problems by selecting the correct items of data from various sources and using them in the correct way to produce a solution. This chapter considers problems where the required data is clearly given (i.e. there is no ambiguity about which pieces of data to use). The problems covered here involve using the data in the correct way to find the solution to the problem. The activity below illustrates this.

Activity

Luiz and Bianca are brother and sister and go to the same school. Luiz walks to school using a footpath, a distance of 900 m, and he walks at 1.5 m/s. Bianca cycles to school along the roads, a distance of 1.5 km, and she cycles at 5 m/s. They both plan on arriving at school by 8.55 a.m. Who leaves home first and by how much?

A Bianca, by 5 minutes
B Luiz, by 5 minutes
C They leave at the same time
D Bianca, by 10 minutes
E Luiz, by 10 minutes

Commentary

The skill in this question is to use the correct pieces of information appropriately and at the right time in the calculation. There are five relevant pieces of data (the two distances, the two speeds and the fact that they arrive at the same time). It is quite clear that the method of solution is to calculate each of the journey times, so in this case there is no method to find. Problems where the method is not clear will be discussed in the next chapter.

Luiz walks 900 m at 1.5 m/s, so this takes him 900 ÷ 1.5 = 600 seconds or 10 minutes. Bianca cycles 1.5 km (1500 m) at 5 m/s, which takes her 1500 ÷ 5 = 300 seconds or 5 minutes. As Luiz takes 5 minutes more, he must leave home 5 minutes earlier, so B is correct. (If you are unsure about relating speed, distance and time, see the advice below.)

This is a multiple-choice question, a type you will see frequently in thinking skills examinations. Some of the activities in this section of the book have multiple-choice answers, as in the examinations. However, many have 'open' answers, where you are asked, for example, to give a numerical solution. This is, in many ways, a better way to learn how to do the questions – you will be able to select the correct multiple-choice answers more easily if you can do the question without needing to know possible answers. If you can come to the solution without looking at the options and then check that your solution is one of the options, this is safer and often quicker than checking the options against the data given. In the case of the example above, it is much better to work out the answer first.

Speeds, distances and times

Many problem-solving questions involve calculating one of the variables speed, distance or time from the other two. If you are uncertain how to do this, the formulae below give the method:

speed = distance/time
distance = speed × time
time = distance/speed

If you are worried about remembering these, there is an easy way. Speed is measured in units such as km/h or m/s. This is a distance divided by a time, which is equivalent to the first formula – the others can be worked out from it.

Always check that you use consistent units in calculations. If the speed is in metres per second and the time is given in minutes, you must first convert the time to seconds (or the speed to metres per minute) before applying the formula.

Also, consider whether your final answer is a reasonable number. If, in the example given above, you had divided Bianca's distance in km by the speed in m/s you would have an answer of 1.5 ÷ 5 = 0.3. A value of 0.3 seconds or minutes would clearly be ridiculous for cycling 1.5 km (and hours did not appear in the calculation) so it is obvious that something is wrong.

Care must be taken when calculating average speeds. Say, for example, that a river ferry travels between two towns 12 km apart, travelling at 4 km/h upstream and 6 km/h downstream. It might seem that the average speed will be 5 km/h, but this is wrong. In order to calculate the average speed, you must divide the *total distance* by the *total time*. In this case, the ferry takes 3 hours upstream and 2 hours downstream – a total of 5 hours. The average speed is, therefore, 24 ÷ 5 or 4.8 km/h.

Activity

Cheng has a garden pond, which he tops up at the beginning of each week from a 200-litre water butt, which is, in turn, filled by rainwater from part of his roof. At the beginning of the summer both the pond and the water butt are full. The average weekly summer rainfall where he lives is 5 mm. The part of his roof from which he collects rain has an area of 6 m^2. He uses 60 litres per week on average to top up the pond.

For how many weeks can Cheng expect to have enough water in the butt to top up the pond fully?

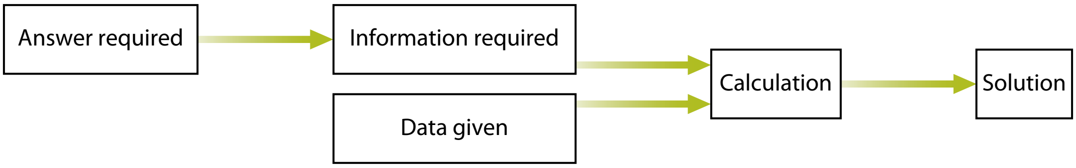
Commentary

This question has a lot of data presented verbally. We must identify the important variables to calculate in order to answer the question. This is done by working backwards: we need the number of weeks the water in the butt will last. This, in turn, depends on the amount of water in the butt at the start (already known) and the average loss of water per week. The average loss of water per week is the amount collected minus the amount used (which we also know). Thus, the only unknown is the amount collected. This is what we need to calculate first.

The weekly rainfall is 5 mm, which is collected on an area of 6 m^2. In consistent units (using metres) the volume collected is 6 m^2 × 0.005 m of rain or 0.03 cubic metres. A cubic metre is 1000 litres, so the volume collected is 30 litres.

As Cheng uses 60 litres per week and collects 30 litres, he loses a net 30 litres each week. Thus his 200-litre butt will last for 5 weeks; at the beginning of the sixth he will have only 50 litres, which is not enough to top up his pond.

This question illustrates one method of approaching problem-solving questions. We know what answer is required, so which pieces of information do we need to come up with to get that answer? This indicates which calculations need to be made on the given data. It may be represented as shown on the following page.



Summary

- We have seen that, for some problems, the important data is given clearly and unambiguously: the skill in finding the correct solution is to use the given data in the correct way.
- We have learned that it can sometimes be useful to work backwards from the answer to identify what needs to be calculated.

End-of-chapter assignments

1 A department store is having a sale. The advertising hoarding for the sale is shown below:

> 30% off if the marked prices total more than $100.
>
> If you buy three items, you get the least expensive free.

This is a bit ambiguous. I don't know whether they give me the free item before they calculate whether it is over $100, or after. Suppose I buy three items at marked prices of $40, $40 and $30. What could I expect to pay under either interpretation?

2 Sylvia Okumbe is trying to break her national record of 14 minutes 35 seconds for running 5000 m (12½ laps of the track). Her average time per lap for the first 5 laps is 1 minute 13 seconds. What average lap time does she need for the remaining 7½ laps?

3 A pancake stall sells sweet pancakes and savoury pancakes. The savoury pancakes can have three toppings (eggs, ham, tomato) which may be used in any combination. The sweet ones come with orange, lemon or strawberry jam with either ice cream or fresh cream. How many combinations does the stall sell?

4 I am going to change my phone contract. The monthly contract I am considering costs $30 per month with 75 minutes of free calls and 100 free text messages. Additional calls cost 10¢ per minute and additional text messages 10¢ each. An alternative is 'Pay as you go' which has no monthly charge but all calls cost 30¢ per minute and texts cost 10¢ each.

I typically make 100 minutes of calls and 60 text messages each month. Which would be the better contract for me and by how much?

Answers and comments are on page 317.

3.5 Finding methods of solution

The previous chapter dealt with problems for which the method of solution was relatively easy to find. In this chapter we are looking at problems where the primary skill in solving them is to develop a method of solution. The way of proceeding to an answer in some problems may not be clear:

a either because it is necessary to find an intermediate solution first,
b or because we need to work simultaneously forward from the data (to identify what can be calculated) and backwards from the required answer (to identify what needs to be calculated).

Having a strategy for approaching such problems is important. In particular, it can be very useful if you have seen a problem of a similar sort before, which you know how to approach – this is where experience in tackling problem-solving questions can be invaluable. Sometimes it may be necessary to try different ways of approaching the problem; it is important to realise quickly if your line of attack is being unsuccessful.

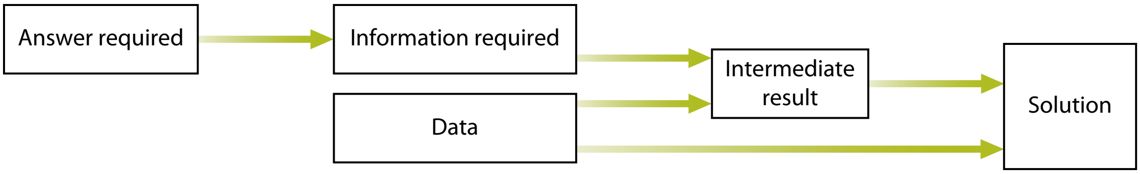
One strategy that can help to solve problems when you are not clear how to proceed is to analyse the problem:

- organise the information you are given
- write down or underline those pieces of information which you feel are important
- simplify (reject unimportant information)
- look at the question and decide what pieces of information could lead to the answer
- make a sketch, list or table.

Sometimes, intermediate answers are necessary in order to proceed to the complete solution. This may be regarded as similar to the identifying of intermediate conclusions in Chapter 2.6. The solution of a problem can be like an argument that first leads to one conclusion, which then, possibly using further information, proceeds to the final conclusion.

This may be illustrated as in the diagram below. Here, the calculation steps are represented by the arrows. Not all these processes are used in all problem solutions.

A problem that may be solved using an intermediate result is given in the example on the following page. This is similar to the question in the previous chapter in that it involves distances, speeds and times but, because of the nature of the question, the method of proceeding is less obvious.



Activity

Amy and her brother David live 400 km apart. They are going to have a week's holiday by exchanging houses. On the day they are starting their holiday, Amy leaves home at 8 a.m. and David at 10 a.m. They both drive at 120 km/h on a motorway that travels directly between their homes.

At what time do they pass each other on the road?

Commentary

From the data given, it is easy to find out when they both arrive at their destinations, but finding when they cross is not so straightforward. The problem can be made much simpler by using an intermediate step. First calculate where Amy is when David leaves. She has been travelling for 2 hours, so she has covered 240 km – that is, she is 160 km from David's house. The problem is now quite easy. At 10 a.m. they are 160 km apart and rushing towards each other at a joint speed of 240 km/h. Therefore, they will meet 40 minutes later (160 km/240 km per hour is $\frac{2}{3}$ hour or 40 minutes). The time they pass each other is 10.40 a.m.

If we had been asked to find the place where they pass, the passing time could have been used as a second intermediate value. David travels 120 km in an hour. 40 minutes represents $\frac{2}{3}$ of this, or 80 km, so they cross 80 km from David's house.

In this case the numbers were very easy but the same method of solution could have been used whatever the distances, times and speeds. The method of solution, which was not immediately obvious, became easier by using the intermediate step.

Activity

Petra's electricity supply company charges her a fixed monthly sum plus a rate per unit for electricity used. In the most expensive quarter last year (January to March), she used 2000 units and her bill was $250. In the least expensive quarter (July to September), she used 600 units and her bill was $138.

She is now adding extra insulation to her home which is expected to reduce her overall electricity consumption by 25%. What can she expect her January to March bill to be next year (if there are no increases in overall tariffs)?

Commentary

This is another problem where an intermediate calculation is necessary. In order to calculate Petra's bill, we need to know the monthly charge and the rate per unit. We know the difference between the two quarters' bills – this difference is only due to the reduced consumption, so 1400 fewer units saved $112. This means units cost 8¢ each.

The three-monthly fixed charge, therefore, is:

$250 – 2000 × 8¢ = $250 – $160
= $90 (or $30 per month)

If Petra reduced her January to March consumption by 25%, this would then be 1500 units, so her bill would be:

$90 quarterly charge plus 1500 × 8¢
= $90 + $120 = $210

In fact, the quarterly charge does not have to be calculated, only the unit rate. The entire process of solving this problem could be speeded up by simply recognising that the relevant three-month bill would be reduced by 500 × 8¢ or $40.

Another way of approaching problems is to lay out the information in a different way. This is especially so when the information is given verbally – and therefore the connection between the different pieces may not be immediately obvious. Consider, for example, the following problem:

Activity

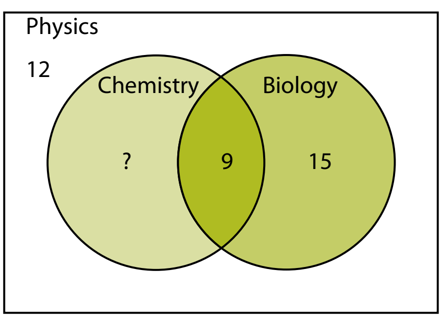
In a group of 45 students at a school, all students must study at least one science. Physics is compulsory, but students may also opt to study chemistry or biology or both. 9 students take all three sciences. 24 take both physics and biology (with or without chemistry). 12 students take only physics.

How many are studying chemistry and physics but not biology?

Commentary

With a bit of clear thinking, this may be solved in a direct fashion by making an intermediate calculation (those not studying biology). Since all students take physics, the situation is simplified. 24 study biology and there are 45 in total, so 21 do not study biology. These 21 comprise those studying physics alone and those studying both physics and chemistry. However, we know that 12 take only physics, so 9 must take physics and chemistry but not biology.

Although this appeared to be an easy calculation, the method of approach was not obvious. The situation can be made a lot easier by using a Venn diagram:



In this diagram, the outer box represents all the students (the universal set); in this case they all take physics. The left-hand circle represents those taking chemistry (and physics) and the right-hand circle represents those taking biology (and physics).

We know that the number taking only physics is 12; this is represented by the area outside both circles. Those taking all three sciences are represented by the intersection of the two circles and shown as 9. The number taking both physics and biology is 24; of these 9 take all three, so 15 take only physics and biology. This is shown by the outer section of the right-hand circle. We can now calculate the number in the area marked by the question mark, as this must be all the students in the class minus the numbers in the other three areas, i.e. 45 – 12 – 9 – 15 = 9. This is the required answer: the number studying chemistry and physics but not biology is 9.

Interestingly, the number studying all three was not used in the original calculation. It is, in fact, not needed to solve the problem. We used it in the Venn diagram solution so we could calculate the numbers in all the areas on the diagram.

Graphs, pictures and diagrams can often be useful in solving problems as they help to clarify the situation and represent the numbers used in a more digestible manner. This is covered in more depth in Chapter 6.2.

The activity above shows that problems can often be solved in more than one way. It is important to keep the mind open to alternatives and not always to pursue a method which is not apparently leading to a solution.

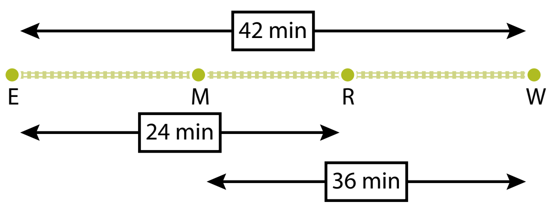
Activity

There is one railway on the island of Mornia, which runs from Enderby to Widmouth. There are two intermediate stops at Maintown and Riverford. The trains run continuously from one end to the other at a constant speed, stopping for three minutes at each station. From departing Enderby to arriving at Widmouth takes 42 minutes. From Enderby to Riverford takes 24 minutes. From Maintown to Widmouth takes 36 minutes.

How long does it take from Maintown to Riverford?

Commentary

A diagram makes this problem much easier to solve.



It can now be seen that if we add the time from E to R to the time from M to W, we get the time from E to W plus the time from M to R. The times from E to R and M to W each include one 3-minute stop, whilst the time from E to W includes two 3-minute stops, so when we subtract the EW time from the sum of the ER and MW times, the stops cancel out. Thus the time from M to R is 24 + 36 – 42 = 18 minutes.

We can now look further at an extra useful element in the solution of problem-solving questions – that is, checking that we have the right answer.

If ER takes 24 minutes (including a 3-minute stop) and MR takes 18 minutes (no stop), then EM (no stop) must take 3 minutes (24 – 18 – 3). Similarly, RW must take 36 – 18 – 3 = 15 minutes. We now have the times for all the sections:

EM = 3 minutes
Stop at M = 3 minutes
MR = 18 minutes
Stop at R = 3 minutes
RW = 15 minutes

The total of all these is 3 + 3 + 18 + 3 + 15 = 42 minutes as expected.

Check that the times from E to R and M to W agree with the answer of 18 minutes from M to R as given.

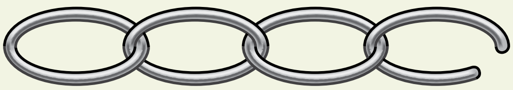
This example once again shows that representing the data in a different way can lead to a simple method of solving a problem that at first appears unclear.

Summary

- We have learned the importance of finding methods of solution for problems for which the way of proceeding to an answer is not necessarily obvious.
- We have found that looking for intermediate results may help to lead to the final answer.
- We also looked at the value of alternative ways of presenting data and considering more than one way of solving problems.

End-of-chapter assignments

1 Aruna's neck chain has broken into two parts. She has lost the broken link and is having it repaired by a jeweller who will open one of the remaining links and use it to rejoin the chain. The chain is made from metal 2 mm thick and each of the broken pieces has a fitting at the end used for closing the chain which each adds 1 cm to the total length.



One of the broken pieces is 34.2 cm long and has 10 more links than the other, which is 26.2 cm long. Excluding the fittings at the ends, how many links will there be in the complete chain?

2 The distance from Los Angeles to Mumbai is 14,000 km. Flights take 22 hours, whilst the return flight from Mumbai to Los Angeles takes only 17 hours because of the direction of the prevailing wind. Assuming the aeroplane would fly the same speed in both directions in still air, what is the average wind velocity?

3 From my holiday cottage by the sea I can see two lighthouses. The southern flashes regularly every 11 seconds. The northern lighthouse, after its first flash, flashes again after 3 and 7 seconds. The whole cycle repeats every 17 seconds.

They have just flashed at exactly the same time, the northern one having just started the cycle described above.

When will they both flash again at exactly the same time?

4 The 23 members of a reception class in a school have done a survey of which cuddly toys they own. Pandas and dogs are the most popular, but 5 children have neither a panda nor a dog. 12 have a panda and 13 have a dog. How many have both a dog and a panda?

Answers and comments are on pages 317–18.

3.6 Solving problems by searching

Some problems may not always be resolved by using direct methods of calculation. Sometimes, problems do not have a single solution, but many, and we need to find one that represents a maximum or minimum (for example the least cost or shortest time for a journey). In these cases we need to have a systematic method of evaluating the data to come up with all (or at least the most likely) possibilities. This is called a 'search'. Once again, with this type of question, it is important to have a way of checking that the final answer is correct.

Here is an example of a problem that requires a search.

Activity

Amir is helping with a charity collection and has gathered envelopes containing coins from a number of donors. He notes that all the envelopes contain exactly three items but some of them contain one, two or three buttons instead of coins. All the coins have denominations of 1¢, 5¢, 10¢, 25¢ or 50¢.

What is the smallest amount of money that is not possible in one of the envelopes?

Commentary

The easiest way to approach this question is to list the possibilities in a systematic order. We know envelopes can contain 0, 1, 2 or 3 coins. The possibilities with one coin (and two buttons) are: 1¢, 5¢, 10¢, 25¢ or 50¢.

That was the easy part. With two coins (and one button), we need to be a little more careful. First consider that the first coin is 1¢, then look at all the possibilities for the second. We can then continue with the first coin as a 5¢ in the same manner (we do not need to consider repeats). The possibilities are:

1¢ + 1¢, 1¢ + 5¢, 1¢ + 10¢, 1¢ + 25¢, 1¢ + 50¢, then
5¢ + 5¢, 5¢ + 10¢, 5¢ + 25¢, 5¢ + 50¢,
10¢ + 10¢, 10¢ + 25¢, 10¢ + 50¢,
25¢ + 25¢, 25¢ + 50¢, and
50¢ + 50¢.

Listing all the totals, we have: 2¢, 6¢, 11¢, 26¢, 51¢, 10¢, 15¢, 30¢, 55¢, 20¢, 35¢, 60¢, 50¢, 75¢ and $1.

Finally, we need to list all the possibilities with three coins. This is slightly more difficult. However, we only need to go on until we have found an impossible amount (you may already have spotted it). The possibilities are:

1¢ + 1¢ + 1¢, 1¢ + 1¢ + 5¢ etc.
1¢ + 5¢ + 5¢ etc.

You should have spotted by now that we have not seen the value 4¢ and that all further sums of three coins (anything including a 5¢ or above) will be more than 4¢. So 4¢ is the answer.

This was actually a trivial example used for the purposes of illustration. There is an alternative way to solve this, which also involves a search. This is to look at 1¢, 2¢, 3¢, etc. and see whether we can make the amount up from one, two or three coins. In this case it would have led to a very fast solution, but if the first impossible value had been, for example, 41¢, this second method would have taken a very long time and we might have been unsure that we checked every possible sum carefully.

The method described above is called an 'exhaustive search', where every possible combination is considered. The alternative given in the above paragraph may be called a 'directed search' where we are looking selectively for a solution and will give up the search once we have found one. In the case above, where we were looking for a minimum, we can reasonably start searching from the lowest value up.

A third alternative may be described as a 'selective search'. In this case we are using a partial analysis of the problem to reduce the size of the search, concentrate on certain areas, or to reject unlikely areas. The activity below illustrates this.

Activity

Try repeating this exercise using coins of denominations 1¢, 2¢, 5¢, 10¢, 20¢ and 50¢ and with 1 to 4 coins in each envelope. This is quite a long search. Consider (and discuss with others) whether there are ways of shortening it.

Commentary

If you start this search you will find it takes a very long time. It is difficult to be absolutely systematic (especially when considering all options for four coins). It is also difficult to keep track of all values that have been covered at any point in the search. It is necessary to look for short-cuts, and out of boredom you will probably have done so.

The denominations 1¢, 2¢ and 5¢ in combinations of 1 to 3 coins can make all the values from 1¢ to 10¢. This means that, by adding to the 10¢ and 20¢ coins, all amounts from 1¢ to 30¢ can be made. After 30¢ it is necessary to use both the 10¢ and 20¢ or a 20¢ and 2 × 5¢. The former leaves one or two extra coins, which can make 1¢, 2¢, 3¢, 4¢, 5¢, 6¢, 7¢ but not 8¢. The latter leaves only 1 coin, which cannot be 8¢, so 38¢ is the minimum that cannot be made from 1 to 4 coins. This method involves analysing the problem, which can be a very useful tool in reducing the size of searches.

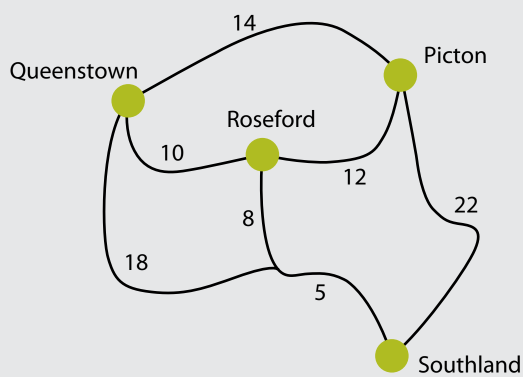
The type of search shown above involves combining items in a systematic manner. Other searches can involve route maps – looking for the route that takes the shortest time or covers the shortest distance, or tables – for example finding the least expensive way of posting a number of parcels.

With all these searches, the important thing is to be systematic in carrying out the search so that no possibilities are missed and the method leads to the goal. The activity below involves finding the shortest route for a journey.

Activity

The map shows the roads between four towns with distances in km.

I work in Picton and have to deliver groceries to the other three towns in any order, finally returning to Picton. What is the minimum distance I have to drive?



Commentary

There are only a small number of possible routes. If these are laid out systematically and the sums calculated correctly, the problem can be solved quite quickly.

The possible routes (with no repeated visits to any towns – you should be able to satisfy yourself that it is never worth retracing your steps) are:

PQRSP
PQSRP
PRQSP
PRSQP
PSRQP
PSQRP

This gives six values. In order to see how they are obtained you may note that there are three pairs, each pair visiting a different one of the towns first (after leaving Picton). The two routes in each pair take the last two towns in opposite orders.

The distances associated with each route are as follows:

59 km
62 km
67 km
62 km
59 km
67 km

Therefore, the minimum distance is 59 km.

If you were particularly astute, you would have noticed that the routes come in three pairs of the same distance (e.g. PQRSP is the reverse of PSRQP so must be the same). This would have saved you half the calculations.

Summary

- We have learned that some problems require a search to produce a solution.
- We have seen the importance of being systematic with a search, in order both to ensure that the correct answer is obtained and to be certain that we have the right answer.
- We also saw that searches do not always have to be exhaustive and how analysis of the problem can reduce the size of the search and time taken.

End-of-chapter assignments

1 The notice below shows admission prices to the Tooney Tracks theme park.

Adult	$12
Child (aged 4–16)	$6
Child (aged under 4)	Free
Senior citizen	$8
Family ticket (for 2 adults and 2 children)	$30
Additional child 4–16 or senior citizen	$5
Additional adult	$10
Family ticket (for 1 adult and 2 children)	$20
Additional child 4–16 or senior citizen	$5
Additional adult	$10

Maria is taking her three children aged 3, 7 and 10 and two friends of the older children (of the same ages) as well as her mother, who is a pensioner. What is the least it will cost them?

2 I recently received a catalogue from a book club. I want to order seven books from their list. However, I noticed that their price structure for postage was very strange:

Number of items	Cost of post and packing
1	45¢
2	65¢
3	90¢
4	$1.20
5	$1.50
6 or more	$3.20

I decide, on the basis of this, that I will ask them to pack my order in the number of parcels that will attract the lowest post and packing charge. How much will I have to pay?

3 Jasmine has been saving all year for her brother's birthday. She has collected all the 5¢ and 20¢ coins she had from her change in her piggy bank. She is now counting the money by putting it into piles, all containing $1 worth of coins. She notices that she has a number of piles of different heights.

If 5¢ and 20¢ coins are the same thickness, how many different heights of $1 pile could she have?

A 5 **B** 6 **C** 10 **D** 11 **E** 20

4 In a community centre quiz evening, teams were awarded five points for a correct answer, no points for no answer, and minus two points for an incorrect answer. The teams marked their own score sheets. I arrived late and the scores after seven questions were shown on the board as follows:

Happy Hunters	28
Ignorant Idlers	18
Jumping Jacks	16
Kool Kats	12
Lazy Lurkers	–1

a One team was clearly not even clever enough to calculate their score correctly. Which one was it?

b Are there any scores, other than those shown above, that would have raised suspicion?

Answers and comments are on pages 318–19.

3.7 Recognising patterns

In Chapter 3.1 we saw that there are three main skills involved in solving problems. We have already dealt with the first two of these (identifying important information and combining pieces of information). This chapter deals with the third skill, that of identifying pieces or sets of information in different forms which are equivalent. In particular, this chapter deals with graphical, verbal and tabular information.

An extension of this skill is to identify possible reasons for variation in data – once again, this springs from past experience as to what causes changes and the types of variation that may be expected. This type of question is dealt with in more detail in Chapter 3.8.

These are best illustrated using examples. The first deals with identifying the similarity between two sets of data.

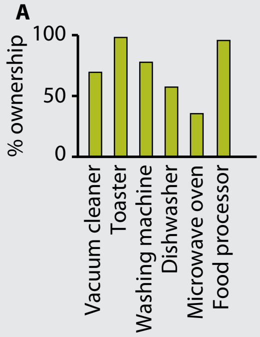
Activity

The table shows the results of a survey into ownership of various household appliances by families who live in a town.

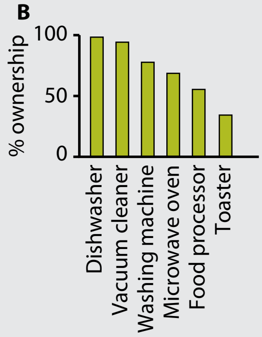
Which of the bar charts accurately represents the data shown below?

Appliance	Dishwasher	Vacuum cleaner	Washing machine	Microwave oven	Food processor	Toaster
% ownership	68	98	77	54	34	92

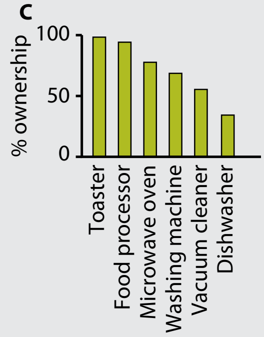
A



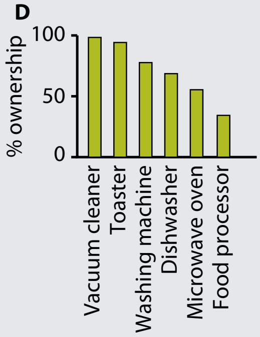
B



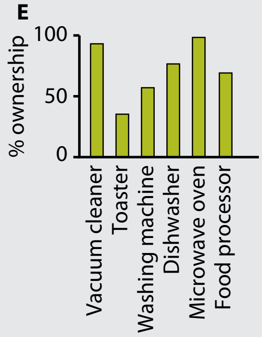
C



D



E



Commentary

This question is actually quite easy. It is only a matter of being careful and matching appliance to bar length correctly. The main complication (and this is a potential trap for those who don't look at the question and the graphs carefully) is that the order of the appliances in the graphs is different in some cases from the order in the table. Also, the exact heights of the bars cannot always be read accurately enough at the scale on which the graphs are drawn, so it is necessary to look at the relative heights of the different bars.

In fact D is the correct graph. The appliances have been put into order by their percentage ownership. A has the appliances ordered as for D but the bars are in the order of the table. The other graphs have similar errors – you might like to identify the error in each case.

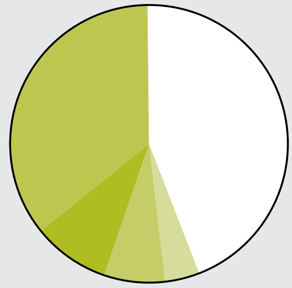
Activity

This activity reverses the skill shown above: the graph is given (a pie chart in this case) and the cost structure it represents has to be identified.

A student is drawing pie charts to show how the various elements of the cost of fuel contribute to the total price in various countries. The data she is using is shown below, with the prices in local currencies.

	Sudaria	Idani	Anguda	Boralia
Crude oil	0.70	18.68	0.40	0.50
Refining	0.02	4.67	0.02	0.02
Wholesale	0.09	3.63	0.05	0.14
Retail	0.06	2.08	0.06	0.05
Tax	0.50	22.84	0.80	0.34
Total	1.37	51.90	1.33	1.05

She drew one pie chart last night, but has not labelled the segments and cannot remember which country it represents. The pie chart is shown below.



Which country is it?

Commentary

The data expressed in local currency is not very useful for a direct comparison. It is easier if the costs are all expressed as percentages so that the appropriate components can be compared. The table is repeated below with the costs as percentages of the totals for each country.

	Sudaria	Idani	Anguda	Boralia
Crude oil	51.09	36.00	30.10	47.62
Refining	1.46	9.00	1.50	1.90
Wholesale	6.57	7.00	3.76	13.33
Retail	4.38	4.00	4.36	4.76
Tax	36.50	44.00	60.27	32.38

In the pie chart, the largest segment is just under half the area. It could, therefore, only be tax in Idani or crude oil in Boralia. We cannot easily distinguish which as the difference in the second-largest segments is not very great. We must, therefore, look at the smallest three segments. Boralia has one (wholesale) three times the size of either of the other two, but in the pie chart they are much closer than this, so the answer must be Idani.

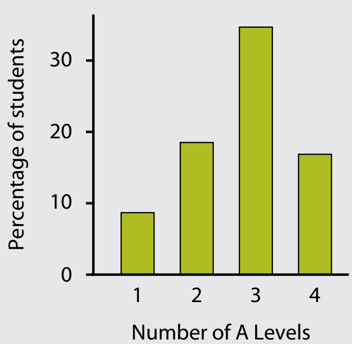
This could, in fact, have been solved without going to percentages by looking at the relative sizes of the components in the table for each country. It would have been quicker to do this, but would have taken more mental arithmetic.

Activity

The table shows the results of a questionnaire, asking the five colleges in a town the proportion of students taking 1–4 A Level subjects.

College	Percentage of students taking number of A Levels shown			
	1	**2**	**3**	**4**
Abbey Road	13	25	42	20
Barnfield	5	18	55	22
Colegate	24	36	28	12
Danbridge	16	18	61	5
Eden House	10	14	48	28

The local newspaper (forgetting that there might be different total numbers of students in the five colleges) just added the numbers together and divided by five to produce a percentage graph for the town as a whole. However, they forgot to add in the data for one college so their percentages did not add up to 100.



Which one did they forget?

Commentary

This question is a little more difficult than some we have seen so far. There are several ways to approach it. We can note that if we knew the actual averages for the four colleges the newspaper *did* include, it might be possible to see if these averages disagreed with an estimated average for the five colleges, and the direction of the error would give some indication of which one was forgotten.

Looking at the 'averages', the approximate values (we have to estimate these from the graph) are 9, 19, 35 and 16 respectively for 1, 2, 3 and 4 A Levels. Multiplying these by $\frac{5}{4}$ (to correct for the fact that they were divided by 5 instead of being divided by 4), we get (approximately): 11, 24, 44 and 20.

If we were being very systematic, we could now compare these with all sets of four averages, but it would take a long time. Instead, let us note that the 11 looks a little low for the average of 1 A Level, as does 24 for the average of 2. 44 for the average of 3 looks very low and 20 for the average of 4 looks far too high. From this, we may suspect that Danbridge has been missed as it is higher than the others for 3 A Levels and lower for 4.

We can check this by averaging one of the columns for the other four colleges (preferably use 3 or 4 A Levels as they look to have the biggest discrepancy) and comparing the results – try this for yourself and see whether you can confirm that Danbridge is the college whose results are missing.

Summary

- We have learned how data may be represented in more than one way and the importance of systematic comparisons between two sets of data in ascertaining that they are the same.
- We saw that reading graphs and tables carefully is necessary in order not to make errors in identifying similarities.

End-of-chapter assignments

1 Look in newspapers (business pages are often useful) or on the internet to find examples of numerical data in various forms (verbal, graphical, tabular). Express the data in a different form. Consider which form makes the data clearest to understand.

2 Four-digit personal identification numbers (PINs) are used to withdraw cash from banks' machines using plastic cards. It can be very difficult to remember your personal number. I have a method of remembering mine. It is the two numbers of my birth date (i.e. the date in the month) reversed, followed by the two digits of my month of birth reversed (using a zero in front if either is a single number so, for example, May would be 05).

Which of the following could not be my PIN?

A 3221 **B** 5060 **C** 1141
D 2121 **E** 1290

3 Four house teams play each other in a school basketball league. The scoring system gives three points for a win, one for a draw and none for losing.

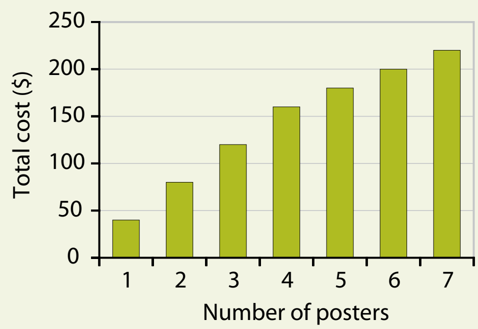
They all play each other once, and the league table before the last round of matches is as follows:

	Played	Won	Drawn	Lost	Points
Britons	2	0	2	0	2
Danes	2	1	1	0	4
Normans	2	1	0	1	3
Saxons	2	0	1	1	1

Which of the following points columns are possible after the last two matches are played? (Hint: you first need to decide which games have already been played, so you know what is left.)

	A	B	C	D	E
Britons	5	3	5	5	2
Danes	5	4	7	5	5
Normans	3	4	3	4	6
Saxons	2	4	1	1	2

4 The graph shows the charges made by a printing company for making various numbers of posters.



Which of the following pricing structures would give the graph shown?

A $30 per poster
B $50 set-up charge + $20 per poster
C $40 per poster for the first four, any extra $20 each
D $30 set-up charge, $30 per poster for the first four, any extra $20 each

Draw the graphs for the other price structures.

Answers and comments are on page 319.

3.8 Hypotheses, reasons, explanations and inference

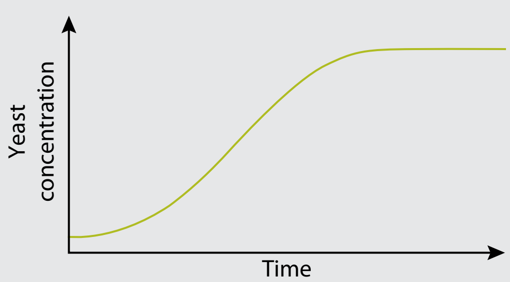
In the introductory chapters we saw that problems involving making inferences from data or suggesting reasons for the nature of variations in the data may appear in either the critical thinking or the problem-solving sections of thinking skills examinations.

Such examples are usually based on quantitative (numerical or graphical) data and may arise from such areas as finance or science. They require analysis of the data given in order to reach some conclusions that may be drawn from the data or to suggest reasons for the nature of the data.

The example below is based on a scientific scenario. While this requires a little understanding of basic scientific concepts, most of the skills involved in coming to a solution depend on clear, logical thinking.

Activity

The graph shows the results of an experiment to determine the growth of a culture of yeast in a nutrient medium. The liquid containing the nutrient was made up and a small amount of yeast introduced. At regular intervals afterwards, the solution was stirred, a small sample taken and the concentration of yeast measured. The graph represents a smooth line drawn through the results.



Which of the following explanations are consistent with the shape of the curve? (Identify as many as apply.)

A Yeast cells divide when they have grown enough, so grow exponentially if they have enough nutrient.

B The rate of increase of yeast cells depends only on the amount of nutrient.

C Eventually, the growth of yeast cells is limited by lack of nutrient.

D Yeast cells die when there is insufficient nutrient.

E The shape of the curve is explained by a linear growth in yeast and a linear decrease in nutrient.

Commentary

Looking at the statements in turn:

A This statement explains the initial increase in growth rate – the increase looks exponential (increasing in size at a constantly growing rate).

B This statement would not explain the initial growth – it would start at a higher growth rate, which would then decrease all the time.

C This statement would explain the drop to zero growth after a time, linked to a lack of nutrient.

D There is no indication of death; in that case the population would fall.

E If both processes were linear (resulting in straight-line graphs), a combination of them would also result in a straight-line graph.

So A and C are the explanations for the shape of the curve.

The activity below is firmly in the Cambridge Thinking Skills syllabus category of 'suggesting hypotheses for variations'. You are given a scenario incorporating numerical data, and asked which, of a number of possible situations, could explain the nature of the data.

Activity

Nikul runs exercise classes at his local gym, and gets there each day by train and bus. Classes start at different times each day, but always either on the hour or at half past the hour. He always gets to the railway station 45 minutes before he is due to start teaching and the train journey takes 20 minutes, after which he takes a bus to the gym, which takes 10 minutes. Trains leave every 20 minutes, starting on the hour. Some days Nikul finds that he gets to work 5 minutes early. On all the other days he finds that he gets there 5 minutes late.

Which one of the following could explain the times that Nikul arrives at the gym?

A The buses leave at 5 and 35 past each hour.

B The buses leave at 15 and 45 past each hour.

C The buses leave at 25 and 55 past each hour.

D The buses leave at 5, 25 and 45 past each hour.

E The buses leave at 15, 35 and 55 past each hour.

Commentary

Nikul arrives at the station at either 15 or 45 past the hour. Therefore, he takes the train either on the hour or at 20 past the hour. He gets to the bus stop at 20 or 40 past the hour. Buses at 5, 25 and 45 past the hour would therefore fit the requirements:

- He would get the bus at 25 past the hour if he arrived at 20 past. This would mean that he arrived at work 5 minutes late.
- He would get the bus at 45 past the hour if he arrived at 40 past. This would mean that he arrived at work 5 minutes early.
- He would never use the bus at 5 past the hour, so it doesn't matter that this one doesn't fit the arrival times.

The correct answer is D. It is also illustrative to see why the wrong answers do not work:

A Buses at 5 and 35 past the hour would always get Nikul to work on a quarter hour which could not be 5 minutes early or late.

B Buses at 15 and 45 past the hour would mean that Nikul was always 5 minutes early.

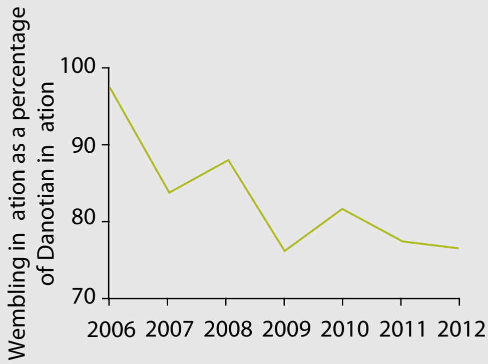
C Buses at 25 and 55 past the hour would mean that Nikul was always 5 minutes late.

E Because Nikul arrives on the train at 20 or 40 past the hour, he would be getting the 35 or 55 past the hour bus. The bus at 35 past the hour would get Nikul to work at 15 or 45 past the hour which is neither 5 minutes early nor 5 minutes late.

Longer questions at A Level can involve analysing quite complex data and determining what conclusions may be drawn from it. The activity below is of this type. It looks at identifying reasons for variations in data.

Activity

The graph shows the inflation rate in the province of Wembling as a percentage of inflation in the country of Danotia as a whole over the period from 2006 to 2012.



Which of the following is the most plausible explanation for the variations shown in the graph?

- **A** Danotian inflation has been high over the period shown.
- **B** Danotian inflation has risen over the period shown.
- **C** Food prices in Wembling have risen less than in Danotia as a whole.
- **D** Food prices in Wembling are subject to higher seasonal fluctuations than in Danotia as a whole.
- **E** The inflation rate in Wembling is falling due to high unemployment.

Commentary

Again we will look at these five answers in turn. It is important to remember that the graph represents the inflation in Wembling as a percentage of the inflation in Danotia, not the *actual* inflation rate in Wembling.

- **A** The graph represents only the ratio between Wembling and the whole of Danotia; high inflation in Danotia cannot explain the shape of a graph of the ratio.
- **B** A rise in Danotian inflation would only cause the fall in the ratio shown if the rise in inflation in Wembling was smaller or negative: we know nothing about this.
- **C** If food prices rose less in Wembling than in Danotia as a whole, this could explain why the ratio fell – even if inflation in Wembling was rising.
- **D** Seasonal fluctuations would only manifest themselves within a year, not between years.
- **E** Even if the inflation rate in Wembling is falling, we know nothing about the inflation rate in the whole of Danotia, so we cannot conclude that the ratio would fall.

Thus C is the only reasonable answer. The others depend either on reading the graph incorrectly or reading more into the graph than we can safely conclude. This illustrates the importance of reading and understanding the information given (both verbal and in other forms) and of reading the question correctly. Beyond that, the deductions that can be made follow from the application of correct logic.

As in the previous chapter, solving these two types of problem also depends on the skill of recognising an identity between data presented in two forms. As with the other skills described here, this comes with practice and it can be useful to look at data in newspapers to see how they are presented and to consider whether they are always presented in the clearest way.

The next activity uses logic based more on manipulating numbers. In this regard, it has elements of the 'finding methods of solution' skill from Chapter 3.5. However, the nature of the question places it closer to the 'suggesting hypotheses for variations' category of questions.

Activity

(Harder task) A tram company runs a service along the seafront from Whitesea to Greylake. Trams leave Whitesea at regular intervals, starting from 9 a.m., taking one hour to reach Greylake. They then turn around and start back 10 minutes after arriving. A driver in the middle of the day, in his journey from Whitesea to Greylake, always sees six trams travelling in the opposite direction. Some of these will have set off from Greylake before he left, and some will have set off after he left.

Which of the following must be true? There may be more than one. If any of the statements are not true, can you correct them?

A It takes six trams to run the service.
B The trams run every ten minutes.
C If I sit on the seafront from 11.15 a.m. to 12.15 p.m., I will see six trams going past.

Commentary

We can test the statements by making a timetable. However, to do this, we need to make an assumption about the departure intervals. It is, in fact, better to carry out a little analysis first.

When a tram driver leaves Whitesea, all those trams which left Greylake up to one hour earlier will already be on their journey. During the hour it takes him to travel to Greylake, more trams will be leaving Greylake. So, in total, he will see all those trams which left up to an hour *before* he left, and also all those which leave up to an hour *after* he left. This means the six trams he sees must have left Greylake in a two-hour period. As they leave at regular intervals, one must leave every 20 minutes, so B is incorrect.

This may be illustrated by looking at some actual times. If trams leave every 20 minutes, they leave Whitesea at 9.00 a.m., 9.20 a.m., 9.40 a.m., etc. They then leave Greylake 1 hour 10 minutes later, at 10.10 a.m., 10.30 a.m., 10.50 a.m., etc. A driver leaving Whitesea, for example, at 11.20 a.m. will see the trams which left Greylake at 10.30 a.m., 10.50 a.m., 11.10 a.m., 11.30 a.m., 11.50 a.m. and 12.10 p.m. – six in total.

Looking at statement A, the first tram leaving Whitesea at 9.00 a.m. reaches Greylake at 10.00, leaves at 10.10 and returns to Whitesea at 11.10, in time to become the 11.20 service. In the meantime, other trams will have left at 9.20, 9.40, 10.00, 10.20, 10.40 and 11.00, i.e. six more, so there must be seven trams to run the service. A is incorrect.

Looking at statement C, if I sit near the midpoint from 11.15 a.m. to 12.15 p.m., I will see the 11.00, 11.20 and 11.40 trams going one way and the 10.50, 11.10 and 11.30 trams going the other way, so I will see six in total. C is correct.

Can you confirm these answers by constructing a timetable?

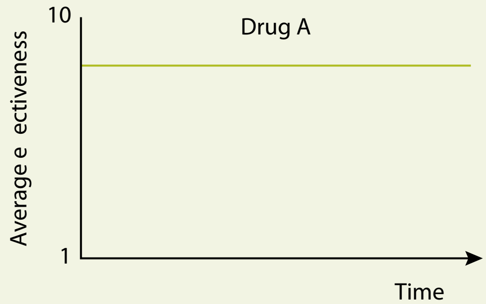
This kind of problem will be revisited in Chapter 6.2 where we look at graphical solutions to problems.

Summary

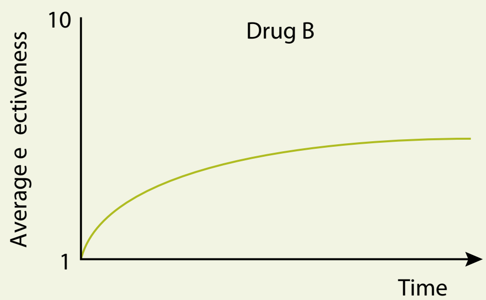
- We have encountered examples where we are required to suggest a hypothesis or a reason for the nature of variation in data.
- The terms hypothesis, reason, explanation and inference are used in exactly the same way in problem solving as in critical thinking, the only difference being that the information given is in the form of data rather than verbal description.
- We have seen that extended examples where more data is supplied can require analysis that may lead to a range of conclusions.

End-of-chapter assignments

1 In order to treat a particular disease effectively, patients are initially given two drugs. Drug A alone has the effect shown on the graph below. (10 = total relief from symptoms. 1 = no relief.)



The effect of drug B showing how it varies with time is shown in the graph below (on the same effectiveness and time scales).



The reason the patients are given two drugs is that drug A, whilst being very effective, has long-term harmful side effects. Drug B takes some time to become effective, and has a lower eventual effect but can be taken indefinitely. The regime used by doctors is to give both drugs starting at the same time, then to withdraw A at a uniform steady rate until, at half the time shown in the graphs, patients have stopped taking it.

Assuming that the effects of the two drugs are independent, what would be the expected shape of the graph of effectiveness for a patient on the regime described?

2 At a local school, 70% of the students studied French and 45% studied German. Which of the following can be confirmed from the information given?

A All students study either French or German.
B $\frac{2}{3}$ of those studying German do not study French.
C 25% of the students study neither French nor German.
D At least 15% of students study both French and German.

3 I was shopping at a market in Northern Bolandia and asked a local how much an orange was. He said that an orange and a lemon together cost \$2. He then further confused me by saying that a grapefruit and a lemon cost \$3 and that all three were different prices.

Based on this rather unhelpful information, which one of the following can be confirmed?

A An orange costs more than a lemon.
B A lemon costs more than a grapefruit.
C A grapefruit costs more than a dollar.
D An orange costs less than a dollar.

4 The Fitland health centre swimming pool is open seven days a week. There are four lifeguards, Liam, Moses, Nadila and Orla, each working four days a week. None works four consecutive days and at least two lifeguards are on duty each day, with three on duty on Saturday and Sunday. We know that:

- Liam doesn't work Monday or Saturday.
- Moses doesn't work Tuesday, Thursday or Friday.
- Nadila doesn't work Wednesday or Friday.
- Orla always works on Thursday.

Which of the following is possible?

A Liam works on Thursday.
B Nadila works on Sunday and Monday.
C Orla works on Monday and Tuesday.
D Three people work on a Monday.

Answers and comments are on pages 319–21.

3.9 Spatial reasoning

Spatial reasoning involves the use of skills that are common in the normal lives of people working in skilled craft areas. Imagine, for example, the skill used by joiners in cutting roof joists for an L-shaped building. This is also necessary for many professionals: the surgeon needs to be able to visualise the inside of the body in three dimensions and, of course, architects use these skills every day of their lives.

Spatial reasoning questions can involve either two- or three-dimensional tasks, or relating solid objects to flat drawings. Thinking in three dimensions is not something that comes easily to all people, but undoubtedly practice can improve this ability.

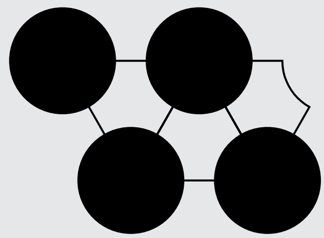
In the simplest sense, a problem-solving question involving spatial reasoning can require visualising how an object will look upside down or in reflection. More complicated questions might involve relating a three-dimensional drawing of a building to a view from a particular direction or the visualisation of how movement will affect the view of an object. This chapter is shorter in terms of description than most of the others but there are more examples at the end; this is an area where practice is more important than theory.

The next example involves a problem-solving task in two dimensions.

Activity

The drawing shows part of the tiling pattern used for a large floor area in a village hall. This is made up of two tiles, one circular (shown in black) and one irregular six-sided tile (shown in white).

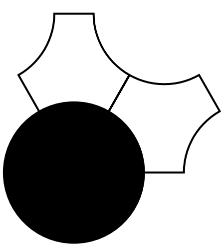
Approximately what proportion of the two tiles will be needed to cover the whole floor?



A three white to one black
B two white to one black
C equal quantities of both
D two black to one white
E three black to one white

Commentary

This seems a relatively simple problem, but the answer is not immediately obvious. This is an example of a tessellation problem. There are various ways to go about solving it – one way is to continue drawing the pattern until you have enough tiles that you can estimate how many of each are needed. Another, more rigorous, method is to identify a unit cell that consists of a number of each tile, which may be repeated as a block to cover the whole area. Such a unit cell for this problem is shown in the drawing.

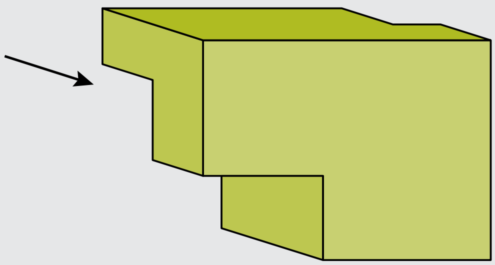


If you now think carefully, you can imagine that this block of three tiles could be repeated over and over again, filling any area without any gaps, to give the pattern shown in the original drawing. So two white tiles are needed for every black tile. B is the correct answer.

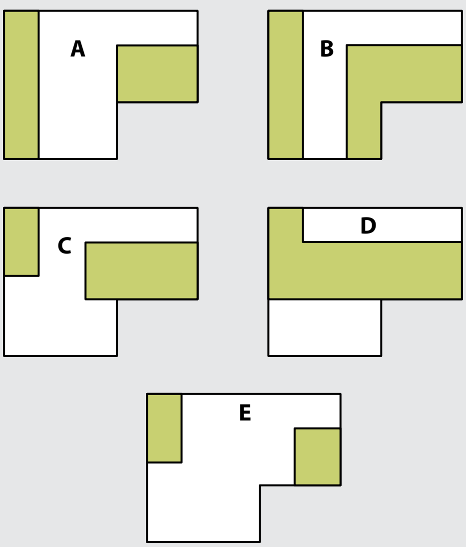
The activity below involves three-dimensional reasoning. Because the drawing is not of a familiar object, there are no short-cuts; you need to work out what the possibilities are for the unseen side.

Activity

The drawing is a three-dimensional representation of a puzzle piece.



Which diagram is *not* a possible representation of how it looks from the back (i.e. the direction shown by the arrow)? The shaded areas are recesses.



Commentary

There must be some sort of recess in the top-left corner (top back right as shown on the 3-D drawing). We cannot tell whether it goes right to the bottom as shown in A and B or just some of the way down as shown in C, D and E.

Similarly, there is a recess that goes through to the right-hand edge (left back on the 3-D drawing). Those shown in A, C and E would all give the same 3-D view from the front. We can now eliminate A, C and E, as both the rear features are shown as being possible. (Remember that we are looking for the diagram that is *not* a possible representation.) D joins up the two recesses – this would also be possible as the join would not show from the angle originally shown.

We have come to the answer B by elimination. This is a completely valid way of proceeding, but it would be useful to check that this is indeed the correct answer. The recess shown in green on the right-hand side of diagram B would be visible at the back of the lowest section of the three-dimensional drawing. So B is not possible.

Once again, this is more difficult than might be expected. We do not actually know what the hidden reverse side looks like – there are an infinite number of possibilities. All we can do is consider what hints are given by the three-dimensional picture. One primary feature of this type of question is that the answer cannot be produced just by considering the information given and the question. This is a *backward* question. The five options must all be looked at and a decision made on whether each is possible. Backward questions are a regular feature of questions on spatial reasoning, and of identifying similarity, which was dealt with in Chapter 3.7. They do not occur very often in the other types of question. The value of elimination was shown in the method of answering this type of question.

Summary

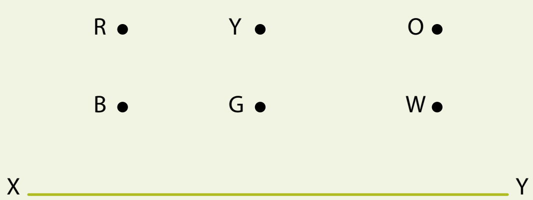
- We have seen the importance of spatial reasoning in many occupations and how problem-solving questions can test this.
- The value of practice in solving this type of question has been emphasised.
- This chapter introduced questions that are *backward* in that the answer must be found from the options rather than just from the information and the question.
- The use of elimination in answering such questions was illustrated.

End-of-chapter assignments

1 Fred wants to write the letters **NSRFC** on his forehead for this afternoon's Northampton Saints Rugby Football Club match. He does it with face paints while looking in a mirror. What should it look like in the mirror?

2 Draw a simple picture of your house or another building with which you are familiar as seen from above and from the front. How much can you tell about the side and back views from your drawings?

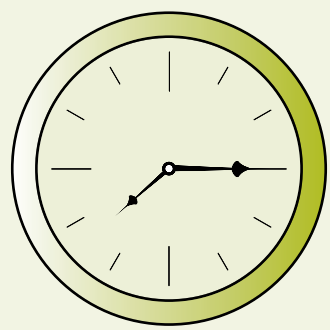
3 Outside the Diorama hotel there is a set of flagpoles, as shown in the drawing. The flagpoles are all painted different colours (red, blue, yellow, green, orange, white).



When they are seen from position X, they are seen in the order (from left to right): R B Y O G W.

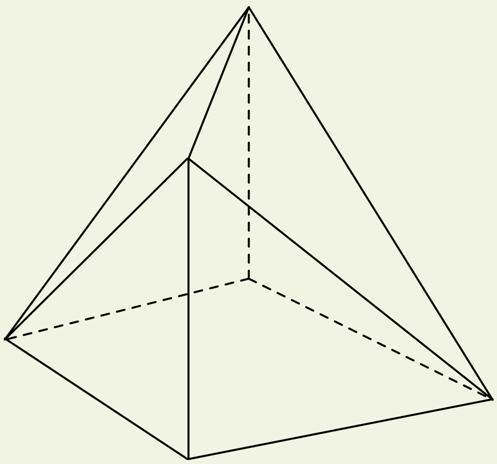
If somebody walks from X to Y, in how many, and what, different orders will they see the flagpoles? (Exclude places where one is exactly hidden behind another.)

4 Our local café has an unusual clock which is upside down. The numbers 3 and 9 are in their conventional places, but 6 and 12 are interchanged. What time is it when the hands are positioned as in the clock face shown?



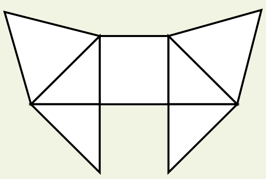
A 2.45 **B** 7.15 **C** 8.45
D 10.15 **E** 10.45

5 The solid shown, which is a cube with two corners cut off, is made from a shaped and folded piece of cardboard. (The dotted lines represent edges which are hidden.)

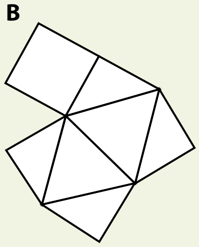


Which of the following pieces of cardboard will fold to make the shape? There may be any number correct from none to four.

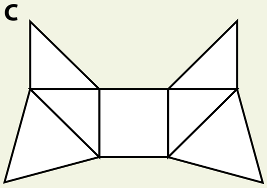
A



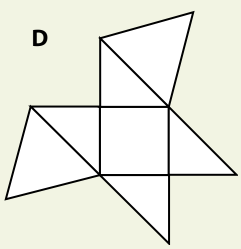
B



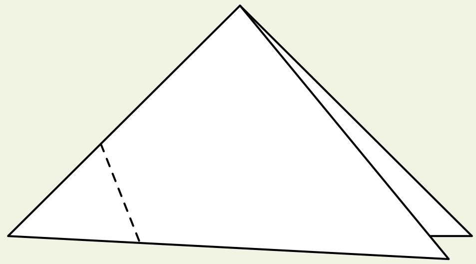
C



D

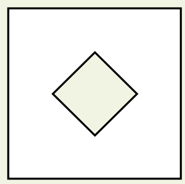


6 Some children are making decorations. A square sheet of paper is folded along a diagonal and then again so the two sharp points meet, as shown. A cut is made through all the layers of paper along the dotted line shown and the small pieces removed. The paper is then opened.

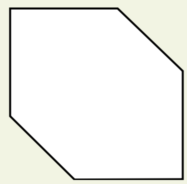


What does it look like? (Try to visualise it in your head before you make a model to test that your answer is right.)

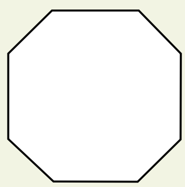
A



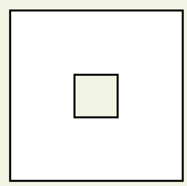
B



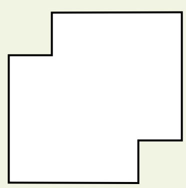
C



D



E



Answers and comments are on page 321.

3.10 Necessity and sufficiency

Another type of problem involves identifying whether there is enough data to solve the problem and, if not, which data is missing. This is a useful building block in problem-solving. It highlights one of the key elements of problem solving, which is to find a way to solve a problem without, in this case, having to do any arithmetic.

The words 'necessity' and 'sufficiency' are used in mathematics but have exactly the same meaning as they do in normal language. An individual piece of data is necessary to solve a problem if we cannot solve the problem without it. A set of data is sufficient to solve a problem if it contains all the information we need.

Identifying which data is needed to solve a problem can save effort in finding unnecessary data or in making unnecessary calculations. Such questions are approached in a manner similar to those described in earlier chapters.

To illustrate the type of question described here, we start with a very simple example. Suppose someone is taking a car journey. We know their leaving time and we know the average speed they will do. We want to know their arrival time. Which other piece of information is necessary for us to calculate this?

The solution is very straightforward: we need the distance of the journey. We can then calculate the journey time (distance divided by speed) and thus the arrival time. All of the three pieces of data we now have are necessary to do this calculation. The three pieces taken together are sufficient.

Here is a slightly more complex example.

Activity

I use the trip meter on my car to measure the distance driven since I last had the car serviced, so that I know when the next service is due. The trip meter can be set to zero by the press of a button and records the kilometres driven since it was last reset.

I set the trip meter to zero after my last service. The next service is due after 20,000 km have been driven. Some time later, I lent the car to my brother. I forgot to tell him about the trip meter; he pressed the button to zero it and drove 575 km. I then started driving again without realising what he had done.

What should the trip meter read when the next service is due?

The above problem cannot be solved with the information given. What additional piece of information is needed to solve it?

Commentary

This question is actually rather easier than it may at first seem. The distance driven from the last service when my brother returned the car was the distance I had driven plus the distance he had driven. I know how far he had driven, so what I need to know was the distance on the trip meter when I gave the car to my brother.

In this case, like the previous example, we were not asked to solve the problem, merely to identify what pieces of information were needed to solve it. In real-life problem solving, the data is not generally given; it has to be found. Having the skill to know which pieces of data are needed can save considerable time and effort. Solving this type of problem does not need particular mathematical skills – just some clear and logical thinking.

Activity

I have a small collection of three types of old coin. The collection contains 15 coins in total. There are more pennies than half-crowns and more half-crowns than guineas.

Which one of the following single pieces of information would enable you to know exactly how many of each type of coin there was?

- **A** There are 4 more half-crowns than guineas.
- **B** There are 5 more pennies than guineas.
- **C** There are 3 more pennies than half-crowns.
- **D** There is one fewer penny than guineas and half-crowns together.

Commentary

In this problem, we are being asked to find which of the options is sufficient (along with the information we have already been given) to solve the problem.

There are 12 ways that 15 can be partitioned into three different numbers:

Guineas	Half-crowns	Pennies
1	2	12
1	3	11
1	4	10
1	5	9
1	6	8
2	3	10
2	4	9
2	5	8
2	6	7
3	4	8
3	5	7
4	5	6

Of the options given, only C gives a unique set. If there are 3 more pennies than half-crowns, there must be 8 pennies, 5 half-crowns and 2 guineas.

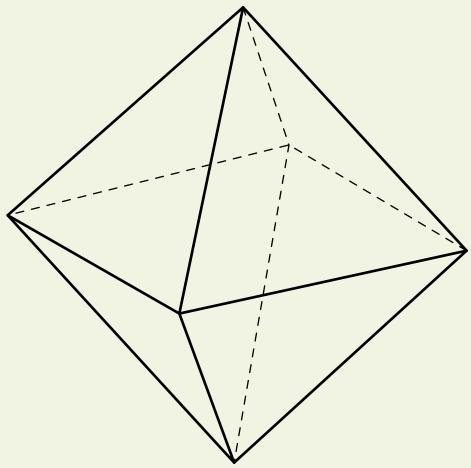
Why do the other options not work?

Summary

- We have met a new type of problem where, rather than being asked to find a solution, we are asked to find what pieces of information are necessary or sufficient to solve it.
- We have also encountered problems where we have to find a solution, but need to identify an additional piece of information which is necessary either to help us with the method of solution or to choose between different possible solutions.
- We have learned the meaning, in this context, of the words ‘necessary’ and ‘sufficient’.
- We have seen various types of problem which require extra data: some needing mathematical solutions; some only requiring us to establish a logical method of solution.

End-of-chapter assignments

1 I have made a dice out of a sheet of cardboard in the form of an octahedron, which has eight faces as shown below.



I now want to number the faces from 1 to 8. The numbers on opposite faces must add up to 9, so when I number a face 1, the opposite face must be 8.

If I start with number 1 and work up, how many faces can I number before I am left with no choice about where to put the numbers?

2 **(Harder task)** George stocks bags of pears and bananas in his shop. Each bag contains either five pears or three bananas. He wanted to know how many to order to keep his stocks up, so he sent his assistant to count the bags. However, the assistant, not being very bright, counted the total number of pieces of fruit instead. George was about to send him back to repeat it when he realised that the number that the assistant had given him was not only sufficient information for him to work out how many bags there were of each, but was also the maximum such number. How many bags of pears and bananas were there?

3 Kuldip told me she had 12 coins in her pocket, all either 1¢, 2¢ or 5¢, with a different number of each denomination. There were more 2¢ than 1¢ coins and more 5¢ than 2¢ coins. She asked me how much money she had in her pocket altogether. I told her that I did not have enough information to answer.

Which of the following additional pieces of information would enable me to know how much money she had in his pocket?

A She had three 2¢ coins.

B The total amount of money was a multiple of 10¢.

C 5¢ coins amounted to $\frac{3}{4}$ of the total value.

D She had two more 5¢ coins than 1¢ and 2¢ together.

Answers and comments are on pages 321–2.

3.11 Choosing and using models

The most obvious and familiar use of the word 'model' is that of a replica of an object, for example a car, at a smaller scale. In this book the word is used in a wider sense. Models can be pictures, graphs, descriptions, equations, word formulae or computer programs, which are used to represent objects or processes. These are sometimes called 'mathematical models'; they help us to understand how things work and give simplified representations that can enable us to do 'what if?' type calculations.

Architects, for example, use a wide range of models. They may build a scale model of a building to let the client see it and to give a better impression of how it will look. Their drawings are also models of the structure of the building. In modern practice, these drawings are made on a computer, which will contain a three-dimensional model of the building in digital form. This may be used to estimate material costs and carry out structural calculations as well as producing a three-dimensional 'walk-through' picture on the screen.

This chapter deals with the recognition and use of appropriate models. A simple example of a model is a word formula used to calculate cost. The amount of a quarterly electricity bill can be described as 'A standing charge of \$35 plus 10¢ per unit of electricity used'. This may equally be shown algebraically as:

$$c = 35 + 0.1u$$

where c is the amount to pay (in dollars) and u is the number of units used.

A more complicated example of a model would be the type that governments set up to simulate their economies. These usually consist of large numbers of equations and associated data, and are implemented on computers. They can predict (with varying success) things such as what will happen to the inflation rate if interest rates are raised. Such models are gross simplifications because there are too many variables contributing to the condition of a national economy and all factors can never be included.

Scientists also use models, for example in predicting population growth. Such a model, for example, to predict fish stocks in fishing areas, can be invaluable as it may be used to control quotas on fish catches to ensure that fishing does not reduce stocks to unsustainable levels.

In both of these cases, the model has been produced as a result of a problem-solving exercise. The actual development of a model to represent a process is beyond the multiple-choice questions in the lower-level thinking skills examinations and will be dealt with in Chapter 5.2. Multiple-choice questions on choosing and using models test some of the basic skills involved in modelling and the extraction of data from mathematical models.

In the following activity you are asked to use different models to compare calculations. This example is close to a real-life situation.

Activity

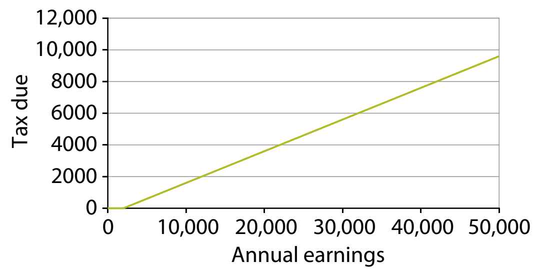
The current structure of income tax collection in Bolandia is that the first \$2000 of annual earnings are tax-free (this is called the tax threshold), then 20¢ of tax is charged on every dollar earned over this (this could also be described as a 20% tax rate).

The government is determined to reduce the tax burden on lower-paid people and intends to bring in a new system, which will mean that the threshold for paying tax will rise to $10,000. They intend that all those below the average earnings of $26,000 will pay less tax, and all those earning more than this will pay more tax. What will be the tax rate on earnings over $10,000?

Commentary

The model of tax used here is quite simple, consisting of a fixed amount of income on which no tax is paid and a single standard rate on earnings above this amount. Currently, those earning $26,000 pay (26,000–2000) × 0.2 dollars, or $4800. If they are to pay the same under the new regime, they will pay tax on $16,000, and the total will be the same as before, i.e. $4800. The new tax rate will be 30¢ on each dollar earned over $10,000 ($16,000 × 0.3 = $4800). This could be done by algebra, but the process is no simpler than that given above, which requires no more than elementary school mathematics. What effect would these changes have on someone earning $50,000 a year?

As an exercise, consider other possible tax structures which might give similar results. Plotting graphs of tax paid against earnings gives a clearer representation of how the various models of taxation work. The graph below shows the tax paid under the current system of tax in the example above.



Add lines for the proposed new system and for any other model you may think of.

The next activity, below, requires you to go some way towards developing a mathematical model of a new situation in order to solve the question.

Activity

My company regularly uses a taxi service to take staff to the airport. If there are several passengers needing to travel from our town at similar times, they combine this into a single journey. They divide the total cost by the number of passengers and invoice each passenger separately. The distance is always the same and the time only varies by a small amount, but I do not know how they work out the charge for the journey.

There are a number of different charging structures they could use. All taxis charge a fixed price per kilometre and per minute of journey time. In addition they may charge a fixed hire fee and an additional charge depending on the number of passengers carried.

What is the charging structure used by this taxi company? What limitations are there to the conclusions we can derive?

Number of passengers	1	2	3	4	5
Charge per journey per passenger	$40.00	$19.98	$14.68	$12.03	$10.38

Commentary

If we just look at the data as it stands the pattern is not clear, other than that the price per passenger drops with the number of passengers. Since we are looking at the charge made by the taxi company, it is preferable to look at the total cost of the taxi in each case. This may be carried out by multiplying the cost per passenger by the number of passengers, as shown in the table below.

The pattern now becomes much clearer. Allowing for some small variations (it was stated that there was a small variation in journey time), the first two values are the same and they then increase by $4 per passenger. We can, therefore, conclude that the taxi company hire fee includes one or two passengers, then there is an extra charge of $4 per additional passenger. The $40 'basic' fee covers the hire charge, the distance charge and an average time charge. We have no information which will enable us to separate these three items. A model can only be as good as the data on which it is based.

A graph can be a very useful tool for analysing data such as in the table below, and can also help in developing models. Try graphing the data, for both the cost per passenger and the total cost per journey. Does this help in clarifying the charging structure?

The activity above introduced the idea that models usually are approximations to the real world. The model used did not allow for variations in the time of the journey. This is why the word 'model' is used. Almost all models are approximate – the model car does not usually have an internal combustion engine. Economic models cannot take into account factors such as the weather.

Many people use models in their everyday lives without even realising it. An efficient shopkeeper will, for example, have a set of rules that tells her how much ice cream to order so she has plenty in the summer months and less stock in the winter.

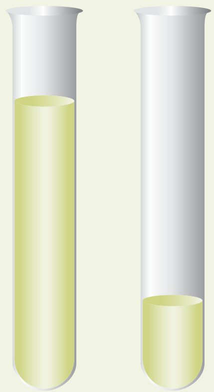
Number of passengers	1	2	3	4	5
Charge per journey per passenger	$40.00	$19.98	$14.68	$12.03	$10.38
Total charge per journey	$40.00	$39.96	$44.04	$48.12	$51.90

Summary

- We have learned how a mathematical or graphical model may be used to approximate real-life processes.
- We have seen how models can be used to simulate changes in cost structure and their effects.
- We have used real data to calculate the constants used in a mathematical model, for example the starting rate and charge per mile for a taxi fare.
- A graph of any sort is a model from which it is possible to get a picture of how variations can occur.

End-of-chapter assignments

1 A novelty marketing company is selling an unusual liquid clock. It consists of two tubes as shown. The right-hand tube fills up gradually so that it is full at the end of each complete hour, and then empties and starts again. The left-hand tube does exactly the same in 12 hours. The time shown on the clock is 9.15.



Draw what the clock looks like at 4.20.

2 Finn walks to school, a distance of 1.5 km which takes him 15 minutes. His older sister, Alice, cycles to school on the same route at an average speed of 18 km/h. She leaves home 5 minutes later than Finn. Does she overtake him on the way and, if so, where? At what time would she have to leave to arrive at school at exactly the same time as Finn?

3 A shop normally sells breakfast cereal for $1.20 a packet. It is currently running a promotion, so if you buy two packets, you get a third free.

Tabulate and graph the price per packet for numbers of packets bought from 1 to 10. How would other special offers (e.g. 'Buy one, get one half price') affect the shape of this graph? If you were working backwards from the graph, how could you determine which offer is currently being used?

Answers and comments are on pages 322–3.

3.12 Making choices and decisions

Many of the problems we encounter in everyday life involve making choices and decisions. To buy or not to buy? Which one to buy? How much to buy? Which train to take? All these are types of choice and decision that contribute to problem-solving processes and involve the use of skills that can be tested by problem-solving questions.

These questions may involve skills that have been covered in earlier chapters: extracting information, processing data and finding methods of solution. The only real difference is that the question asks for a decision to be made. The following is an example of such a question.

Activity

My local shops all have different discounts on jars of coffee. Which of the following represents the best value for money?

A Everlo: \$1.29 for a 150 g jar
B Foodland: \$2.89 for 200 g, buy one get one free
C Springway: \$3.36 for 300 g, buy one get the second half price
D Superval: \$1.57 for 150 g, 50¢ voucher off the next 150 g (one voucher per customer)
E Massive: \$1.57 for a 150 g jar with 50 g extra coffee free

Commentary

In this case it is easiest to express all the prices to a 300 g equivalent – you may see that this requires fewer calculations than, for example, converting them all to 100 g.

A is 2 × \$1.29 = \$2.58 for 300 g
B gives 400 g for \$2.89, so 300 g is 0.75 × \$2.89 = \$2.17
C gives 600 g for 1.5 × \$3.36 = \$5.04 or 300 g for \$2.52
D first 150 g is \$1.57, second is \$1.07, so 300 g is \$2.64
E \$1.57 for 200 g or \$2.35 for 300 g

So B is the best value.

This is quite straightforward; no skills that have not already been introduced are involved. It is just necessary to work efficiently and correctly, finding the most effective way of approaching the problem.

The activity below involves making a decision.

Activity

Eve buys chickpeas in bulk to sell in her shop. They come by volume and each drum can contain between 10 and 12 kg. She repackages them to sell in 0.5 kg packs. She gets a delivery on a Monday morning and sells anything from 8 to 15 packs in a day. She is open 7 days a week and on Sunday night has 14 packs left and half a drum of bulk chickpeas.

How many drums should she order to make certain that she has enough for the next week?

A 2 **B** 3 **C** 4 **D** 5 **E** 6

Commentary

This is a maximum and minimum type problem: we have to combine the most chickpeas Eve could sell with the least she has in stock and minimum in the drums she buys.

The least she has in stock is half a drum, which might be as little as 5kg; this will make up a minimum of 10 packs. She has 14 packs in stock, so at least 24 in total. She needs at most 7 × 15 = 105 packs, so may need as much as 81 × 0.5 kg = 40.5 kg. At 10 kg per drum, she will need 5 drums to be sure she can last the week.

You might also like to work out what is the fewest drums she might need.

This illustrates a particular type of decisions question – where the decision is based on the minimum (or sometimes the maximum) to fulfil a criterion.

Summary

- We have seen how problems may involve making choices and decisions.
- Decisions may involve selecting one item from a number of options or making a decision on an action.
- In solving these problems it is important to choose an efficient way of working so the correct answer is obtained and calculations are carried out in the most effective and simplest way in order to reduce the chance of error.

End-of-chapter assignments

1 In a game of pontoon dice, you continue throwing a single die until the sum of all your throws exceeds 21 (bust) or you decide to stop. You win plastic counters depending on the score you stop at.

Stopping score	Counters won or lost
1 to 12	0
13 or 14	Win 1
15 or 16	Win 2
17 or 18	Win 3
19	Win 6
20	Win 8
21	Win 10
Over 21	Lose 4

After four throws you have scored 17. Should you throw once more? Consider the chances of getting different scores and how much you will win or lose. What is best on average?

What should you do if you have a score other than 17?

2 Clyde's local supermarket has an offer on petrol, depending on the amount you spend in the store. If you spend \$20–\$30 you get a voucher that gives you 2¢ per litre off petrol; if you spend \$30–\$50, you get 3¢ per litre off; and if you spend over \$50 you get 4¢ per litre off. Clyde's car will take 30 litres of petrol.

Consider for what range of total purchase prices in the supermarket it is worth his buying a small amount extra, so that the reduction in the petrol cost will make his total bill smaller.

3 Students at a school have to decide what subjects they are going to study next year. English, science and mathematics are all compulsory, but they can choose the remaining four subjects.

The table shows how the choices can be made. Students must choose one subject from each column. The fourth subject may come from any column.

1	2	3
geography	French	history
technology	German	religious studies
art		physical education
music		Latin

Which of the following combinations would not be allowable?

A French, geography, physical education, art
B French, German, Latin, music
C technology, German, art, history
D French, German, geography, music
E geography, music, French, religious studies

4 The country of Danotia prints stamps in the following denominations:

1¢, 2¢, 5¢, 9¢, 13¢, 22¢, 36¢, 50¢, \$1.00, \$5.00

A mail order company sends out equal numbers of three sizes of package, which incur postal charges of 34¢, 67¢ and \$1.43. They want to stock as few different denominations of stamp as possible (but not only 1¢ stamps as sticking lots of these on envelopes would be a nuisance!).

Which ones should they stock?

Answers and comments are on page 323.

4.1 Inference

The verb 'infer' means to draw a conclusion, usually from some factual (or supposedly factual) information. The information provides the *grounds* for the inference. If the grounds are good, we say that the inference is sound, or reliable. Another word that is often used is 'safe'. If the grounds for an inference are poor, we have to say that the inference is unsafe: it cannot be relied upon.

Judging whether an inference is safe is therefore similar to judging whether an argument is sound. The main difference is that in a standard argument the conclusion is stated. In many texts and documents, however, there is no explicit conclusion. There may be claims; there may be information. But unless some further inference is drawn from the information, there is no argument. Sometimes there may be an *implicit* conclusion, where the information is clearly leading in one particular direction, and it is obvious what we are *meant* to infer. There is an example in Chapter 2.4, item [7] (page 34). Here is another example (the subject may sound familiar):

[1] These banknotes all have the same serial number. All genuine banknotes have different numbers.

This time the conclusion has been left unsaid. But it is clear what the author is getting at. If asked what can be inferred from [1] most people would probably answer that:

[2] The banknotes are not all genuine.

Moreover this would be a safe inference, because [1] provides very good grounds for [2].

Of course [2] is not the only inference that could be drawn from [1]. Nor is it the only safe one. [1] also gives good grounds for inferring that I would be committing an offence if I tried to spend these banknotes. I would need to know that passing false currency is illegal, but that is such common knowledge that it practically goes without saying.

It would be fairly safe, too, to infer from [1] that

[3] The banknotes are forgeries,

on the reasonable assumption that only forgeries could have duplicate numbers. By contrast it would be entirely *unsafe* to infer that

[4] The banknotes are the work of terrorists, intent on destabilising the economy.

Inferring [4] from [1] would be a blatant example of jumping to a conclusion.

Inference and science

Drawing inferences, and judging the reliability of inferences, are especially important in scientific contexts. Scientists typically base their claims to knowledge on the information they collect from observation and experiment. Because we cannot confirm the truth of such claims without supporting evidence, we have at least to be sure that the evidence is strong and the inference is reliable. Assessing what can and cannot be inferred from a given document is therefore a key component of critical thinking.

Here is a short introductory example:

DOC 1

Ice ages last for roughly 100,000 years, going by the record of the past half-million years. The warm phases in between are called interglacials. The standard view, until quite recently, has been that we are coming to the end of the present warm phase, which has already lasted just over 10,000 years. Indeed, data from Antarctic ice cores* indicated that the previous three interglacials have lasted between 6000 and 9000 years which, if repeated, would have seen parts of Europe, Asia and North America covered in ice since before the rise of the Roman Empire. The most recent Antarctic ice cores have revealed that the warm phase before that lasted for 30,000 years. It is known, too, that the Earth's alignment relative to the Sun during that long interglacial was similar to its alignment at the present time.

* An ice core is a sample obtained by drilling down into the ice cap. The state of the ice at different levels provides a climatic record that can extend over hundreds of thousands of years.

Activity

From the information in Doc 1, which of the following can reliably be concluded?

A Another ice age is due any millennium now.
B The standard view is wrong.
C The present warm phase is set to last another 20,000 years.
D According to the recent geological record, ice-age conditions are the norm, and it is Earth's present climate which is unusual.
E Global warming is delaying the start of the next ice age.

Give a brief reason for each response.

Commentary

We'll consider the inferences in turn. The first one, A, as well as being a bit vague, has little support from the text. If we take 'any millennium now' to mean in the next one or two – which is its natural meaning – then A would mean sticking with the standard view, despite the most recent findings casting doubt upon it. The standard view is that the present warm phase should be reaching its time limit; but according to the last three warm phases the limit has already been passed. The latest ice core also suggests that some interglacials could last as long as 30,000 years. Evidence that the Earth's present alignment with the Sun's rays resembles that of the last *long* interglacial, pours even more cold water on A. (Interestingly, even without the most recent evidence, the grounds for A would still be weak: half-a-million years is a blink of an eye in geological terms, and the sample of just three warm phases is too small to call a reliable trend.)

B might seem consistent with what has just been said about A. If we are not near the start of a new ice age, the standard view must be wrong. But there is a lot of difference between saying that an inference is unsafe, and declaring it false. B is a much stronger claim than can be supported by the data in Doc 1. We *might* be near the end of a warm phase: one that is longer than the last three and shorter than the one before that. There is little or no positive evidence for such a claim; but nor is there proof that it is false. Remember the significance of strong and weak claims (see Chapter 2.2). A strong claim requires much more to justify it than a more moderate claim. Had B asserted that the standard view is now less plausible than it was, instead of plainly false, that would have been defensible.

C suffers from the same fault as B: it, also, is too strong. A single example of a 30,000-year warm phase, coinciding with a particular alignment of the Earth and Sun, is far from adequate to justify an out-and-out prediction that the present interglacial will also last that long. No serious scientist would go so far. The author of the Geological Society webpage from which the information is Doc 1 was sourced claims only that:

F On these grounds, even without human intervention, another 20,000 years of warmth may be expected.

Compare this with C. Although it is drawing broadly the same conclusion, it expresses it in a careful way which would permit its author to fend off objections. Another 20,000 years *may be expected*, on the evidence supplied. That claim does not cease to be true, at the time of making it, even if in 10,000 years' time it turns out to have been optimistic. Nor will F be falsified if new, contrary evidence comes to light, because the author has qualified the claim by prefacing it with: 'On these grounds . . .' You should remember these details of presentation when you are constructing your own arguments. They may seem like purely linguistic points; but they can make the difference between inferring something that can be substantiated, and something that cannot.

Whoever inferred D has taken the right sort of care in expressing it. It is preceded by the phrase: 'According to the recent geological record . . .' and draws a conclusion that the record firmly supports. Recent ice ages (geologically speaking) have lasted between 3 and 17 times longer than interglacials. So glacial conditions have been the prevailing ones during that period, and it is the Earth's present climatic state that is unusual. D is a safe inference.

For E to be true it would mean that the standard view was correct after all, and/or that the trend of the last three glaciation cycles was continuing. It would also require global warming to be taking place; and to be capable of delaying the onset of an ice age. E is not impossible, but it would take a lot more than the claims in Doc 1 to make it true. In a word, it is unsafe.

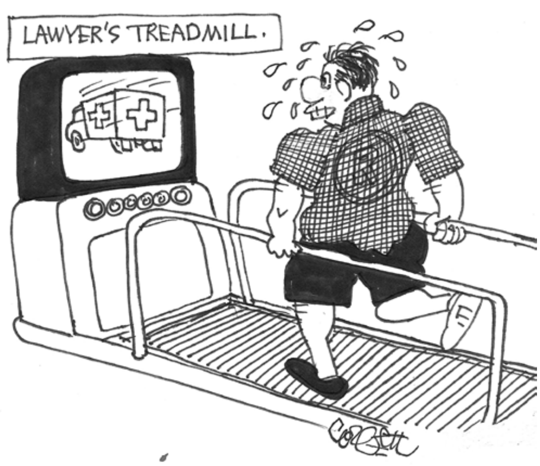
Why do we use the word 'safe'?

The practice of calling some inferences unsafe is a recognition of the importance of reasoning carefully. What makes an inference unsafe is not just that it may be wrong, but that it may have consequences, sometimes very serious ones. Perhaps the most obvious illustration is a criminal trial, where a verdict must be reached on the basis of the evidence. A trial-verdict is a particularly serious kind of inference, on occasions a matter of life or death. As the result of a faulty inference, an innocent person might go to prison for a long time. On the other hand, a not-guilty verdict passed on a guilty person may leave him or her free to commit a further atrocity. You will sometimes hear the expression, 'That's a dangerous inference to make!' We can easily see why that is entirely appropriate.

But even if there are no obvious dire consequences, it is still important to reason well rather than badly, because it gets us closer to the truth. Judging when an inference is safe or reliable is therefore a key element in critical thinking.

Assessing inferences

With this in mind, read the following passage (Doc 2) and make a mental note of the information that it contains. It introduces a topic which will occupy the rest of the chapter, and feature in the next two.



www.CartoonStock.com

DOC 2

The industry surrounding large compensation claims following accidents and personal injury has attracted much media attention, with stories of millions of dollars being awarded in damages to successful claimants, and of compensation being sought for every kind of injury or loss. There has also been a dramatic increase in law firms – or 'claim management firms' that act as intermediaries – canvassing for accident victims by advertising on television, the internet or on the street. (The phrase 'ambulance chasers' is often used to describe them.) People complain of getting unsolicited calls or text messages asking: 'Have you had an accident that wasn't your fault? . . .' or words to that effect. Television – especially daytime television – has a high proportion of such ads.

At the same time it has become commonplace for lawyers to offer their services on a no-win no-fee basis, which allows people on low or moderate incomes to go to court with no risk of running up expenses they couldn't otherwise afford. This is also known as a 'conditional-fee agreement'. Lawyers earn nothing for unsuccessful claims but are entitled to charge up to twice their normal costs if the claim is successful. This practice, known as 'uplift', can add thousands to the bill the losing side then has to pay out.

Activity

For each of the following statements in turn, assess whether or not it can reliably be concluded (inferred) from the above passage.

A Lawyers are self-serving and unscrupulous.

B Without the no-win no-fee option there would be fewer claims for personal injury.

C Advertising by law firms and/or claim management firms encourages clients to exaggerate or invent injuries.

D A no-win no-fee arrangement benefits the lawyer much more than it does the claimant.

E As long as they win more than 50% of conditional-fee cases, a law firm need not be out of pocket.

F Claims being pursued for personal injury have increased significantly since the introduction of conditional-fee agreements.

Commentary

Although the passage has a somewhat negative tone it is not openly judgemental in what it claims. We have to be careful, therefore, what we *read into* the passage and what we infer from it. If someone has a poor opinion of lawyers generally, or of 'ambulance chasers' in particular, it would be easy to be led by prejudice into thinking that this document supports those viewpoints. It does not.

You cannot, for example, infer from it that lawyers are bad people, even if you happen to think that they are. If you included A in your list of safe inferences, then either you weren't taking the question seriously, or you were simply giving a view based on prejudice, or other data that is not provided here. In fact there is practically never sufficient hard evidence for any claim as strong, as sweeping, or as judgemental as A. A is not even the *kind* of claim that can be ordinarily inferred reliably from purely factual information.

What about B? This certainly seems a more reasonable inference, and a less opinionated one. There is, too, a widely held belief that the number of claims has risen significantly since no-win no-fee agreements were introduced, and probably because of them. Since conditional-fee agreements give people on low or middle incomes the chance to pursue expensive legal actions at no financial risk to themselves, it is natural to think that there would be a surge in claims. But the very fact that B seems so reasonable, and happens to be widely believed, is precisely why we need to approach it critically. If you already assume that no-win no-fee arrangements have resulted in more personal injury claims, you are likely to see the passage as grounds for believing it. But interpreted neutrally, the passage neither supports B nor disputes it. All that we are told in Doc 2 is that there are no-win no-fee arrangements on offer, and how they work. We are also told that this has prompted stories in the media, but with no comment on the truth or falsity of these, or even what they actually claim. There is no information about the effect the arrangements have had on numbers or attitudes. If we stick faithfully to what is contained in Doc 2, B must be seen as leaping to an unjustified conclusion.

C, likewise, may seem like a very believable consequence of advertising for victims, especially if the advertisements emphasise the possibility of making big money out of an accident. Obviously, the worse the harm that has come to the claimant, the more money the court is likely to award in damages. So there would be a temptation for a dishonest person to cheat by exaggerating or inventing an injury. But that is very different from saying, as C does, that the advertising by law firms and intermediaries *encourages* cheating. That is a serious allegation. It is also a little hard to believe, if it is taken to mean that lawyers actively encourage dishonesty in the way they advertise. But even if we interpret C more charitably to mean that the advertising has the *unintended* effect of giving some people the idea of cheating, there is still no evidence of any such connection in the passage.

Note that even if there were statistics showing that since lawyers started advertising, more dishonest claims have been lodged, that would not permit an inference that the advertising was the cause, or that it gave encouragement (see cause–correlation fallacy in Chapter 2.10). In Doc 2, however, there is not even a correlation. We are not given any data on numbers of dishonest claims before or after advertising began. C is definitely not a safe conclusion.

D claims that no-win no-fee arrangements, which on the surface look quite advantageous for the client, are actually of more benefit to the lawyers. Assuming that 'benefit' means financial benefit, it is clear from Doc 2 that lawyers do get some benefit from the way in which the system works. They may lose money when the case is unsuccessful, but they have bigger costs awarded when they do succeed. Provided they win more cases than they lose, they should be better off than if they didn't take the case at all because the client could not afford the fee. On the other hand, the client benefits too, either by winning, or by having nothing to pay if the case fails. The question is whether the lawyer benefits more than the client; and again we find the passage uninformative. There is simply no data in Doc 2 by which to quantify the gains comparatively.

However, Doc 2 does give strong support to E. This is because the information in Doc 2 is mainly explanatory. In particular it explains how lawyers can afford to take on cases without charging a fee. When they win a case they get back around twice their normal costs, to make up for the fee they would have been paid by the client, win or lose, under the old system. It is simple mathematical fact that so long as they don't lose more cases than they win, they are not out of pocket. If they win more cases than they lose, they make money. We cannot infer that they do better out of this arrangement than the clients, as claimed by D. But if Doc 2 is factually correct, we can quite safely infer E.

If you were really alert you might have added that E carries the implicit assumption that a lawyer's cases all have the same value, and require the same level of input in terms of hours worked, and other costs. Arguably a lawyer could lose a small number of very complex cases and still be out of pocket because they were worth more in fees than the income generated from cases he or she won. Strictly speaking E is safe to infer only if *on average* all cases cost about the same to pursue.

And so we come to F: 'Claims being pursued for personal injury have increased significantly since the introduction of conditional-fee agreements.' By now it should be clear that this cannot be inferred from Doc 2 either. You may have thought that F could be inferred because it seems so *likely* to be true, given that conditional-fee agreements allow people to make claims without having to pay anything. In that respect F is like B. But F can only be inferred if it is also assumed that there have been no other changes which might have had a reverse effect. Nor can F be inferred without assuming that lawyers now take on more cases than they did before the introduction of no-win no-fee. Doc 2 provides no information to justify either of these assumptions. In fact, the note about E in the previous paragraph suggests that lawyers may be much more selective than they were, since they have more to lose. F could, quite realistically, be false, and public opinion seriously flawed.

The clear warning to take from the discussion above is that many seemingly reasonable inferences were in fact unsafe. Some of the claims may be true, and may be found to be so after further investigation. But Doc 2, as it stands, lacks the hard data we would need for drawing conclusions such as B, C or F.

Popular opinion

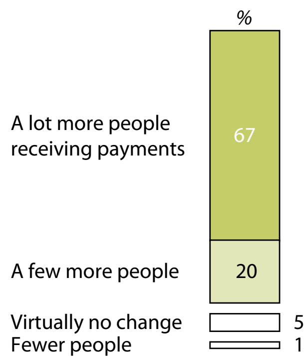
Many people hold the opinion that there is a growing 'compensation culture', with many more claims being made for injuries – real or otherwise – than there were, say, five or ten years ago. Many also take the view that a lot of the claims are bogus, or fraudulent, especially the infamous 'whiplash' injury, following quite trivial car accidents. The writer Andrew Malleson wrote a book called *Whiplash and Other Useful Illnesses*. The content is serious, but the book's tongue-in-cheek title tells its own story.

Here is another document, this time graphical. It consists of three bar charts. (See Chapters 3.7 and 5.4 for further questions of this type.) The data is from an official questionnaire conducted among 509 randomly selected adults living in the UK. However, it probably reflects opinion in many developed countries.

DOC 3

CHART 1

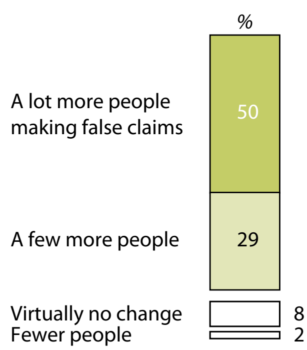
Compared with ve years ago, do you think there has been a change in the number of people receiving compensation payments for personal injuries?



** Respondents who answered 'Don't know' have been excluded from chart.*

CHART 2

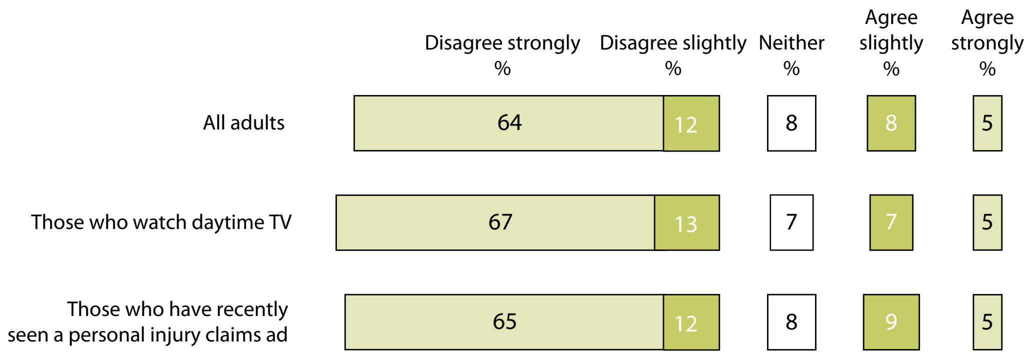
Compared with ve years ago, do you think there has been a change in the number of people making false claims?



** Respondents who answered 'Don't know' have been excluded from chart.*

CHART 3

'I might be tempted to make an exaggerated claim for a personal injury, even if I didn't have a strong case for compensation.' To what extent do you agree or disagree with this statement?



**Respondents who answered 'Don't know' have been excluded from chart.*

Take some time to think about and/or discuss the following questions, before reading the commentaries that follow.

Activity

1 Which of the following are supported by the information in Chart 1? (The answer may be any, none or all.)

Assume that the data is accurate and that the sample of people questioned is representative of the population.

A There is a widespread and strong belief that more people are receiving compensation for personal injuries now than five years ago.

B 87% of those responding to the survey believe that there are more people receiving payments for personal injury compensation than there were five years ago.

C Claims being pursued for personal injury have increased significantly in the past five years.

Commentary

The first point to make is that the data concerns public opinion. The first of the three claims is therefore clearly supported by the data. (As a matter of interest, it is the conclusion that the researchers drew themselves.) Only 1 in 20 people thought that there had been no change. Well over half thought that the change was substantial in favour of more people receiving payments. This indicates a 'widespread and strong belief', and makes A a safe conclusion. The question of whether the sample was representative need not concern you, as you were told to *assume* that the figures are accurate and well researched.

Claim B is a more direct interpretation of the data, and simple arithmetic shows that it too is a safe inference. Claim A is true because claim B is true.

Claim C is more complex and more interesting. It would seem reasonable to argue that if there are more people receiving payments, there are more claims being made. But ultimately C rests on the assumption that there really are more payments being made; or, in other words, that the widespread *belief* expressed by those questioned is correct. It is this step which is the problem: reasoning from the evidence that most people believe something, to the conclusion that it is true or probable, is a classic fallacy known by the Latin *argumentum ad populum*. If you prefer more modern names there are plenty to choose from: appeal to popular opinion, appeal to consensus, appeal to the majority, the authority of the many over the few. The weakness of this argument is nicely captured by the old joke that 40,000 lemmings can't all be wrong. (The joke is that every so often whole colonies of lemmings are believed to run to the edge of the nearest cliff and plunge to their deaths!) C therefore is not a safe inference.

Activity

2 Suppose the majority view represented in Chart 1 is correct. Would it follow that there has been a change in the number of people making false claims?

Commentary

This is another complex question. What makes it so is that it is *hypothetical*. We don't know whether the opinions represented in Chart 1 are true or not. The question is: *If* the sample of public opinion is right and there has been a big increase in claims, can we infer that a significant number of people are making *false* claims?

Why might this be true? Well, if it is correct, as it is widely believed, that there are more people getting money for injuries, others may see this as a way of getting some money themselves. It is a sad fact that there are dishonest people who will seize such opportunities. Then again, it might be the other way round: that with the help of no-win no-fee agreements more people have begun putting in false claims, and that would explain the rise in the number of claims generally.

These are plausible hypotheses, but they are poorly supported by the data in the charts. The fact that one thing *would* explain another *if* it were true, does not not permit us to infer that it *is* true. In some circumstances this can be a powerful argument; but as you discovered in Chapter 2.10, it can also be a dangerous one. (Remember the Bayside fish restaurant. Just because food poisoning *would* explain why a restaurant has closed, it does not follow that food poisoning occurred or came from the restaurant.) In the present case, just because cheating would explain rising claims, it does not mean there is widespread cheating. There are many other equally plausible reasons why claims could be increasing in frequency, if they are.

Activity

3 Does the data in Chart 3 contradict the data in Chart 2?

Commentary

A comparison of Charts 2 and 3 is very interesting. According to Chart 2, most people evidently believe that there is an increased level of dishonest claiming going on, and half of those questioned believe that there is a big increase. But if Chart 3 is anything to go by, very few people say that they would so much as exaggerate a claim. Even those who watch daytime TV (which, we were told, carries a lot of advertising by the so-called 'ambulance chasers'), or who have seen a claims advertisement recently, say they are no more likely to claim than the general sample of the population. Remember, too, that being tempted to do something and actually doing it are two different things. Of the 13% of all adults who said they might be tempted at all, how many would have gone as far as *making* a false claim?

We simply cannot say. We also have to wonder whether those questioned in the Chart 3 survey would all have answered truthfully.

There is no contradiction here. If 13% of all adults are willing to admit to being tempted to exaggerate a claim, that could result in a considerable and increasing number of actual claims. It could certainly be a sufficient number to explain why many believe that there was a significant increase in dishonest claims. But even if that widespread belief (shown by Chart 2) is unfounded, it is not because the data in Chart 3 contradicts it. It is perfectly rational for people to hold the two views: (1) that there are more cheats than there were; but (2) that they wouldn't cheat themselves. These do not conflict; so nor do the two sets of data.

Before answering the next question, there is a further short document to read:

DOC 4

File Edit View History Bookmarks Help

How did no-win no-fee change things?

Ten years after the introduction of no-win no-fee agreements the UK Compensation Recovery Unit reported that the number of cases registered to the unit had remained relatively stable. In 2000/1 there were 735,931. The number in 2007/8 was 732,750.

For example, clinical negligence cases notified to the unit fell from 10,890 in 2000/1 to 8872 in 2007/8. Accidents at work cases fell from 97,675 in 2000/1 to 68,497 in 2007/8. Only motor accident claims have risen rapidly, rocketing from 403,892 cases in 2004/5 to 551,899 cases in 2007/8.

In its 2006 report on the 'compensation culture', the House of Commons Constitutional Affairs Committee heard evidence that personal injury claims had gone up from about 250,000 in the early 1970s to the current level, but that the introduction of no-win no-fee had coincided with this levelling off.

Lawyers dispute the claim that no-win no-fee inevitably leads to more frivolous claims and more cases generally. They say the solicitor acts as a filter, knowing that every case that doesn't make it to court or a settlement is a financial loss to the firm.

BBC News Magazine

Activity

4 Can it reliably be concluded from the information in the three charts and Doc 4 that public perceptions about false or exaggerated compensation claims are seriously mistaken?

Commentary

This is a more open question than the others, and consequently there is more than one direction that your discussion could have taken, and more than one decision you could have reached. What matters most is not which answer you gave, but why you gave it; how you interpreted the evidence. You could, for example, have noted that there is something of a contradiction between what the majority think (Chart 1) and the official figures (Doc 4, paragraphs 1 and 2). Those figures reveal that the number of claims overall has 'remained relatively stable', or even fallen slightly over the period in question, with examples of medical claims and work-accident claims both being down. If the total number of claims has fallen, it seems groundless to infer that the number of false claims has risen. You might also have added the point, already made in the comments after Activity 3, that Chart 3 casts some doubt on the belief that false claims are soaring. Your answer could therefore have been that the public perception is simply false.

However, Doc 4 also reveals that the small reduction in claims generally contrasts with a massive increase in motor accident claims in particular. If that rise is the result of an increase in false and exaggerated claims, then the public perception could be justified. A good answer to the question will therefore recognise that the evidence is inconclusive, with some of the facts pointing in one direction, some in another. This need not stop you drawing a conclusion, but it should not be too strong or overstated. If you inferred that the majority were clearly correct in their opinions, or completely wrong, without acknowledging the room for doubts, your conclusion would be unsafe.

Summary

- When presented with a source of information, whether in text form or numerical or graphical, we often draw conclusions / make inferences.
- A 'safe' (reliable, sound) inference is one that has strong support from some or all of the available data, and is not obviously contradicted by other data.
- To be 'safe' an inference or conclusion must be more than just plausible or reasonable. It must follow from the data.

End-of-chapter assignments

1 Based on the discussions you have had and the commentaries you have read, write a short answer to each of the following questions. (These are good preparation for some of the questions in Cambridge Thinking Skills Papers 2 and 4.)

a Does advertising, especially on daytime television, encourage people to make dishonest claims for personal injury?

b Does the statistical information in Doc 4 on page 134 contradict the view that claims for personal injury are on the increase?

2 How much can be inferred, reliably, from a photograph such as the one here?

Without some background information it would be unsafe to say that this photograph was *conclusive* evidence that a crime was being committed. On the other hand there is enough detail in the picture to raise suspicions. Assuming that this was a genuine action-shot, and not posed, consider these four possible accounts of what is happening in the photograph:

A The person on the left of the picture is snatching a bag from the shoulder of the other, and is about to run off with it.

B The person on the left is attempting to take something out of the bag.

C The person on the left has accidentally made contact with the person on the right.

D The two people in the picture are friends walking together by a lake or river.

Then either decide which of the above explanations can most safely be inferred, or if you think none of the above is a safe inference, suggest one that is.

Write a short justification for your conclusion, based on clues you can find in the picture. (For convenience call the person on the left of the picture 'L', and the one on the right 'R'.)

3 'It is believed in many countries around the world, including the UK, that there is a damaging "compensation culture".'

How much support is there for this belief in Docs 2–4? Your response should take the form of a short written essay.

Answers and comments are on page 323.

4.2 Explanation

In this chapter we return to an important concept that was introduced in Chapter 2.8, namely explanation. Explanation, like argument, involves giving reasons. But explanatory reasons do not lead to conclusions, as reasons do in arguments.

Examine the following short passages.

[1a] Seawater is salty. This is because the river water that drains into the oceans flows over rocks and soil. Some of the minerals in the rocks, including salt, dissolve in the water and are carried down to the sea.

[1b] The river water that drains into the oceans flows over rocks and soil. Some of the minerals in the rocks, including salt, dissolve in the water and are carried down to the sea. Consequently seawater is salty.

These are both explanations. To be more precise they are the same explanation, with slightly different wording. Typically, explanations tell us *why* something is as it is, or *how* it has come about. The explanation here consists of two reasons: (a) that rivers flow over rocks and soil; and (b) that the rocks and soil contain minerals that dissolve in the water. These two reasons, between them, explain a fact, the saltiness of seawater. But the saltiness of seawater is not a conclusion or inference drawn from [1a] and [1b]. Most of us don't need any argument to convince or persuade us that seawater is salty. We have the evidence of our senses. We can taste it, which is a good enough reason to take it as fact.

This is the key difference between an argument and an explanation. Arguments are meant to give us reasons to believe something which we did not know, or were less sure of, before hearing the argument. That is what we call a 'conclusion'. Explanations work in the opposite direction: they take something that we know or just *assume* to be true, and help us to understand it. Explanation plays a very important role in science; and it is easy to see why. One of the main goals of science – if not *the* main goal – is to discover how and why things are as they are: what causes them, what makes them happen. Once we can fully explain something, such as the saltiness of seawater, we can go on to predict or infer all sorts of other related facts or phenomena.

Need for explanations

Explanations are particularly useful when there is something surprising or puzzling that needs to be 'explained away'; or where there is a discrepancy between two facts or observations; or where there is an anomaly in a set of facts. (An anomaly is an exception: something unexpected or out of the ordinary.) If a patient's blood pressure is being monitored, and on a particular day it is much higher or lower than on all the other days, that would be classed as an *anomalous* reading, and might well lead the doctor to look for or suggest an explanation.

Here is an observation that would seem to be at odds with [1a] and [1b]:

[X] River water does not taste salty.

We are told by the scientists that seawater gets its saltiness from the rivers that flow into it. So why can we not taste the salt in the river? Unless you know the explanation, there appears to be a discrepancy here: if one tastes so strongly of salt, why does the other taste fresh? By analogy, if you poured some water from a jug (the river) into an empty bowl (the ocean), and then found that the water in the bowl tasted salty, but the remaining water in the jug did not, you would be right to feel puzzled. You would probably infer that there had been some trick, since fresh water cannot turn into salt water just by being poured!

Activity

Give a concise explanation for the fact that rivers taste fresh and the sea salty. You may know the reason, in which case just write it down as if you were explaining for someone who did not know. If you don't know the reason, try coming up with a hypothesis; then do some research, on the internet or in the library, to find out if you were right.

Commentary

The scientific explanation is as follows. The water that flows into the oceans does not all remain there. The sun's energy causes it to evaporate, after which it condenses again and falls as rain or snow. The rainwater finds its way back into the rivers and carries more salt down to the sea. This process goes on in a continuous cycle (part of what is called the 'water cycle'). The key to the explanation is that when the seawater evaporates, it leaves the salt and other minerals behind, so that over an extended period of time (millions of years) the salt becomes increasingly concentrated in the oceans. The relatively small amounts of salt that dissolve in a volume of river water as it flows to the sea aren't enough to give it a taste. Besides, rivers are constantly being refreshed by new rain and melting ice or snow.

The analogy of the jug and the bowl is therefore a bad one. It misses out the key factors of evaporation and the large timescale. To explain why seas are salty and rivers are fresh, you have to include the fact that the process has taken a very long time.

Suggesting explanations: plausibility

In the previous example the explanation is grounded on good scientific evidence. If there were any doubt about it, scientists could measure the minerals that are dissolved in rivers; they could test rainwater and confirm that it is pure, and so on. But not all facts or happenings can be explained with the same confidence, either because they are more complex, or because there is limited available data.

Science is not the only field in which explanation is needed to account for facts. Historians, for example, do not just list the things that have happened in the past, any more than scientists just list observations and phenomena. Like scientists, historians try to work out why events happened, what their causes were. For example, take the following piece of factual information:

DOC A

> In October 333 BC, Alexander's Macedonian force confronted the Persian king Darius III and his army at Issus. The Macedonians, though more disciplined than the Persians, were hugely outnumbered. Yet, surprisingly, in the furious encounter that followed, it was Darius's massive force that fled in defeat, leaving Alexander victorious.

This is neither an argument nor an explanation. It is simply a series of informative claims; a statement of historical fact. However, it is a fact in need of an explanation because, as the text says, it is a *surprising* fact. Normally, if one side in a battle hugely outnumbers the other, the larger army wins, unless there is some other reason for the outcome. If the larger army wins no one is very surprised. No one is likely to ask: How did such a big army beat such a small one? Usually it is only when the result is unexpected that we want to know why.

With this case, as with many other historical events, we don't know for certain why or how Alexander turned the tables on Darius. But there are many possible explanations which, if true, would explain the outcome of the battle, against all the odds. Alexander may have used better tactics. He may have had better weapons. He may have been a more inspiring leader than Darius. The small numbers may have made the Macedonian army more mobile, easier to command. The Persians may have been tired, or sick, or suffering from low morale. They may have been overconfident because they had more soldiers and were taken by surprise by the ferocity of their enemy, and so on. One or more of these possibilities could have been sufficient to change the course of the battle away from the foregone conclusion that most people would have predicted. We cannot say which, if any, really was a factor, still less the *decisive* factor, on the day. All we can say with certainty is that there are competing hypotheses. But we can make some valid judgements: we can assess the competing explanations in terms of their plausibility. We can ask, of a proposed explanation: Would it, *if true*, have explained why the battle went Alexander's way? If the answer is yes, it is a plausible explanation, even though we cannot infer that it is *the* explanation.

Conversely, we can say that certain statements would *not* adequately explain the outcome even if known to be true. The fact that Alexander's soldiers were Macedonian is not an adequate reason, though it is a fact. It might be adequate if we also knew that Macedonians were particularly skilled or ferocious or dedicated fighters; but on its own the fact of being Macedonian does not explain their victory. Similarly, if we were told that Alexander later became known as 'Alexander the Great', that would not explain the victory. It is his victories which explain why he was called 'the Great'. Nor would the fact that Darius's soldiers fled when they realised they were beaten count as an explanation: it would just be another way of saying that they were defeated, not a reason why.

Don't jump to conclusions

You saw both in the previous chapter and in Chapter 2.10 that some of the worst reasoning errors come from jumping to conclusions. This is a particularly strong temptation when inferring causal explanations. Suggesting explanations is fine. Assessing their plausibility is fine. But just because an explanation is plausible it doesn't follow that it is true. If it were a fact that Darius had a huge but poorly trained army, that *could* explain the Persian defeat at Issus. But so might good training explain the Macedonian victory.

Moreover, it might be neither of these. It might be something quite *unlikely*. Conceivably the battle was determined by Alexander's mother casting a magic spell, or laying a curse! This may seem fanciful and implausible. We don't really believe these days in spells and curses as real causes. But sometimes the wildest theories turn out to be correct. It is fairly well documented that in ancient times people were much more superstitious than they are today. Oracles and soothsayers were taken seriously and consulted before decisions were made; witches were burned for the evil powers they were thought to possess. Had a spell been cast, and *believed*, the psychological effect could have been quite potent. It might have filled one side with confidence, and/or the other side with terror.

Judging alternative explanations

It is all very well to say that if something *were* true it would *explain* a fact. The mistake is to move too quickly from the discovery of a satisfying and credible explanation to the inference that the explanation *is* true. Explanations need to be evaluated just as critically and carefully as the reasoning in an argument. In the case of explanations we are looking for the *best*. What makes one better than another?

There are two useful tests for judging the effectiveness of an explanation. One is to question its *scope*; the other its *simplicity*. The 'scope' of an explanation is just shorthand for how much it can explain. Staying with ancient history for a while longer, some serious defect among Darius's troops, on that fateful October day, could explain wholly why Alexander won, without requiring any extraordinary brilliance from his enemy. Perhaps half the Persian soldiers had dysentery; or there was a mutiny. These are singular explanations which, if true, would explain a singular event. But if Persian weakness was the whole explanation, it would be difficult to explain how Alexander's elite force won so many other battles, across most of the then known world, and against armies that frequently outnumbered them. By most accounts he was never defeated (at least until he reached India, the limit of his empire). It is highly implausible that each time there was some different, unique reason for victory.

Far more plausible is that Alexander, and/or his army, was immensely talented. We say that this explanation has 'scope', because as well as explaining the outcome at Issus, it explains countless other victories. Nor does it require his enemies to be defective: if Alexander was superior that was enough. Someone who wished to detract from his achievements might come up with a different explanation for each of his successes, always suggesting there was some failure in his opponents. But the standard historical claim, that he was an amazing general and brilliant tactician, is a far simpler account, as well as explaining much more.

Anomalies

Look again at Doc 4 in Chapter 4.1 (page 134) about compensation claims for injuries. In the second paragraph of the document we read that out of all the different kinds of claim only motor accident claims have risen; all other categories fell. That is to say, motor accident claims represent an *anomaly*: they 'buck the trend'. If a table were created to match the data for the years in question, it would look like the following.

Category of claim	2000/1	2004/5	2007/8	Change
Clinical	10,890		8872	down 2018
Work related	97,675		68,497	down 29,178
Motor		403,892	551,899	up 148,007
ALL CLAIMS	735,931		732,750	down 3181

What would explain this? Why would one category of claim have risen sharply (up by 37% in just four years) when all the others declined? One obvious answer is that the number of accidents had risen. If that was true it would certainly be a plausible explanation, and therefore a reasonable hypothesis.

The trouble is, it is not true. An official government report in 2011 states that:

DOC B

> . . . over the past two decades, the number and severity of accidents has reduced. Compared with the 1994–98 average, in 2010 there were fewer people killed or seriously injured in road accidents (–49%) . . . and, the slight casualty rate was lower (–39%).

Plainly the hypothesis is dead in the water. If the number of accidents explained the number of claims, the trend should have been down as sharply as it was in other categories!

Activity

Suggest and assess one or more alternative explanations for the anomaly shown in the table above.

Commentary

There are various explanatory avenues which can be explored. One is that people really are faking or exaggerating injuries, and in very large numbers. Another is that although there were more accidents in the past, people were not bothering to claim. Yet another is that advertising by law firms and others has encouraged people to claim who would not have done so in the past, because they would not have thought they had a strong enough case. The trouble with all of these is that we still have to explain why claims are down in all but this one category of motor accidents. So below are two suggestions. (There may be other plausible suggestions, besides these.)

Suggestion 1: To make a claim for an injury, you have to be able to pin the blame on someone else. In the case of a motor accident, it is usually quite easy to prove whose fault it is. (It may be very much harder to prove that a doctor or employer was negligent.) So claimants, or their lawyers, go for the easiest type of claim. That's a possibility.

Suggestion 2: The public are very aware of the incidence of road accidents. Most people have either seen one, experienced one, or know someone who has had one. Because the other categories of accident are less common, there is less awareness of them, and so people are less likely to think of making a claim for, say, a workplace accident or poor medical treatment.

There is a third possibility, of course, and that is that the best explanation is a combination of these factors. Jointly they may be more plausible than either one on its own.

When in doubt

It is often quite easy to think up a plausible explanation, or combination of plausible explanations, for some observed fact. But it is often very difficult to come to a confident decision as to the best explanation. Moreover, as we have just seen, even the most plausible explanation, which seems to tick all the boxes, can turn out to be factually untrue, thus ruling it out.

Here is another interesting example. The English word 'posh' is widely believed to be an acronym, P.O.S.H, formed from the phrase:

Port **O**ut, **S**tarboard **H**ome

This phrase, it is claimed, dates back to the 19th century, when people travelling to India and the Far East would normally go by sea. Wealthy European passengers, it was said, demanded the more expensive cabins on the port side of the ship travelling east (out), and on the starboard when returning (home), because they were cooler in the hottest part of the day. The request was allegedly written on the tickets of these passengers using the initials only. Hence the word 'posh' entered the language as a description for persons of wealth and position who could afford such a luxury.

It is a very satisfying, pleasing theory, and one which seems *too* plausible to be wrong. However, there is not a shred of hard evidence for it: tickets, for example, with the initial letters on them. Most experts (lexicographers, etymologists and so on) dispute it. There are other explanations offered, but ultimately the origins of the word are not known for certain. At any rate the acronym hypothesis looks like being a myth, sadly. But it serves as a useful warning. The port-out-starboard-home explanation is so plausible, and so pleasing, that once people have heard it, they want it to be true; and they are disappointed when they find out that it is at best dubious, or worse still false.

The danger of believing what we want to believe is a serious one. It is also one of the reasons why critical thinking is so essential to serious inquiry and the acquisition of knowledge.

Summary

- Explanations differ from arguments, despite resemblances.
- There may be many possible explanations for an outcome or event, though some are more plausible than others.
- The best explanations are those that are simple and that explain the most (have the widest 'scope').
- Even the best explanations may be wrong: they are, strictly speaking, hypotheses.

End-of-chapter assignments

1 Which of the following short passages are arguments, and which are explanations?

A Icebergs are formed from glaciers breaking off into huge chunks when they reach the sea. The process is known as 'calving'. The glacier is formed from snow, so it consists of freshwater ice. The oceans consist of brine (salt water), which has a significantly lower freezing point than fresh water. Therefore the sea around icebergs remains in a liquid state.

B Ice is less dense than liquid water. Consequently, ice forms on the surface of lakes and ponds, instead of sinking to the bottom.

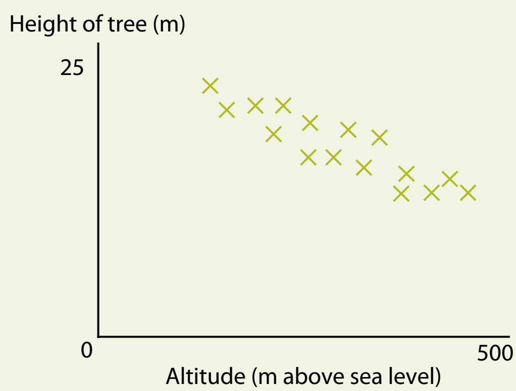
C In our ordinary everyday lives we use the word 'weight' as if it meant the same as 'mass'. For example, we 'weigh' cooking ingredients in the kitchen to tell us how much to use, not to measure how much downward force they exert on the scales. But there is a distinction, and in science-teaching it must be preserved and stressed. A bag of flour on the surface of the Earth has a different weight from the same bag on the moon: here it is approximately six times heavier. And in an orbiting spacecraft we would say it was weightless. But in all three locations the amount of matter remains the same, and this constant amount is what is meant by its mass.

D He did it for sure, because no one else had the opportunity, the motive or the training to do such a thing.

E He did it because he needed the money and because an opportunity came his way.

2 Study the following information and then answer the question that follows.

Measurements were taken showing the growth of 16 fir trees planted at the same time but at different altitudes on a hillside. The results were recorded as shown in the graph.



Which of the following, if true, would be a plausible explanation for the data recorded in the graph? (There may be more than one.)

- **A** The fir trees planted at higher altitudes tended to be shorter.
- **B** The higher up a hillside you go, the poorer the soil tends to be.
- **C** Air temperature decreases with altitude.
- **D** The higher a tree is planted, the smaller its growth.

3 Re-examine the data in Doc 3 in Chapter 4.1 (pages 131–2) and the statistics discussed in this chapter from Doc 4 (page 134). Suggest one or more explanations for the widespread public perception (shown by the charts) that claims for injury are up, when official statistics suggest that they are stable or falling. Which is the best explanation, in your view, and why?

4 **(Harder task)** Study the following short article and complete the task below.

> In 1953, New Zealander Edmund Hillary and the Nepalese Sherpa Tenzing Norgay became the first climbers to reach the summit of Mount Everest and survive. That was then. Now Sir Edmund has come out in forthright criticism of some 40 climbers who passed a dying man on the upper slopes of Everest, and left him there to die. Leaving a climber to his fate, just to get to the summit of a mountain, was unthinkable in Hillary's day. Then people climbed as members of large, organised expeditions and knew each other as friends and colleagues. Not all of them were expecting, nor even attempting, to reach the summit, because it was the purpose of the expedition just to get one or two climbers to the top. It was a team effort, and the credit was shared. Once the mountain was beaten they could all go home, satisfied that they had achieved their shared goal. Today Everest is besieged by swarms of individuals who have paid thousands of pounds for their one chance to make it, personally, to the top. No wonder traditional mountaineering morals have been thrown to the 80-mile-an-hour winds.

Is this passage an argument or an explanation? (Give a reason or reasons for your interpretation.)

- If it is an argument, identify its conclusion and summarise the reasoning.
- If it is an explanation, state what is being explained, and what the explanation is.

Answers and comments are on page 324.

4.3 Evidence

Practically anything can be evidence: a footprint, a bloodstain, a written or spoken statement, a statistic, a chance remark, an email, some CCTV footage . . . the list could run to pages.

There is good and bad evidence, just as there are good and bad reasons (for a conclusion). Judging whether or not a piece of evidence is 'good' depends on what it is being used as evidence *for*. There is nothing good or bad about a percentage of people saying they think that false claims for personal injury are on the increase. That is just raw data; a fact. It becomes evidence when it is used as a reason for some conclusion or verdict; or, to put it another way, when something is inferred from it.

From this we can see that 'evidence' and 'reason' have some overlap in meaning. However, there are subtle differences in the way we use the terms in connection with arguments. Recall once more the evidence underlying the discussions in the previous two chapters. The charts in Doc 3 in Chapter 4.1 (pages 131–2) could be cited as *evidence* for the claim that:

[A] The vast majority of people believe that more compensation payments are being made than previously, and that more false claims are being made.

But we could just as well say that the data in the charts gave *reasons* for inferring [A]. It is useful therefore to think of the numbers and percentages reflected in the charts as raw evidence (or raw data) which has been extracted and processed into the statement, [A]. [A] expresses the evidence in a form that could be used as a reason (or premise) in an argument. It also *interprets* the data, by summarising the figures and connecting the information from Charts 1 and 2.

Similarly, the figures in Doc 4 in Chapter 4.1 (page 134) are evidence for the claim that:

[B] Overall, more than 3000 fewer claims were notified in 2007/8 than in 2000/1.

These two claims between them could then be used to *argue* that, for example:

[C] The public perception of a dishonest 'compensation culture' is completely mistaken.

Expressed as an argument:

[1] According to a report by the UK House of Commons Constitutional Affairs Committee the vast majority of British people believe that more compensation payments are being made than previously. However, the Compensation Recovery Unit reported that over 3000 fewer claims were made in 2007/8 than in 2000/1. The widespread perception among the British public that there is a growing, and increasingly dishonest, compensation culture is completely mistaken.

Whether we want to call [A] and [B] 'evidence' for [C] or 'reasons' for [C] is a matter of preference. They are evidence because they are factual and statistical; they are reasons because they are used in support of a conclusion. The distinction is maintained when we say that the reasons (or premises) in [1] are *based on* the evidence provided by two sources. If [A] and [B] are warranted by the evidence from those sources, then [1] is well founded, and strengthened. Hence the evaluation of an argument like [1] ultimately comes down to evaluating the evidence from which it draws its premises. However good the reasoning may be, if the evidence base is false, then the argument is groundless.

Types of evidence

As stated at the start of the chapter, evidence can take many forms. We have been looking at one kind, namely statistical evidence. Evidence can usefully be subdivided into two categories: *direct* and *indirect*. Direct evidence, as the name suggests, is first-hand, and immediate. The most direct form of evidence is what we experience with our own senses. If I see something happening in front of my eyes, that is direct evidence – for me, at least – that it has taken place. Of course there are occasions when we are mistaken or confused about what we see or hear. Also we may misremember some of what we have experienced when we try to recall it later. But it remains true that personal experience is the most direct contact that we can have with the world and what happens in it:

Testimony

'Testimony' means giving an account. A witness statement is testimony. So long as it is an account of something that the person has witnessed or experienced at first hand, it too counts as direct evidence. This is in contrast to what is known as 'hearsay evidence'. The difference is clearly illustrated by the following statements by two witnesses:

W1: 'I know Janet Winters personally, and I saw her punch the receptionist.'

W2: 'I found the receptionist crying and she said that Janet Winters had punched her.'

It is obvious why this distinction matters. So long as W1 is telling the truth, and is not mistaken about what she saw, then Winters did punch the receptionist. W2 on the other hand may also be telling the truth about what she was told, but the receptionist may not be telling the truth about what happened. Of course, either of the two witnesses might be lying or mistaken. But in the second case there are two ways in which the evidence may be unreliable; in the first case only one.

Circumstantial evidence

By 'circumstantial evidence' we mean a fact, or set of facts, which may be used to support a conclusion or verdict *indirectly*. The facts themselves – the *circumstances* – are not in question. What is in question is what they signify, or permit us to infer. Wherever an inference is needed to get to the truth, the evidence cannot be accepted as direct, even if it is strong.

The classic example is the 'smoking gun'. A detective rushes into a room after hearing a shot. He sees a body on the floor and a man standing holding a gun with smoke still coming from the barrel, indicating that it has just been fired. The natural assumption is that the man holding the gun is the murderer. The detective testifies at the trial, reporting exactly what he has seen. The suspect pleads not guilty because, he says, he too heard the shot and rushed into the room, and picked up the still-smoking gun from the floor where it was lying. The facts – the gun, the smoke, the man holding the gun, the body on the floor – are identical. The inferences are totally opposed.

'A likely story!' you may say of the suspect's explanation. But in the absence of any other evidence, even the smoking gun is insufficient for a conviction. It is (merely) circumstantial.

Corroboration

If, however, it were also known that the suspect knew the dead man, that in the past he had threatened to kill him, that he owed the dead man money, and/or that he had recently visited a gun shop, then his guilt would be rather more probable. Each of these on its own is another piece of circumstantial evidence, but now the various items *corroborate* each other, and together provide overwhelming evidence of guilt. In fact, the smoking gun would then be virtual proof of guilt; the other evidence – without the smoking gun – would be very much weaker. For that reason the expression 'smoking gun' has come to be a metaphor for evidence which would finally settle a case. An investigation may be getting nowhere through lack of conclusive evidence, until the so-called 'smoking gun' turns up in the form of an incriminating email, or revealing photograph, or something of the kind. On its own it would not be proof of the desired conclusion; but on top of other corroborating facts it removes any lingering doubt.

The student demo

Here is a fictional scenario which will illustrate some of the concepts that we are considering.

> An unpopular congressman, visiting a university, was greeted by a large student demonstration. As he was stepping out of his car a raw egg thrown from the midst of the crowd struck him on the side of the head and broke, followed by a second and third. Soon the politician was cowering under a hail of missiles. As the crowd surged forward, he was helped back into the car by security officers and driven away.
>
> A 20-year-old sociology student, Amelia Jackson, was arrested soon afterwards. She had been seen in the crowd, and was caught on surveillance cameras shouting angrily and holding a large placard on a pole.
>
> Jackson was wearing a backpack containing some provisions she said she had bought in the market that morning. Among them was a cardboard egg box with spaces for ten eggs, but with only six eggs in it. She was taken into custody for questioning and later charged with assault, on the grounds that she had thrown one or more objects at the congressman with intent to injure or intimidate.

Activity

Discuss how strong this evidence is. On the charge of assault, as described, would you say Jackson was:

- **A** guilty?
- **B** *probably* guilty?
- **C** probably *not* guilty?
- **D** none of the above?

Commentary

The evidence available is entirely of the kind we call circumstantial. However, as circumstantial evidence goes, it looks fairly damaging. There is no *direct* evidence that Amelia Jackson did anything more than attend the demo and express her feelings. No one reports seeing her throw anything. But together with that is the fact that she had bought some eggs, and some appeared to be missing from her bag. There is therefore an accumulation of evidence. Firstly, she was present at the scene; secondly, she was actively demonstrating. Thirdly, eggs were among the objects thrown at the congressman; and fourthly – the nearest item to a 'smoking gun' – there were empty compartments in the egg box she was carrying. Do these corroborate each other sufficiently to answer the question above with A, B or C?

Not strictly. B is the nearest one could come to incriminating Ms Jackson, but D is the safest answer. Clearly there is insufficient evidence for A: guilt would require evidence that put the verdict beyond reasonable doubt. However difficult it may seem to explain away the empty places in the egg box, it is not impossible that it had nothing to do with the assault on the congressman. Plenty of other people were throwing things: Amelia Jackson may just have gone there to protest, angrily perhaps, but not violently.

On the other hand it is very *plausible*, given the circumstantial evidence, that Jackson was guilty as charged. Because of that, C would be a strange inference to make. She is no more likely to be innocent than she is to be guilty.

Additional evidence

Amelia's statement

When she was questioned, Amelia stated that she lived in lodgings with two other students and it was her turn to buy food and cook the evening meal. She had bought six eggs so they could have two each. She always bought eggs at a market stall, where they were sold singly. It was cheaper than buying ten. And she took her own cardboard container so that they would not break.

Stallholder's statement

The owner of the stall where Amelia claimed to have bought the eggs stated that he did not recognise her when shown a photograph of her. But he did make the following statement:

> 'A lot of the students buy their eggs loose. If they want a box they have to buy ten. I sell loads of eggs that way every day.'

Flatmates' statements

The two students with whom Amelia Jackson shared an apartment were questioned separately, and asked the same three questions. Both gave the same answers:

Q: 'Whose turn was it to cook that day?'
A: 'Amelia's.'
Q: 'Do you know where Amelia was going when she left the apartment that day?'
A: 'Shopping. Then to the university.'
Q: 'Was she planning to attend the demonstration?'
A: 'She didn't mention it.'

Eyewitness account

58-year-old Rajinder Choudhury, a retired headteacher, picked Amelia Jackson out of a police line-up.* He said:

> 'She's the one. She was up ahead of me in the crowd, right where the stuff all came from. She jumped up and down, and did a high five with the kid next to her. They were loving it. Then she ducked down and picked something up. The crowd rushed forward then and I lost sight of her, but later I saw her get arrested, and saw her face close up. It was her all right. Later I heard the police were asking for witnesses, so I came forward.'

** This is also known as an 'identification parade': a number of people form a line and the witness points out the one he or she claims to have seen. If the suspect is identified in this way, that is a form of direct evidence.*

Activity

Discuss whether Amelia's story is plausible (or is it far-fetched?). Is it corroborated by any of the other evidence and, if so, how strongly? Is it seriously challenged by any of the other evidence?

Commentary

It is a reasonably plausible story. Anyone who has been a student, or knows students, would agree that most of them tend to shop as economically as they can, and if eggs can be got more cheaply by taking a container and buying them loose that makes sense. What is more, if there are only three residents in the flat (or apartment) then it also makes perfect sense to buy multiples of three, and not ten. This does not prove Amelia was innocent, but it goes some way towards tipping the balance back in her favour.

What is more, there is considerable corroboration from both the stallholder and the other students with whom she shares the flat. Of course the flatmates might be protecting her by answering as they do. They were questioned separately, so the fact that they gave exactly the same answers could mean they were telling the truth. But it could also mean they had prepared what they would say. As far as the stallholder is concerned, he has no reason to say anything which would assist Amelia. Evidently he doesn't even know her.

You may have answered these questions slightly differently, but you should have registered that the circumstantial evidence against Amelia now looks less threatening. It fits just as well with *her* statement as it does with the charge made against her. What has always to be remembered with circumstantial evidence is that if it can be explained away, and the explanation is not far-fetched, no safe conclusion can be drawn from it. An evaluation of the evidence in this case would not be nearly strong enough to justify a conviction because any number of students, or others, could have bought eggs, and could have thrown them. Amelia is no longer in a special position, but is one of many potential suspects.

What about the 'eyewitness' statement? Prima facie (meaning 'on the face of it') this may seem to count *against* Amelia. However, there are a number of weaknesses in Rajinder Choudhury's evidence that you should have noted. Firstly, he did not see Amelia actually throw anything; all he saw was her reaction. The claim that she was enjoying what was going on does not mean she actively took part in it. Besides, his identification of Amelia is practically worthless, for reasons which will be discussed in the next chapter. You may also have detected a possible tone of disapproval in his statement, for Amelia or for student demonstrators generally, which could be interpreted as prejudice. He might *want* her to be guilty, for one reason or another.

Summary

- Evidence takes many forms.
- The terms 'evidence' and 'reason' have some overlap in meaning when used in the context of arguments, and care must be taken to use them appropriately.
- Evidence can be divided into two main categories: direct and indirect (or circumstantial). Circumstantial evidence requires an inference to be made from the facts to the conclusion.
- Evidence is strongest when it is corroborated by other evidence.

End-of-chapter assignments

1 Explain the difference between direct and indirect evidence, giving illustrative examples.

2 Imagine an investigation that turns on whether a certain person, whom we'll call Mr White, visited another person, Mr Green, one Saturday afternoon. Mr Green is accusing Mr White of coming to his house and assaulting him.

- A witness, Mrs Short, who lives in the flat below Mr Green, says that she saw a man answering White's description arriving by car at the house on that Saturday. Later, when she went out to the shop, she noticed the car again, and thought she saw a parking ticket on the windscreen.
- White says he was nowhere near Green's house, and produces a second witness – a restaurant owner – who testifies that White was in his restaurant on the Saturday in question, and that he stayed there all afternoon; and that his car – a white Peugeot – was in the restaurant car park the whole time. White and the restaurant owner are old friends and business partners.
- On the Sunday evening a third witness, Mr Long, who lives opposite Green but doesn't know him or White or the restaurant owner, comes forward and states that he had seen a white Peugeot parked outside his (Long's) house the previous day. He couldn't be sure of the time. The Sunday papers had printed the story of White's arrest, with a recent photograph of him getting out of the same white car at a friend's wedding.

  a How strong is the evidence provided by Mrs Short? Does it count as corroboration for Mr Green's accusation?

  b How reliable is the restaurant owner as a witness?

  c What problems are there with Mr Long's evidence?

  d Where would you look for further evidence if you were investigating this case?

3 **(Harder task)**

> 'Because of the compensation-claim culture which has grown up in many countries, advertising by lawyers and conditional-fee agreements for personal injury cases should not be permitted.'

Write a short evidence-based argument supporting or challenging this recommendation. Base your argument on the evidence found in Docs 3 and 4 in Chapter 4.1 (pages 131–2, 134), and give an assessment of how strongly you think this supports your conclusion.

Answers and comments are on page 324.

4.4 Credibility

Whilst we are often unable to say with confidence whether or not a claim is true, we can make a judgement as to its *credibility* – how justified we are in believing it. Credibility is determined by two main factors. The first is the plausibility of the claim itself. A wildly improbable claim is less credible than an unsurprising claim that fits in well with our other beliefs. But, as we all discover from time to time, something wildly improbable can on occasions be true, and something highly plausible can be false.

You may recall your role as the imaginary time-traveller in Chapter 2.3, attempting to convince a pre-Copernican population that the Earth is not a flat dish but a large ball whirling like a bucket on an invisible rope around a distant nuclear furnace . . . You can imagine their incredulity, given their other beliefs at that time. The account of the solar system that we now regard as fact was once so far beyond people's understanding as to be fantastical. If the Earth were a ball, surely the people on the sides and underneath would fall off! Isaac Newton's theory of universal gravity was not yet formulated; and that too was treated with derision when it was first announced.

Likewise some of today's new scientific theories seem improbable. Some of the implications of quantum physics are more like science fiction than science fact, especially to a non-scientist. They don't make *ordinary* sense, any more than the solar system made ordinary sense in the middle ages. The point of this is that plausibility and justification do not always correspond. Just because a claim seems implausible we should not reject it out of hand; nor should we accept a claim just because it seems plausible. We need methods of evaluating claims that are more critical than merely relying on common sense.

The sources of claims

A second factor in judging the credibility of a claim is its *source*. If the claim comes from a trusted source, we have more grounds for believing it than if we do not know where it comes from. 'Source' in this context may be an individual making an assertion; or it may be a book, an article in a newspaper, a website; or it may be a publisher. If you have found two conflicting claims, one from a book published by, say, Harvard University Press, the other from a blog or tweet by some anonymous individual, you would be likely to put your trust in the former rather than the latter.

When deciding the extent to which we can trust a source, we are looking for qualities such as honesty and possession of knowledge. There are other qualities, but those are probably the most important. We need the first for obvious reasons: we cannot trust a known liar. But however honest an author may be, we also have to be assured that he or she is well informed. An honest mistake is no more true than a deliberate lie, even though one may be more excusable than the other.

Judging credibility

However, there is an obvious problem when it comes to judging *who* to believe. It is no easier than judging *what* to believe. Suppose someone says to you: 'Look, I'm telling you the truth and I know what I'm talking about.' This is just a claim like any other. To believe in the source of the claim, you have to believe the claim; and to believe the claim, you have to believe the source. All you are doing is going round in circles! What is needed is a set of objective or independent criteria for judging a source's credibility.

What are the options? A good place to start is *reputation*. Generally speaking, a witness or claimant with a reputation for honesty, good education, status in the community, and so on, is a safer bet than someone with no such reputation – or, worse still, a negative reputation. A criminal with a record for fraud is less likely to be believed than a law-abiding citizen with a responsible job; and with good reason. It is reasonable to believe that the probability of obtaining the truth from a reputable source is greater than it is from a disreputable one.

But, as stated, this is a generalisation. Under certain circumstances it may be more rewarding to consult a convicted criminal than an ordinary citizen. If, for example, the subject of inquiry is criminality, a person who has committed crimes and knows the criminal world is likely to be better informed than someone who has no such experience. The risk that the fraudster may lie is balanced by his or her access to direct evidence. There is therefore a second criterion that we can apply, namely *experience*, or *expertise*. Ideally, of course, we would hope to find sources that are reputable *and* informed. So, for instance, a qualified researcher who has made it her business to investigate crime and criminal activity, study statistics, talk to criminals and law-enforcement officers, and analyse and verify her findings is arguably the best source of all.

Another point to be borne in mind about reputation is that it may not be deserved. You don't have to read very many newspaper articles before you come across a story of someone who has held a highly respected position but betrayed the trust that comes with it. No one's occupation or rank is a guarantee of credibility. Every so often a doctor, police officer, teacher or priest will be discovered to have acted dishonestly or stupidly. Conversely, there are countless people with no special status in society who are honest and clever. Reputation is a guideline, but it is no more than that. Cast your mind back to the eyewitness, Mr Choudhury, in the previous chapter (page 147). He was a retired headteacher, and as such would have been expected to be fair-minded and honest – especially towards students. Yet his testimony was less than wholly reliable. Maybe he was mistaken about what he saw; maybe he was a supporter of the visiting politician and took a dislike to Amelia for showing pleasure at his ill-treatment. Maybe none of these was the case, and he was telling the unvarnished truth. The point is that, although reputation is not irrelevant, on its own it does not guarantee credibility. It is one factor among many.

Choudhury's evidence is interesting for another reason. He identified Amelia. He recognised her in a line-up as the person he had seen throwing eggs. Here is his statement again:

> 'She's the one. She was up ahead of me in the crowd, right where the stuff all came from. She jumped up and down, and did a high five with the kid next to her. They were loving it. Then she ducked down and picked something up. The crowd rushed forward then and I lost sight of her, but later I saw her get arrested, and saw her face close up. It was her all right. Later I heard the police were asking for witnesses, so I came forward.'

Activity

In legal terms Choudhury's identification of Amelia Jackson would be 'inadmissible evidence'. Why is this?

To put this another way: Why is Choudhury not a *credible* witness?

Commentary

This question was partly answered in the previous chapter. Choudhury did not claim to have seen Amelia actually throw anything. He just said (twice): 'She's the one.' The most that could be pinned on her was showing excitement, and bending down to pick something up. What she picked up the witness does not say, raising the question of how he could be sure she picked anything up.

But there is another weakness in Choudhury's supposedly 'eyewitness' account. Whoever he saw in the crowd, it was from behind; and he lost sight of her in the crowd. He saw Amelia's face close up only when she was arrested. *That* was the face he picked out of the line-up, but whether or not the two women were the same we can't be sure. If Choudhury had not seen the arrest, would he have identified Amelia in the line-up? Again, we can't be sure. The credibility of Choudhury as a witness ultimately comes down to his ability to see what, and who, he *claims* to have seen.

A person's ability to apprehend information is thus another important factor in assessing certain kinds of evidence. Imagine a witness who claims to have overheard every detail of a private conversation at another table in a busy restaurant. The credibility of the claim could be tested by asking her to sit at the same table and repeat what she hears in similar, or more favourable, circumstances. If she cannot hear the words spoken in the test, she can hardly claim to have heard every detail of the alleged conversation. Her credibility as a witness would come down to her ability to hear what she says she heard, just as Rajinder Choudhury's comes down to his ability to see.

Neutrality

As noted at the end of the last chapter, there is a possibility that Choudhury may have formed a dislike for Amelia. He seems quite eager to point the finger at her, even though he has little hard evidence; and there is something in the tone of his testimony which hints at disapproval. If this were the case, it would further undermine confidence in the evidence. As well as being able and informed, a reliable source should, as far as possible, be neutral. Even the possibility of bias or prejudice is enough to lessen a source's credibility. A newspaper that has known political affiliations – as have many if not most newspapers – may report an event, or give an account of something, in a way that another publication, with different affiliations, flatly contradicts. A third commentator may give yet another version of events, different from either of the others. Any one of the three may be correct, but without any way of judging which one it is, we tend naturally, and justifiably, to place most trust in the one that has no 'axe to grind' – as the saying goes. Neutrality, therefore, is another criterion for assessing credibility.

Vested interest

One of the main reasons for doubting a source's neutrality is the discovery of a *vested interest*. Vested interests may take many forms, the most familiar being financial interest. Take, for example, the following scenario: an oil company wants to sink an exploratory well in a region where there is some alleged risk of environmental damage, and possible harm to wildlife. Environmentalists have voiced strong opposition; the oil company has hired a team of 'independent' experts to assess the risks and report on their findings. After some time the team produce a statement that there is practically no risk of contamination or other damage, and the oil company gets the go-ahead. Then just before the drilling is due to start two of the experts on the team are found to have substantial shares in the oil industry. Had the report been negative, they would have lost a lot of money; as it stands, they will make a lot of money instead.

Obviously the report is discredited, not because it is necessarily false, but because of the vested interest of two of its authors. This is an extreme example, and a stereotypical one. But it is illustrative. The general question that we have to ask is therefore this: Does the author of the claim have *any reason to make the claim*, other than believing it to be the truth? If the answer is yes, truth may not be the author's highest priority.

Corroboration

Each of the criteria that we have discussed affects how we judge a claim. Yet none of them, on its own, is sufficient to put a claim beyond reasonable doubt. A claim is, by its nature, uncertain, whoever has made it and however plausible it may be. Corroboration has been discussed at various points already, so that it doesn't need any further explanation. Of all the criteria for assessing credibility, it is perhaps the most potent. This is hardly surprising, since it is not really a single reason to believe a claim, but a combination of reasons supporting and endorsing each other.

The simplest form of corroboration is agreement – though it must be agreement between *independent* sources. If two or more people make the same claim, or express the same opinion, there is more reason to believe it than if one person alone has made the claim. It is crucial to add the word 'independent' here, because if it is found that one person has influenced the others, the added credibility is cancelled, for they are effectively making a single, repeated claim rather than several separate claims which genuinely corroborate each other. You may recall that in the previous chapter, the police interviewed Amelia Jackson's flatmates separately. The fact that they still gave the same answers added to the credibility of what they said, but there was still the possibility that they had conferred in advance, and anticipated the questions. Indeed, if it is known that they had conferred, that would actually *detract* from their credibility, for it would have to be explained why they had conferred. If they were both simply telling the truth, there would be no need to confer.

Corroboration is at its most potent when there is agreement between different kinds of evidence: for example, when statistical evidence bears out what several independent witnesses have said, and the circumstantial evidence all points in the same direction. By the same token, credibility is at its lowest when there is a lack of corroboration, or disagreement.

Summary

- In the absence of knowledge or certainty about the truth of some portion of evidence, we often have to rely on its credibility.
- There are a number of criteria by which we can judge credibility:
  - the plausibility of the claim or claims themselves
  - the reputation, expertise, independence and/or neutrality of the source
  - the ability to have seen or perceived what is being claimed
  - the absence of vested interest (or motive for saying one thing rather than another)
  - corroboration by other evidence or from other sources.

End-of-chapter assignment

This assignment can be completed individually in writing, or as a group discussion. (If you choose the second of these, you should also make notes on what you discussed, what decisions you came to and, most importantly, why you reached those decisions.)

Read the following passage carefully and answer the questions that follow.

PARTYTIME STAR ACCUSED OF STEALING SONG

by Jan Ewbank, Arts and media correspondent

The superstar band Partytime, and their lead singer Magnolia, came under more fire yesterday when it was alleged that their number one hit, *If You Knew*, was originally written by an unknown schoolteacher who has never received a cent in recognition.

The disclosure came hot on the heels of criticism that Magnolia has cashed in big-time on her much publicised, so-called charity visits to developing countries last year.

Now, if the latest accusations are true, her most famous song isn't even hers to sing. It appears that the tune and chorus of *If You Knew* were written ten years ago by Sarah Berry. Sarah had worked as a volunteer in Africa before training as a teacher. At college she met Magnolia, then Maggie Coleman.

'The college did a charity concert, and we were both in it,' she recalls. 'I wrote a song for it, and Maggie sang it. I didn't think it was all that good, and never gave it another thought afterwards. It was only when I heard *If You Knew* that I recognised Maggie – and my song.'

Magnolia hotly denies the claim. 'I don't even remember anyone called Sarah Berry,' she says. 'I wrote *If You Knew* because I was fed up of hearing rich people whingeing when there's real hardship and suffering in the world, like we saw in Africa. Whoever she is, she's on the make. If she's got any proof she ought to produce it – or otherwise shut up.'

Partytime's road manager Paco added: 'I was around when Mags was writing it. It came straight from her heart after the tour. We write all our own songs. People are always coming out of the woodwork accusing stars of plagiarising – you know, stealing their songs – once they're famous. This Berry woman's not the first and won't be the last.'

I visited Sarah in her rented one-room apartment. She dug out an old photograph album and scrapbook. In it was a picture of a very young Magnolia fronting a student band. Under it were the names of the group, including 'Maggie Coleman'. There was also a handwritten song with guitar chords, but no tune. The chorus runs:

'If you'd been to the places I've been / And seen the things that I've seen / You wouldn't be sighing that life is so trying . . .'

Magnolia sings the chorus of *If You Knew* in front of a big screen showing harrowing images. Her chorus goes: 'If you knew the things that he's seen / Been to the places she's been / You'd have less to say in your self-centred way . . .'

When I confronted her with this evidence, Magnolia said: 'OK. Maybe this woman did stand on the stage with me once when we were at college. Maybe we sang a song together and some bits of it stuck in my mind. That doesn't mean she wrote it, whatever she pasted in her scrapbook. It's so long ago I just don't remember. As for the tune, that was all mine, and that's what really counts.'

I next visited Professor Jon Rudenko, who has been called as an expert witness in many high-profile plagiarism wrangles. He told me the chord sequence in Sarah's scrapbook would fit the melody line of *If You Knew*, although it would not be impossible for the same chords to fit two quite different tunes. Asked to estimate the odds against two tunes having these same chords by chance, he said: 'Upwards of twenty to one. Not huge. It's quite a common sequence in popular music.'

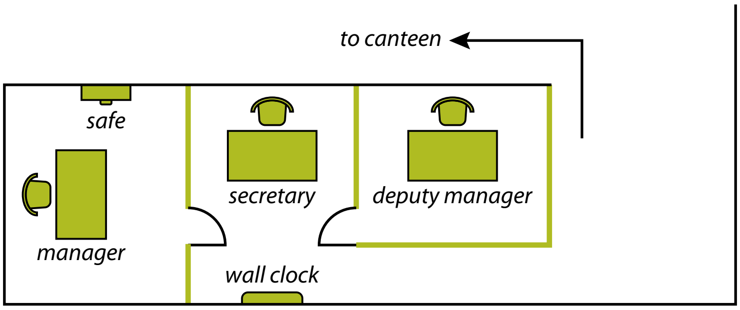
The jury is out on this one, but whatever the verdict, it's another unwanted smear on Magnolia's already tarnished reputation.

1 Assuming it has been fairly represented by the author, decide how credible is the testimony given by each of the following:
   - Magnolia
   - Sarah Berry
   - Paco
   - Jon Rudenko.

   Base your assessments on the criteria discussed in the chapter.
2 Identify and assess one or more pieces of circumstantial evidence reported in the article.
3 As a source of information, how reliable do you consider Jan Ewbank's article to be in its reporting of the dispute? On what grounds might someone question its reliability?
4 Imagine you were an informal jury considering the evidence contained in the article. What would your verdict be, and why?
5 Assess the language used by the author Jan Ewbank. Do you consider it to be a fair and neutral report, or judgemental, perhaps even biased? What evidence is there, if any, of partiality towards one side or the other?

Answers and comments are on page 324.

4.5 Two case studies



Case One: Who's telling the truth?

The diagram is a plan of the Management Suite on the first floor of a firm's premises. Some money, in a brown envelope, has gone missing from the safe, and an investigation is underway.

General facts

Three people are employed in the Management Suite:

- the manager (Mrs Mann)
- the deputy manager (Mr Depp)
- the secretary (Rita).

Only the manager knows the safe combination.

Secretary's evidence

'I took the manager her morning coffee at 9.30. I noticed the safe was open and the brown package was visible inside it. I took her the mail at 10.00 and it was still open. Immediately after that the manager left her office and went straight along the corridor. She was away about 20 minutes. Mr Depp, the deputy manager, came out of his own office and visited the manager's office twice that morning: once at about 9.45 and again while the manager was away – I couldn't say the exact time.'

Manager's evidence

'I was away from the office for about 20 minutes. I didn't lock the safe. I quite often don't lock it in the daytime, and nothing has ever gone missing before. I am fairly certain the deputy manager's door was open and his office was empty when I left, and it was still empty when I returned. It was when I got back that I realised the money was missing.'

Deputy manager's evidence

'I went into the manager's office only once, and she was there at her desk. At around 10.00 I went to the canteen because there was a driver who had a problem to discuss – an argument he had had with another worker. It took over half an hour to sort out.'

Driver's evidence

'I was with Mr Depp in the canteen from around 10.00. We talked for quite a long time. I didn't notice how long. We were sorting out a personal problem.'

Activity

Following on from the discussions in the previous chapter, assess the evidence given above. Use it to ask yourself who, if anyone, is *not* telling the truth.

Commentary

What we have here are two conflicting stories. The secretary, Rita, claims that the deputy manager went into the manager's office twice, once while the manager was in there and once after she had left. The deputy manager, Mr Depp, confirms that he went into her office the first time, but denies the second. He claims that during the time he was alleged to have entered the manager's office he was in the canteen talking to a driver. At some time during all this, some money went missing from the safe. The secretary's statement, if true, casts considerable suspicion on Depp.

We will start by considering the witnesses themselves. The three occupants of the Management Suite are the manager, the deputy manager and the secretary. The driver is also a witness. Their ranking in the company is probably in that order. So does this mean we should rank the reliability of their evidence in the same way: the manager's more than the deputy's, the deputy's more than the secretary's, the driver's least of all?

In a word, no. In some cases there may be more reason to trust a manager's judgement over a junior employee's, on the grounds of their respective qualifications and experience. But we are not talking about *judgement* here, only about honesty and accuracy. You may argue that a manager has more to lose than a secretary. But it would be quite unjustified to assume that *therefore* the secretary is more likely to be dishonest. It would be even more unjustified to assume that the secretary was less likely to be accurate in her statement. If you looked carefully at the evidence you will have seen that it is the secretary who is the most exact in the information she gives, the manager the most vague and imprecise. And it should not be overlooked that the manager left the safe unlocked, suggesting some absent-mindedness or carelessness on her part.

What about the statements themselves: are they equally *plausible*? On the face of it, yes. There is nothing improbable about Depp going into the manager's office, or about his going to talk to the driver. They are both normal, unsurprising events in a typical office day, and there is no obvious reason to believe one rather than the other. It is only because they conflict that we would question them at all. But since they do conflict, we have to question them.

Corroboration

Where Depp's statement scores over Rita's is that it gets some measure of corroboration both from the driver and from the manager herself. Rita has no witnesses or circumstances to corroborate her counter-claim. However, the corroborating evidence is not 100% solid. The manager says that she is 'fairly certain' the deputy manager's door was open and his office was empty when she left. The driver, too, gives rather vague estimates: 'I was with Mr Depp . . . around 10.00. We talked for quite a long time.' Conceivably, by this reckoning, the meeting could have ended in time for Depp to go back to his offices before Mrs Mann returned. So, although the corroboration of two other witnesses adds to Depp's credibility, it does not by any means remove all doubt about his version of events.

Suppositional reasoning: 'What if . . . ?'

So far it looks very much like a case of one person's word against another's. But there is a way forward. It involves a very useful technique known as suppositional reasoning. Suppositional reasoning typically starts with phrases such as 'Supposing . . .' or 'What if . . . ?'

For example, suppose that the secretary is right: that Depp did go into the manager's office while she was away, which was also during the period when the money went missing. What would follow from this? It would mean, of course, that Depp had an opportunity to take the money. It would also mean that he was lying when he said he was away from the offices throughout the manager's absence, unless he had mysteriously forgotten where he had been that morning. And it is hard to understand why he would lie unless he had something to hide. But would he really have walked into the manager's office, taken the money and walked out again with the secretary sitting at her desk, then simply denied it in the hope that he would be believed and not her?

If the secretary is right it also means that the manager wrongly thought the deputy's office was empty when she passed it on two occasions; and that the driver's statement is questionable. In other words, we would have to disbelieve *three* people's statements in order to believe the secretary's statement. For them all to be wrong would be quite a coincidence. For them all to be lying would require some mysterious explanation.

So although the secretary's story seems credible enough in itself, when we subject it to this kind of critical examination, it turns out to have some unlikely consequences. A consequence is something that follows from something else. If we find that a certain claim, or version of events, would have puzzling consequences, that must throw some doubt on the claim.

What if we accept the deputy manager's account? First of all it is consistent with what two other witnesses are saying, and that has to be in Depp's favour, even if their statements are a bit vague and uncertain. But, of course, it means that Rita is lying. It also means that Rita was alone in the Management Suite for about 20 minutes when the money went missing. She therefore would have had a much better opportunity than Depp to steal and hide the money with no one around to see her. If she did steal the money, she also had a motive for trying to pin the blame on someone else.

If you compare the two suppositions, Depp's story has much more believable consequences than Rita's. This does not put it beyond reasonable doubt that the secretary is a thief and a liar, but it does make her story harder to swallow.

Suppose the deputy manager planned the theft with the driver. He waited for the manager to leave her office, walked in there as the secretary reported, took the money, and later slipped out to give it to the driver and tell him to say they had been in the canteen all the time. So that the manager would think he was not in his office he left the door open and hid behind it as she passed. Is this all *possible*? Yes, it's possible. But it is unlikely. For a start, how would Depp know when the manager was going to leave? This, added to the fact that the secretary would see him, makes such a possibility too remote to take very seriously.

On balance of probabilities, it seems that the secretary's version of events is altogether less credible than Depp's. And that is the most rational conclusion.

Case Two: Collision course

Two drivers – Ed Farr and Ray Crowe – collided and spun off the track in heavy rain in the last race of the season earlier today. Neither driver was injured, but the incident put both cars out of the race, leaving Crowe as World Champion for the second year running. Before the race there was just one point between the two drivers. If Farr had finished the race ahead of Crowe, he would have moved into first place and taken the title.

After the race an inquiry was called for into allegations that Ray Crowe had *intentionally* collided with his opponent's car. The following items of evidence were noted:

[1] **Farr's team manager** reacted furiously by claiming that Crowe had deliberately swerved and forced their driver off the track as he tried to overtake on a notorious S-bend* known as the Slide. 'It was no surprise, either,' she added. 'With Ed out of the race, Crowe knew he had won the championship. Of course he meant to do it.'

[2] A **television camera team** filmed Crowe walking away from his wrecked car. He appears to be smiling as he removes his helmet. He says to reporters: 'I hope you're not all going to blame this on me. I just held my line**, and that is completely within the rules.' Later he added: 'It was all Ed's fault. He could have killed us both. It was a crazy place to try to overtake. He has only himself to blame.'

[3] **Ed Farr** stated: 'There was plenty of room to get past if Crowe had held his line**. He waited till I came level, then drove into me.'

[4] Today's race winner **Waleed Akram**, who was just behind the two cars at the time, commented: 'That's motor racing. Ray had earned his one point lead, and he was just defending it. If it had been the other way round, Ed would probably have done the same. Everyone was expecting something like this to happen.' Asked if he had seen Crowe swerve, he said: 'Maybe not a "swerve" exactly, but he could have avoided the crash. Anyway, it stands to reason that he would take Ed out of the race if he got the chance. It's not the first time he's done something like that.'

[5] **Computer-generated images (see right)** were made from trackside cameras, recording the positions of the cars just before, and just as, they made contact.

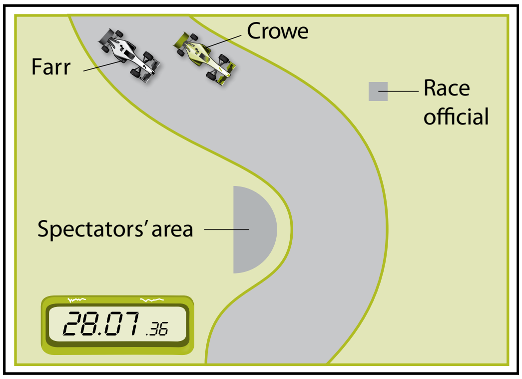
[6] **A race official**, stationed on the bend, reported: 'There was a lot of spray as the cars rounded the bend. Farr tried to cut through on the inside. He was almost past when the two cars touched. They both spun and ended up on the verge opposite. It is hard to tell, but to me it just looked like an accident.'

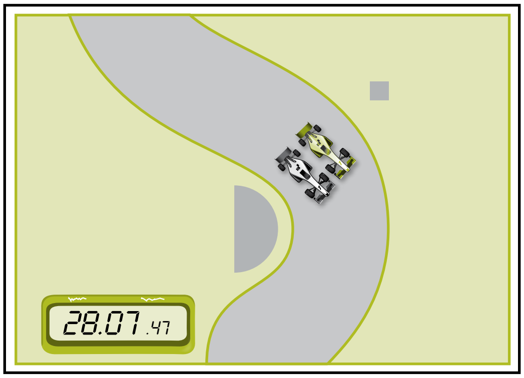
[7] **Journalist Gudrun Brecht** added to the controversy by reporting that she had been at a party two days before the race and that she had heard Crowe openly boasting that he would 'do anything necessary to win the championship'. She wrote: 'I know Crowe well, and he makes no secret of his determination to win, whatever it takes.'

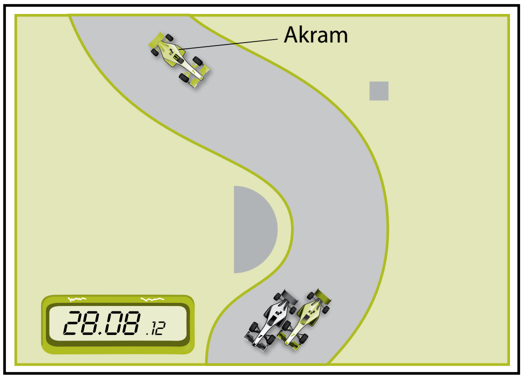
[8] **On record**: Crowe was involved in two similar controversies in previous seasons, but on both occasions he was cleared of any blame.

** S-bend: a double bend in a road or track, shaped like the letter S.*

*** Holding your line: staying on your chosen course, not swerving or cutting across another driver. The rules of the sport permit a driver to choose his line through a bend, but not intentionally to cause a collision.*







Answer each of the following questions and compare your answer with the commentary that follows. The questions are similar to those set in Cambridge Thinking Skills Paper 2.

Activity

1 What is the team manager's argument for blaming Crowe for the incident? How strong is her statement as evidence against Crowe?

Commentary

The manager's argument is based on what she sees as Crowe's motive. She is pointing out a *fact* when she says that with Ed out of the race Crowe would win the championship. But she infers too much from it. Besides, she is probably biased and sounds angry. As Ed Farr's manager she has a vested interest in the outcome of the race. We say someone has a vested interest in an outcome if they are likely to benefit, financially or otherwise, if the decision goes one way rather than the other. Crowe, Farr and the manager all have an obvious vested interest in the outcome of this case. The other witnesses may or may not, but there is no reason to think they have.

We don't know if the manager actually witnessed the incident first-hand, but even if she did, it would be very hard to say that one of the drivers had acted intentionally. She uses the tell-tale phrase 'of course' to show that she is *assuming* there was intention on Crowe's part because it would be to his advantage.

On its own this is not strong evidence. The fact that someone stands to gain from some act or other does not mean he or she will commit that act. However, taken together with other evidence, motive does add *some* weight to the argument. Let's put it this way: if he didn't have a motive, there would be much less reason to think Crowe caused the crash deliberately.

Activity

2 How reliable is Akram as a witness? Consider what he has to say in the light of other information and evidence available. What impact should his statement have on the outcome of the inquiry?

Commentary

Akram claims to be an eyewitness. However, given what the race official says, and taking into account his (Akram's) position on the track when the collision occurred, it is doubtful whether he could have seen very much. Like Farr's manager, Akram bases his assessment of what happened partly on Crowe's motives, but also on his past record. He says 'it stands to reason' that Crowe did it on purpose.

Unfortunately, it doesn't really stand to reason at all. Akram is unable to say that Crowe actively 'swerved', yet he is prepared to say he allowed the crash to happen. As a professional racing driver, we can give Akram credit for having the expertise to make such a claim: he would know better than most people if an accident could have been avoided or not. But that is not to say that Crowe let it happen intentionally. It could just have been carelessness that caused it, or poor visibility. Akram is not really in a position to make such a judgement objectively.

Activity

3 How seriously can you take the evidence provided by Gudrun Brecht?

Commentary

This evidence cannot be taken very seriously at all. It is a classic case of hearsay evidence: she 'heard him' boasting that he would do anything necessary to win. We don't have any means of knowing if these were his exact words, or if they were a journalist's colourful way of presenting them. Besides, even if they were his exact words, they don't really tell us how far Crowe was prepared to go. Maybe he meant he would try as hard as he could, but would draw the line at risking his life and the lives of others just to get the title.

Also, Gudrun claims, 'I know Crowe well.' She doesn't say whether she likes or dislikes him, but from the statement she makes it is more likely that it is dislike. If she were fond of him, she would hardly imply so strongly that he was prepared to cheat. This makes her a less reliable witness, since her neutrality is in question. As sports-page gossip, what she says is of some interest, but it ought not to count for much as evidence of guilt in an official inquiry.

Activity

4 Can you draw any conclusions from Ray Crowe's behaviour and his comments as the camera team filmed him walking away from the crash site?

Commentary

Crowe's actual denial counts for very little, for obvious reasons. If he had collided with Farr in order to win the championship, he would be just as likely to deny that it was intentional. It could also be said that he was very quick to deny it, doing so even before he had been asked about it. On the other hand he may have expected a hostile reaction from the media, whether he was guilty or not, especially given his apparent reputation.

The smile he appears to have as he takes off his helmet may be a smile of satisfaction, or of relief. It may even be a sarcastic smile, at seeing the cameras and the television crew appear so quickly. Smiles and other facial expressions are often seized on by the media, and conclusions drawn, but it would be wrong to interpret Crowe's apparent smile as a sign of guilt.

As for his own defence, which takes the form of a pre-emptive attack on Farr, there may be some justification for what he says. We do not have a great deal to go on other than the three computer-generated images of the incident. These are the focus of the next question.

Activity

5 What evidence can be found in the images to support either of the two sides involved in the dispute?

Commentary

Unlike almost all the evidence supplied by witnesses, the images are hard evidence. The saying 'the camera never lies' is often challenged because nowadays almost anyone can fake or 'doctor' a photograph. But it is still true that the camera itself doesn't lie: it is what is done with the photographs afterwards that can create deception. Anyway, we will assume these images are an accurate reconstruction.

One way to approach this question is to draw on the picture the line you think Crowe would have chosen through the S-bend. Obviously racing drivers like to steer through bends by the fastest route, but if other cars are in their way they have to go wide to get round them. Remembering what the rules are, do you think Crowe keeps strictly to a natural line, or does he steer over into Farr's path as he comes level and so cause the collision?

Read again what the two drivers had to say and what the race official saw, and, on the strength of the pictures, decide whose story is more believable. There is no right or wrong answer to this: you have to draw your own conclusions – and support them with the evidence as you find it.

End-of-chapter assignments

1 On the basis of the evidence, can it be concluded that Ray Crowe intentionally collided with Farr? Give a short, reasoned argument to support your answer.

2 The principal of a college is investigating allegations that one of the students, Corinne Blake, has cheated on multiple occasions by: copying essays found on the internet; asking friends to write assignments for her; and taking revision notes into an exam. Corinne denies all the allegations and says that the other students are accusing her out of spite.

The evidence in front of the principal consists of three items, all messages:

A An anonymous email sent to the principal. It reads: 'I heard Corinne Blake tell a friend she had downloaded stuff off the internet and got an A for it. They were both having a good laugh about it. I thought you should know.'

B A statement by a student saying that she had been sitting behind Corinne in an exam and watched her unfold a page of notes and read it under the desk before answering one of the questions. She could not say what the notes were about specifically.

C An intercepted text message from a postgraduate st udent to Corinne's phone, saying: 'Cant believe u r bribing me. Wot kinda friend r u!!! Write your own essay.'

Rank these three items according to the weight you would give them, stating reasons for your assessments.

3 Comment critically on the following further item of evidence given to the principal investigating the allegations against Corinne Blake. It is from a report by an educational psychologist who interviewed Corinne:

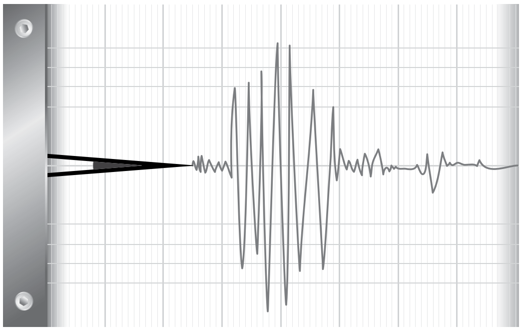
> 'Miss Blake seemed agitated and anxious. Her mannerisms and body language were consistent with the behaviour of someone who has something to hide. When asked to repeat the answers she had given to some of the questions in the exam she gave a number of incoherent responses which suggested to me that she had less knowledge of the subject matter than her written answers might have indicated. I do not believe she could have given those answers without external help of some sort.'

4.6 Critical thinking and science

Science is a highly disciplined form of critical thinking. This is not surprising, since science is a methodology that is reliant upon *evidence*, in particular the evidence provided by observation and experiment. Scientists make observations and use them both to construct and to test their theories. A scientific theory is only as good as the evidence on which it is based and the reasoning by which scientists proceed in drawing their conclusions. All that has been said about not leaping to conclusions, or making unwarranted assumptions, applies with particular relevance to science.

An *observation* in scientific terms is any fact that can be verified by experience: for example, evidence of the senses. It means more than just visual data. If I suddenly sense the ground trembling beneath my feet, or hear a rumbling sound, or see a cup fall off a shelf, these are all observations. I may not know what has caused them: they may be indicative of an earthquake, or just a heavy vehicle passing on the road, or a controlled explosion in a nearby quarry. Without further evidence I have no way of inferring which, if any of them, is the correct interpretation. But the experience itself – the observation or sensation – remains the same whatever its cause turns out to be.

Of course, people can be mistaken about what they experience. We sometimes imagine things, or misremember them. A reliable scientific observation is therefore one which cannot be dismissed easily. If many people describe having had the same experience at the same time, that is better evidence than one person's word. The term we use for this, as introduced in Chapter 4.3, is 'corroboration'. Observations may be even more trustworthy if they are detected and recorded by instruments or sensors. Moreover, instruments can often pick up information that human senses cannot detect. They can make measurements of things where humans can only estimate crudely. A seismometer, for instance, is a device for measuring earth tremors. It can give accurate readings of movements far below the ground that no human would notice or find significant. Such readings are also 'observations'.



If they are made accurately, these are *facts*; but without accuracy they remain observations. Their importance, scientifically, lies in the use they can be put to as evidence for hypotheses or predictions: for example, the causes of earthquakes, or the risk of earthquakes in a given region. For such purposes *single* observations are rarely sufficient for establishing conclusions. A large part of scientific inquiry therefore involves the analysis of collections of data to identify patterns and correlations. Observations on their own can be thought of as 'raw' data. To function as evidence this raw data generally has to be collated and interpreted, often in the form of tables, graphs, reports and so on. A critical question therefore arises as to whether the processed data is fair and objective, or whether it distorts the facts in one direction or another. For instance, if the observation concerns a sample of data, is it a representative sample; or is it selective, exaggerated, biased or misleading in any way?

Good science is self-critical on just these points. Not only do serious scientists, whose aim is to discover the truth, check their own findings with care and make every effort to avoid reasoning errors, they check each other's work critically – a procedure known as 'peer review'. Among the flaws that they look for are two which have been discussed in previous chapters: over-generalising from limited examples, and confusing correlation with cause. Both are easy errors to make.

Scientific method is not only of interest within science. Any evidence-based reasoning should be subjected to the same critical standards as good science. We see scientific methods being applied in subjects as diverse as history, economics, sociology, psychology and education, and many more.

An example: social networks

A field of study in which many modern scientists have developed an interest is social networking, especially with the coming of phenomena such as Facebook, Twitter and so on. Are these purely modern and human inventions, or are they products of our natural animal evolution? A key question is:

Do other animals, besides humans, form 'social' networks?

Activity

Take some time to think about and/or discuss the question above. You do not need any specialist knowledge to do this: it is an *open* discussion, an exploration of ideas. However, you should try to bring some examples or evidence into the discussion. You can use your own observations and experiences as evidence – for example, documentaries you have seen of animals in the wild, and the way they behave. Think, too, about what is meant by 'social' in this context.

Commentary

As stated, this is an open discussion, so there is no single right way to tackle it. The only stipulation is that you should provide more than just opinions. If all you say is that you think animals do behave like humans and form social networks, or that they don't, this would not be a critical response. Nor would it be a scientific one. For the response to be critical it would have to include reasons as well as opinions and judgements. For it to be scientific it would have to have some evidential basis.

You are also asked to consider the meaning of the term 'social'. It's all very well to say that many animals live in groups – herds, shoals, flocks, packs, colonies, etc. – but it is another thing altogether to assert that these are *social* groups. On the other hand it is unjustified to claim that social groups belong only to humans unless you can say what you consider so special about human groups. Recognising and defining key terms in a text is one of the essential skills of any critical thinking assignment. In this case it is very obvious that the whole discussion turns on the definition of a 'social' group. For example, compare a group of friends or work colleagues, or a military unit, with a herd of wildebeest or with a shoal of fish. Clearly these are all groups of one kind or another. But what, if any, are the key differences? It is generally argued by zoologists and others that herding is an instinct for self-preservation by the individuals in the group. If a wildebeest strays from the herd it is more likely to be singled out for attack by a predator. A lone animal is easy prey. The best place for a wildebeest to be is near the middle of the herd, so wildebeest have developed a herd instinct for reasons of survival. There is no obvious evidence that within the herd wildebeest form relationships, and less still that fish form relationships within the shoal. If all that is involved in herding is each individual's instinct for self-preservation, there is nothing 'social' about that.

Now that you have had a chance to discuss and think about the issues and terms involved, we can turn to a text which deals with the subject on a more scientific level.

A scientific study

A scientist who has undertaken extensive research in this area is Robin Dunbar, Professor of Evolutionary Anthropology at the University of Oxford. His research focuses on the evolution of sociality in the primates: the order that includes apes, monkeys and humans. He is particularly interested in the structure and dynamics of human social networks. The following extracts are from an article published in *New Scientist*. Although they all come from the same article, they are presented here as four separate documents to make them easier to refer to in the activity which follows.

DOC A

We tend to think of social networks as being distinctly human. In fact, they occur wherever animals live in 'bonded' groups – where individuals gather together because of their personal relationships rather than being forced to by environmental factors such as a food source or safe sleeping site. Bonded groups are found among all primates and a few other mammals . . . Such networks have benefits, but they are also costly to maintain and are only an option for the smartest of species.

DOC B

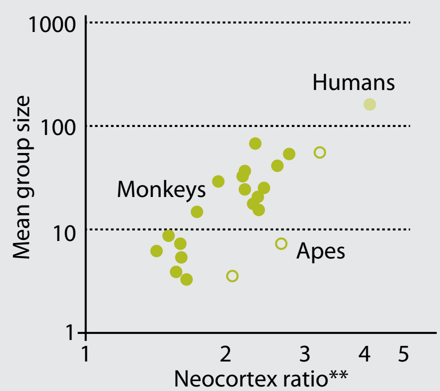
Monkeys and apes create and nurture social relationships by grooming* each other. The physical action of being groomed is rather like massage and triggers the release of chemicals called endorphins. This creates a light euphoria that seems to make it possible for animals that groom each other to build a relationship based on friendship and trust. The average time spent grooming by members of a species correlates with the size of their social group. Those, such as gibbons, which typically live with only three or four others, groom for 5 per cent of their day at most. Baboons, meanwhile, live in groups of 50 or more and can spend as much as 20 per cent of their time grooming. However, as group size and time spent grooming increases, this social effort is concentrated on fewer and fewer partners.

Although we use grooming in intimate relationships, the very intimacy of the activity makes it ineffective as a tool for bonding our large social groups. Instead, we have evolved alternative ways to create the same endorphin surge on a bigger scale. One of these is laughter, another is communal music-making. Language, too, plays an important role – not only can we speak to many people at the same time, we can also exchange information about the state of our networks in a way that other primates cannot. Gossip, I have argued, is a very human form of grooming.

** 'Grooming' means tidying, removing dirt or nits from fur, etc.*

DOC C

Primates with a large social network have bigger brains*

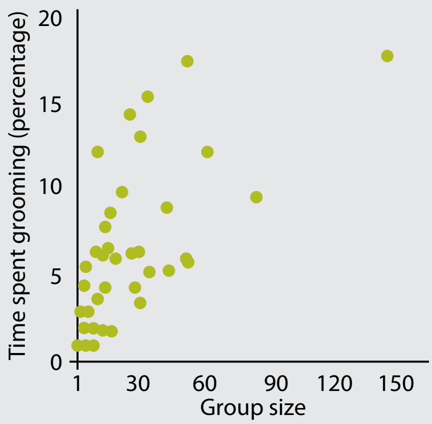


* In Doc C 'bigger brains' means more than just brain volume. It is the proportion of the whole brain that is associated with higher functions like perception and communication. This is called the 'neocortex'. In humans the neocortex is the part of the brain which enables language, reasoning and conscious thought.

** Neocortex ratio = neocortex volume divided by volume of the rest of brain

DOC D

The larger a primate's group size, the longer they spend grooming to cement bonds



The four documents above – two textual and two graphical – are typical of those used for critical thinking questions in many examinations. Once you are familiar with the content, have a go at answering the questions below, each of which is followed by a short commentary, discussing the question and suggesting a suitable answer (or answers).

Activity

1 In the paragraph marked Doc A, what viewpoint is the author challenging, and on what basic grounds does he make the challenge?

Commentary

This is a very straightforward question. In Doc A the author sets out his target for what follows: the view that social networks are distinctly human. He challenges this view by claiming that social networks occur wherever there are 'bonded' groups, defining bonding as gathering together for more than just physical reasons such as food and security. This is the key difference between a social gathering and a mere herd or pack. According to Professor Dunbar, these bonded groups occur among many animals, including all the primates – apes, monkeys, humans, etc. – and some other mammals too.

You are not asked to assess the evidence, nor to evaluate the argument. To do that you would need to have read more widely. But it is clear that if the author is right in saying that primates form groups that are bonded by relationships, rather than mere environmental factors, then there are grounds for the claim that social groups are not distinctly human.

Activity

2 Does the data in Doc C support the view that a species' average group size tells us something about how 'smart' (i.e. intelligent) it is?

Commentary

We will begin by saying something about the data itself. Doc C is a scatter graph. Scatter graphs are intended to show correlations. Here the correlation being investigated is between brain size (the horizontal axis) and average group size (the vertical axis) in primates. 'Brain size', as explained in the notes, is a shorthand for something rather more complicated, namely the amount of an animal's brain that is associated with higher levels of intelligence. It is measured as a ratio, and obtained by dividing the volume of the whole brain by the volume of the neocortex. In humans the neocortex is over four times the volume of the rest of the brain, making the human brain the 'biggest' in the defined sense.

You may have noticed the somewhat unusual scale that has been used on the graph, especially on the vertical axis. The lowest band shows group sizes between 1 and 10, the second between 10 and 100. Mathematicians among you will recognise this as a logarithmic scale. It is a useful device when the range of values is large, as it is in this case. Group sizes start at about 3 and rise to around 150 (in humans). With an ordinary scale the graph would either have to be very tall, or the dots would be packed so tightly together that they would be difficult to tell apart.

Each dot or circle on the graph represents one species. The pattern of the dots suggests that the primates with bigger brains tend to form larger groups. Most of the monkeys with a brain size rated at less than 2 live in group sizes smaller than 10. Those with brain sizes between 2 and 3 form much larger groups: anywhere between 10 and 100. With apes, too, there is a correlation between brain and group size, although their groups are slightly smaller in relation to their brain size. Only humans form groups of more than 100.

So, to get back to the main question, the graph does show a general correlation between brain size (as it is defined) and group size, both in monkeys and in apes. Humans top the table on both counts, and humans are very smart – or so we tell ourselves. Therefore it could be argued that group size is an indicator of smartness: the larger the group, the greater the intelligence. The author even offers an explanation for this in Doc A. Social networks, he says, are ‘costly’, and only the smartest species could manage them. (By ‘cost’ he probably means the time and effort that they take up, which could be spent eating or hunting instead.)

But there is a proviso. Yes, the data on group size and brain size does tell us something about the smartness or intelligence of a species, but only if the groups in question are ‘bonded’ or ‘social’ groups. We know from the earlier discussion that big herds, shoals and so on don’t count as social groups. If they did then there would be some animals (e.g. some fish) that have very small brains but gather together in groups of thousands. The graph *on its own*, therefore, is selective. It relates only to primates, which are already understood to be at the smarter end of the scale of animal intelligence. You may want to qualify your answer by saying that the graph tells us something about smartness and *bonded* groups.

Another point you might make is that the graph tells us only about the correlation between group size and brain size (or *neocortex ratio* to be precise.) Does this permit us to make the further claim that animals which form bigger groups are ‘smarter’? To put it another way, is there an assumption that brain size equals smartness? The problem is that we need a definition of smartness that connects it with brain size. Without that it would be jumping to a conclusion to say that group size – even bonded-group size – indicated intelligence.

Another point still that you could raise is that although there is a general match between group size and brain size, there are some exceptions. As we observed earlier, the three ape species apparently form smaller groups than many monkeys with similar-sized or even smaller brains. If apes are more intelligent because their brains are larger, why would they live in smaller groups? This at least requires some explanation if we want to make the connection between group size and smartness.

So a good answer to a question like this is more than simply yes or no. You may be satisfied that the graph does tell us something about the smartness of a species, but you must be able to say why you reached this judgement. You should also be prepared to qualify your answer by adding reservations, or acknowledging the assumptions that have to be made, or further questions that have to be answered. Likewise, if you decided that the graph does not tell us anything about smartness, you would need to give your reasons, and to acknowledge what it does tell us as well as what it does not.

Activity

3 In the first sentence of Doc B, the author claims that monkeys and apes develop social relations by grooming each other. How well does the rest of the document, and the information in the second graph (Doc D), support this claim?

Commentary

Firstly the author explains how grooming may account for the building of relationships within a group. It is known that naturally produced chemicals called endorphins can cause a pleasurable (euphoric) feeling in humans. We know that among the 'triggers' which release endorphins is massage, which is very similar to grooming. Laughter, music-making and so on have similar effects. If people share these pleasurable experiences it tends to bind them together as friends or partners. It is a plausible hypothesis that grooming has a similar effect among animals, and results in bonding between individuals within the group.

As we have seen several times in previous chapters, being plausible is not enough to make a hypothesis true. But it is enough to make it worth investigating further. This brings us to one of the key features of scientific reasoning: the need to *test* hypotheses by looking for further evidence which either corroborates or disproves it. The methodology is this: we suppose that the hypothesis is correct and ask ourselves what else would be true or probable as a consequence. In this case the question would be: If the grooming theory is right, what else would we expect to find?

One quite obvious expectation would be that animals with large social groups would do more grooming than those which form very small groups. In Doc B Professor Dunbar provides some data which suggests that this is indeed the case: gibbons, which don't form large groups, spend much less time grooming than baboons, which form groups of 50 or more. Of course, two favourable examples do not prove the theory correct, or even give it much support. Doc D, on the other hand, provides many such examples. And, as in Doc B, the trend does support the hypothesis: time spent grooming does show a tendency to increase with group size. There are a few 'outliers', as they are called: one species which grooms more than most but has a group size of around 10; and the primate with the second-largest group size grooms less than many which live in smaller groups. (These are 'outliers' because the points on the graphs lie furthest from the centre of the bunch.) You can single out for yourself other examples which are not typical. The question you must ask is whether these *anomalies* are enough to discredit the theory, or whether they can be ignored, or explained (see Chapter 4.2, pages 140–1).

You might also have picked up on the fact which Professor Dunbar makes at the end of the first paragraph of Doc B: 'As group size and time spent grooming increases, this social effort is concentrated on fewer and fewer partners.' This may seem puzzling. It may even seem to contradict the main idea that group size goes with more grooming. For both reasons, it calls out for an explanation, which takes us on to our next and final question.

Activity

4 What explanation could be given for the fact that in large groups grooming is concentrated on fewer partners?

Commentary

There may be a number of plausible explanations which you could give, so do not be concerned if your answer is different from the one here. It is a suggested answer, not the only correct one. The clue is in human behaviour, and is discussed in the second paragraph of Doc B. Humans form large groups, compared with most if not all other primates – 150 on average (Doc C). Humans, as we know, use physical grooming only in very intimate relationships. With less intimate acquaintances, Dunbar argues, grooming takes more varied and more acceptable forms such as laughing, singing and gossiping. The explanation we are looking for may therefore be that other more advanced primates, with larger group sizes, and with brain sizes approaching those of humans, also reserve grooming for their most intimate partners. Perhaps they too have other ways of interacting with the wider group, as humans do. That would account for the concentration of grooming on small numbers of partners. If this is the right explanation, it would also support Dunbar's claim that social groups are not a purely human phenomenon.

Summary

- Scientists make observations and use them both to construct and to test their theories.
- Critical thinking has much in common with scientific thinking.

End-of-chapter assignments

1 Is there enough evidence in the extract you have read to conclude that some animals form social groups similar to those of humans? Write a short reasoned case to support your answer.

Questions in this form occur regularly in Cambridge Thinking Skills Paper 2.

2 Find out more about the research of Robin Dunbar. Identify one of his theories and one or two items of evidence he gives in support of it.

4.7 Introducing longer arguments

We return now to arguments, but to longer and more challenging texts than you have been working on so far.

Start by reading the passage below. It is followed by a number of questions that will help you to engage critically with the article and the reasoning in it. As in the past, you should try answering the questions yourself before reading the commentaries.

THRILL OF THE CHASE

In crowded cities across the country there has been a growing number of crashes as a result of police officers pursuing stolen cars. Tragically, many of these high-speed chases end in death, not just of the car thieves but also of innocent bystanders or other road users. The police should be prohibited from carrying out these car chases. If someone dies as a result of police activity and the fatal weapon is a gun, there is rightly a huge outcry. But if it is a car, that seems to be accepted as an unavoidable accident.

The police say that they are not putting the public at unnecessary risk, because their policy is to stop the chase when the speed becomes too high for safety. This merely emphasises the stupidity of carrying out the chases. Either the policy is adhered to, and the car thieves escape, or the policy is ignored, and injuries or deaths result. Not only is it obvious that this policy is ineffective – otherwise the crashes would not have happened – but it is also easy to understand why.

The police officers will find the chase exciting, since it is a break from routine, and gives them the chance to feel that they really are hunting criminals. Once the adrenaline is flowing, their judgement as to whether their speed is safe will become unreliable. Car chases can be huge fun for all the participants.

Moreover, those police officers who are trusted to undertake car chases are the most experienced drivers who have had special training in driving safely at high speed. The car thieves, however, are almost all young men with very little driving experience. By the time the police driver judges that his speed is unsafe, he will have pushed the pursued driver well beyond his limit of competence.

The police may say that if they were not allowed to chase car thieves, this would encourage more people to commit more of these crimes. Would it be so terrible if this did happen? Surely saving lives is more important than preventing thefts of cars, and the police would be more profitably employed trying to catch serious criminals rather than bored, disadvantaged young men who steal cars for excitement. In any case, there are other ways of stopping stolen cars. For example, a certain device has been developed which can be thrown onto the road surface in front of the stolen car in order to bring it safely to a halt. And sometimes the chases are unsuccessful – the car thief succeeds in evading the police, abandons the car, and escapes.

Activity

1 What is the main conclusion of the passage?

Commentary

The conclusion is in the first paragraph, and you should have had no problem identifying it: 'The police should be prohibited from carrying out these car chases.' The two sentences before the conclusion are introductory and explanatory.

Activity

2 Identify three or four of the main reasons which the passage offers to support the conclusion that car chases should be banned.

Commentary

You could have chosen any or all of the following as the main reasons offered in support of the conclusion:

- Car chases have led to the deaths of car thieves and innocent bystanders.
- The police drivers' judgement as to whether their speed is safe will become unreliable.
- By the time the police driver judges that his speed is unsafe, he will have pushed the pursued driver well beyond his limit of competence.
- Saving lives is more important than preventing thefts of cars.
- The police would be more profitably employed trying to catch serious criminals.
- There are other (safe) ways of stopping stolen cars.
- Sometimes the car chases are unsuccessful.

Note that these reasons have simply been extracted from the passage and listed. A list like this doesn't show how the argument is structured, or how the reasons are grouped together to form sub-arguments within the whole argument.

Nor does the list show all the claims that are made in the passage. For example, it doesn't include the claim that car chases can be fun (paragraph 3). This is because it is not one of the *main* reasons. Yes, it contributes to the argument by helping to explain why police drivers may drive too fast for safety, namely because they enjoy it. But by itself it does not provide any grounds for believing that car chases should be banned. We would therefore classify the claim about car chases being fun as an *indirect* reason, leading to an intermediate conclusion, rather than directly to the main conclusion.

Similarly, the last half-sentence, after the dash, explains in what sense car chases are sometimes unsuccessful. It is the claim that they are sometimes unsuccessful (as well as dangerous and time-wasting) which is a *main* premise here and therefore makes it into the list.

Finally, of course, there are some claims that are not reasons at all, or conclusions, but have other functions in the passage. The first sentence of paragraph 2 is a good example. It offers no support at all for the conclusion, either directly or indirectly. Its role is to set up an objection that an opponent – in this case the police – might wish to make. The objection is that they, the police, have a policy of stopping the chase if it becomes too fast for safety, and that therefore they are not putting the public at unnecessary risk. The author claims that the policy is both ineffective and stupid, and devotes the middle three paragraphs of the passage to supporting these claims. The next pair of questions focuses on this section of the argument.

Activity

3 What grounds does the author have for saying that the police policy 'emphasises the stupidity' of car chases?
What two explanations does the passage offer as to why the policy is 'ineffective'?

Commentary

The author uses quite an ingenious piece of reasoning to criticise the policy. She considers the possible outcomes. Firstly, she considers what will happen if the policy is observed ('adhered to') by the police. Then she considers what will happen if it is ignored. If it is observed, says the author, the thieves will get away, presumably because the police will have to give up before the thieves do. If it is ignored, then accidents will continue to happen, just as they have happened in the past. And since they have happened in the past, it is obvious that the policy does not work as it is claimed to.

The question also asked you to identify the *explanations* that are offered for the policy's failure to work. There are two of these. The first is that police officers find the chase exciting, and that this affects their judgement about safety. The second is that whereas the police driver is likely to be competent to drive safely at high speed, the pursued driver has little driving experience, so that the officer will overestimate what is a safe speed for the car thief. The author concludes that not only is the policy ineffective, but that it is 'easy to understand why'.

How successful is this reasoning? (This was not part of the question you were asked, but it is part of the next one.) Like all arguments, its success depends not just on what is stated but also on what is assumed, and whether the assumptions that the argument rests on are warranted assumptions.

Activity

4 Are there any assumptions that are not stated in the passage but that the author appears to be making in connection with the claims made in paragraph 2?

Commentary

Yes, there are. The most significant assumption is that it is not possible for the police officer to catch the thieves without driving too fast for safety. The author claims that if the policy is adhered to, the thieves will get away; and if it isn't, accidents will result. In so doing she overlooks a third possibility: that some police drivers may be sufficiently skilled to remain within safety limits *and* to keep up with some of the thieves. She paints it as a so-called 'no-win situation', but is it? Without some statistical evidence it is hard to know what grounds the author has for predicting that the policy will inevitably fail one way or the other.

There is another assumption, too, although it is a lot less obvious. It is that if the stolen car were not being pursued, its driver would not drive unsafely anyway. The author wants to persuade the reader that there is no overall benefit to the public from chasing car thieves, only increased danger. That implies that the danger to the public comes only, or mainly, when car thieves are pursued. If they were left to drive around the streets unpursued, can we be sure there would not be just as many accidents – or even more, if would-be thieves get the idea they won't be chased and arrested? Again, the author is making a prediction on the basis of no hard evidence. Her prediction may be right – the policy of pursuing cars may prove ineffective – but it doesn't follow from the reasons she gives unless she makes these two major, and questionable, assumptions.

‘Restricting the options’

What we have exposed in the above discussion is a very common reasoning error: one to add to your catalogue. It is sometimes called ‘restricting the options’, because it consists in claiming or implying that there are fewer possibilities to consider than there really are. This is easier to understand by seeing an example of an argument from a different source that commits this error:

[1] When you go into business either you can adopt ethical practices or you can make a profit. Herbco has declared itself to be an ethical company, so if you want to see good returns, you really need to invest your money somewhere else.

On the face of it this looks like sound advice, given the two premises. If it really is true that you must choose between ethics and profit – and it often is – then surely it is not a good plan to invest money in an ethical company if your aim is just to get a good return.

But, like the author of ‘Thrill of the chase’, the speaker here is restricting the options to just two, and assuming that there are no others. Yes, you *can* choose between ethics and making a profit, as the first premise says. But you don’t *have to* choose between them unless they are the only choices. By drawing the conclusion that it does, argument [1] clearly makes the assumption that it is a straight choice between ethics and profit with no other options. But it is not a straight choice: Herbco could operate ethically *and* make a profit – for example, if it became very fashionable to buy goods produced by ethical companies.

The same sort of restriction is imposed in considering the police driver’s options. The driver can either obey the rules and let the thief escape, or drive dangerously and capture him. The possibility of obeying the rules *and* catching the thief is not openly or fairly considered.

Of course, you may happen to agree with the author, even after recognising that she has restricted the options. Like her, you may feel that there really are only two possible outcomes of the policy because there is no way of *partly* observing the rules: either you do or you don’t. And if you do, you have to let thieves escape, which makes it pointless, and if you don’t, you put the public at risk. By saying that an argument rests on an assumption that there are only two options, you are not necessarily saying that it is unsound. If you consider the assumption to be a fair one, then you can still accept the argument and the conclusion.

So in the end there is still room for agreement or disagreement, and scope for further argument. It is a piece of further argument that we turn to in the next question.

Note: when only two options are involved, the above fallacy is sometimes called ‘false dilemma’ or ‘false dichotomy’. (A dichotomy is a division into two.)

Activity

Here is a point someone might raise on reading ‘Thrill of the chase’:

> ‘Some of those who steal cars are attempting to escape after committing other serious crimes.’

Does this statement, if true, strengthen or weaken the argument (or neither)? Give your reasons.

Commentary

If someone said this in response to the argument it would be natural to think it was meant as an objection. It would be hard to interpret it as supporting the argument, or even as a neutral remark. Almost certainly it is picking up on the author’s claim that: ‘saving lives is more important than preventing thefts of cars, and the police would be more profitably employed trying to catch serious criminals rather than bored, disadvantaged young men who steal cars for excitement’.

In fact, the comment suggests that there is a fault in the argument very similar to the one we were discussing in the last question. The author is assuming that there is a choice between using police time to catch 'serious' criminals (whatever that means) and chasing 'bored young men'. And there is a further assumption that the latter are not serious criminals. Again, we have to ask whether this is a straight choice. The objection implies that it is not, suggesting that there may be some circumstances in which the car thief *is* a serious criminal: for example, an armed robber using a stolen car as a getaway vehicle.

As this possibility could be used to support a conclusion that car chases should not be banned altogether, it does to some extent undermine the argument. However, it is not a particularly difficult challenge to counter. There are several ways this could be approached. One is to say that the argument is mainly directed at the large number of cases in which the car theft itself is the only crime. Car theft in connection with more serious crimes such as murder or armed robbery is rare and a special case, and could be given special treatment without altering the author's general conclusion. Another, more robust, reply would be that it doesn't matter how serious a crime is, catching the criminal is never a good enough reason for endangering the lives of innocent bystanders. And finally the author can fall back on her last-but-one premise: that you don't have to chase stolen cars, because there are other, safer ways of stopping them.

Taken together, these responses to the statement take most of the sting out of it. The best assessment is therefore that if it weakens the argument at all, it does so only slightly.

Using analogy

The last feature of this argument we are going to examine is found in the first paragraph. It is called *arguing from analogy*. Used well, it is a very powerful tool. However, it is often used badly or misleadingly, in which case it creates a flaw in the reasoning, not a strength.

An analogy is a comparison. For example, suppose you are arguing about what it is to be a good leader, and how a good leader should behave towards the people he or she has been chosen to lead. One approach is to compare the nation-state to a family, so that being a ruler is analogous to being the head of a family. If we accept this broad analogy we can draw certain conclusions from it. An obvious conclusion is that a ruler does not merely have authority over the citizens but also a duty of care towards them, just as a parent has a duty of care towards his or her children. If you want to say that an authoritarian but uncaring parent is a bad parent (as most people would) you are also committed to saying that – by analogy – a purely authoritarian ruler is a bad ruler. This kind of reasoning is what is meant by argument from analogy. It stands or falls on whether the analogy is a fair one or an unfair one; and that is what you as the critic have to decide.

But what is a 'fair' analogy? Obviously the two things being compared are not exactly the same, or you wouldn't need to draw the comparison. What an analogy does is to say that two things are alike in certain relevant respects. In the analogy above, the role of a ruler is being likened to that of the head of a family. There is a difference in that the citizens are not the ruler's own offspring or close relatives, and of course there is a difference in the size of the 'family'. But by using the analogy for the argument you are not suggesting that the two roles are exactly the same: only that they are sufficiently alike – *in the relevant respect* – for the same kind of duties and responsibilities to apply.

Most people would probably agree that the nation–family analogy was a fair one if it were used to support the conclusion that rulers should not treat their citizens more brutally or unjustly than they would their own children; or simply that rulers have a 'duty of care' similar in certain respects to that of a parent. If, on the other hand, the argument was that a good ruler has to treat every citizen like his or her own child, that would be taking the analogy too far. In other words the fairness of an analogy depends upon the use it is put to in a particular argument.

Activity

An analogy is used in the first paragraph of 'Thrill of the chase'. Identify the two things that are being compared; and assess how successful the analogy is in the context of the argument.

Commentary

The comparison is between deaths resulting from the police action of chasing stolen cars and deaths resulting from police action involving a gun. In order to give support to the argument, the analogy has to compare things that really are similar in ways that are relevant. It also has to be true that there *should* be an outcry if police action resulted in deaths from firing a gun. The author clearly assumes that there should by using the word 'rightly' when drawing the analogy.

The similarities are fairly obvious. Guns and car chases both kill. And if things go wrong, both of them kill innocent bystanders as well as criminals and suspects. It is often said that a car is potentially a lethal weapon and this is very much what the analogy is saying here. Is it a fair comparison? As far as the consequences go, yes, it seems very fair. Why should we disapprove of a shooting accident, but shrug our shoulders at a driving accident, just because the 'weapons' used are different?

But there are dissimilarities, too, and they cannot all be brushed aside. A gun is *designed* to be a weapon, whereas a car is not. Also, when a gun is fired by a police officer it is with the *intent* to kill or wound someone, whereas generally the driver of a pursuit vehicle kills by accident. Of course, this doesn't make an accidental death arising from a police car chase any less painful for the bereaved relatives. But it does explain the attitude to which the author is objecting: the attitude that 'if (the weapon) is a car, that seems to be accepted as an unavoidable accident'.

Does the analogy successfully support the argument? Not entirely. Although the similarities seem quite striking, they are undermined by significant differences. A gun is *primarily* a weapon; a car is primarily a transport vehicle, and becomes a weapon only if it is misused. Also, if you place too much weight on this analogy, where do you draw the line? Do you want to say that *any* police action that results in tragic accidents should be banned, whatever the instrument – batons, riot shields, water hoses, tear gas . . .? If we completely disarm the police of all 'potentially lethal weapons', how can we ask them to protect the public from criminals who could harm them? It is a genuine dilemma, and it cannot be solved by judging all actions by their sometimes-tragic consequences.

Summary

- 'Thrill of the chase' is not a bad argument. It tackles a difficult and controversial subject and draws a conclusion that many people will have sympathy with. But it does not have all the answers. In this unit we have looked at the strengths and some of the weak points in the reasoning, so that an informed and considered judgement can be made as to whether its conclusions are acceptable. Or you may decide that there is more to be investigated and more argument to be had.

End-of-chapter assignments

1 In paragraph 3 of 'Thrill of the chase' it is observed that car chases can be fun for all the participants. In paragraph 5 it is implied that car thieves are predominantly bored young men looking for excitement. How could these claims be developed to counter the argument of some police officers that banning police pursuit would lead to an increase in car theft?

2 Find an example of an argument based on analogy – or write one yourself. Critically examine it, like we examined the example in the 'Thrill of the chase' passage, and decide whether or not it does its job successfully.

Answers and comments are on page 325.

4.8 Applying analysis skills

In the previous chapter you looked at a longer piece of text and answered some searching critical questions. Some of them were about analysis, some about evaluation and some about objections and further argument. In this chapter, and in the next two, we will examine two new articles, applying each of these skills in turn. We start, in this chapter, with analysis.

The text on the next page is an argument about criminals who become celebrities. Read it through twice, once for general meaning, then again for more detail. Then answer the following questions.

Activity

1 What is the main conclusion of the passage?

Commentary

Although arguments like this are longer and more involved than the ones you have been used to, the strategy for analysing or interpreting them is much the same as it was for the short, illustrative examples in Unit 2. When seeking the main conclusion, first look for a likely candidate – perhaps some recommendation or prediction or verdict – and ask yourself if other parts of the argument are reasons for making such a claim, or not. If not, look for another candidate.

It should be fairly obvious what this passage, 'Time to get tough', is leading up to. It claims that the legal principle of no profit from crime should be extended to cover celebrity criminals. And it claims that, on principle, income from criminal celebrity should be confiscated. These two claims between them summarise the author's main contention. If you had to pick one as the last word, it would be the second, the recommendation to confiscate income, since this follows from the more general claim that the law should be extended.

You might have been tempted by the last sentence of paragraph 3, which claims that there is no real difference between direct and indirect profit from crime. This certainly is a conclusion, as the word 'therefore' would suggest, and it follows from the reasoning in the third paragraph. But establishing this conclusion is only one step in the argument, and it is not the final step. It is therefore an intermediate conclusion, not the main one.

Best answer: 'If the principle of not benefiting from crime means anything, all income, direct or otherwise, should be confiscated from anyone whose criminal past has helped them to get rich'; or the same statement in your own words.

Activity

2 Two objections, or counter-arguments, are considered in the passage. What are they? Why does the author raise them? How does he deal with them?

Commentary

The counter-arguments are contained in the third and fourth paragraphs. They are recognisable from the use of the words 'protest' and 'object(ed)', but also from the obvious fact that they challenge the author's conclusions.

Why should an author include in a text a challenge to his own conclusions? Doesn't that weaken the argument? No, it strengthens it, because it shows that the author has an answer to the challenge. Imagine you were in a

TIME TO GET TOUGH

It is an established legal principle, in almost all parts of the world, that convicted criminals should not profit from their crimes, even after serving their sentences. Obviously offenders such as fraudsters and armed robbers cannot be allowed to retire comfortably on the money they made fraudulently or by robbing banks.

But the law does not go far enough. It should also apply to the growing number of notorious criminals who achieve celebrity status after their release from jail. Ex-convicts who become television presenters, film stars or bestselling authors often make big money from their glitzy new careers. But they would never have had such careers if it weren't for their crooked past.

The producers, agents and publishers who sign the deals with celebrity criminals protest that the money does not come directly from a convict's previous crimes, but that it is a legitimate reward for their redirected talent, and for the audiences they attract. But this is an unacceptable argument. Firstly, the producers and others take a big cut of the profit, so obviously they would say something of that sort. Secondly, a notorious gangster needs no talent to attract an audience: their reputation is enough. Therefore, whether the income is direct or indirect, it is still profit from crime.

It is often objected that once a person has served a sentence, they should be entitled to start again with a clean sheet; that barring them from celebrity careers is unjust and infringes their rights. This is typical of the views expressed by woolly-minded liberals, who are endlessly ready to defend the rights of thugs and murderers without a thought for their victims. They forget that the victims of crime also have rights. One of those must surely be the right not to see the very person who has robbed or assaulted them, or murdered someone in their family, strutting about enjoying celebrity status and a mega-buck income. Moreover, victims of crime do not get the chance to become chat-show hosts, or star in crime movies, because being a victim of crime is not seen as glamorous.

If the principle of not benefiting from crime means anything, all income, direct or otherwise, should be confiscated from anyone whose criminal past has helped them to get rich. After all, no one is forced to become a big-time crook. It is a choice the individual makes. Once they have made that choice the door to respectable wealth should be permanently closed. It's the price they pay. If would-be criminals know they can never profit in any way from their wickedness, they might think twice before turning to crime in the first place.

debate and it is your turn to speak. Even before the opposition have their chance to raise an objection, you have anticipated it and responded to it. It is sometimes called a *pre-emptive* move: dealing with a point before it has been made.

Take the first 'protest' that producers and others allegedly make. The objection is that the money ex-convicts make from acting, writing, presenting and so on is due to their talent and comes only *indirectly* from crime, not *directly* like the money from fraud or bank raids. The reply, not surprisingly, is that this is unacceptable. Two reasons are given: firstly, that the producers '*would* say something like that, because they take a cut of the profits; secondly, that gangsters need no talent: their criminal reputations are enough to draw an audience. From this the author concludes that whether the income is direct or indirect, it is still profit from crime.

You may already have noticed that paragraph 3 is itself a complete argument: a sub-argument within the overall argument. Here it is in standard form:

Target: the counter-argument

But . . .

R1 Producers would say something like that because they take some of the profit.

R2 Notorious gangsters need no talent; their reputation is enough.

IC Indirect income is still profit from crime.

C This (counter-argument) is unacceptable.

The next objection that the author anticipates is that ex-convicts have the right to start again. It is dismissed as a 'woolly-minded' argument, and as one that ignores victims' rights and feelings. It also points out an unfairness in that criminals gain from their crimes whereas victims have no such opportunities.

These responses lead directly to the *main* conclusion that *all* income from crime should be confiscated.

Activity

3 As well as the responses to objections, what other reasons are given in support of the conclusion?

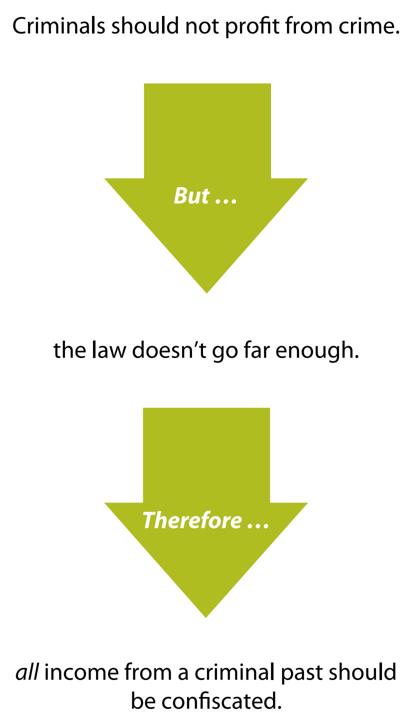
Commentary

The final paragraph adds a further set of reasons that directly support the conclusion. They are: (1) that criminals make a choice; (2) that if they make that choice, the door to respectable wealth should be closed; and (3) that if would-be criminals know they will never be able to cash in on their crime, they may think twice before choosing to be criminals.

What about the first paragraph: where does it fit in, and what is its function? It states that there is an established legal principle, namely that crime shouldn't pay, and provides two examples of unacceptable income that nobody could really argue with – profit from fraud and from bank robbery. So, should any of this have been included in the list of reasons; or are these just introductory sentences? You may have interpreted this part of the argument as a premise (reason), on the grounds that, without the principle, the argument wouldn't really make a lot of sense; and that, in a general sort of way, it does support the conclusion that profit from crime should be confiscated.

But on closer inspection this is not the best and clearest interpretation of what the author is aiming to achieve. For his argument is not really about crimes such as fraud and bank robbery. In fact, it is more or less taken for granted that the profits from these crimes should be forfeited if the criminal is convicted. No supporting reasons are given and none are needed. The *real* argument begins with the word 'But . . .' at the start of paragraph 2. Reading it that way, the first paragraph can be seen more as an introduction than as part of the reasoning.

The shape of the whole argument is:



Mapping the structure

The previous diagram gives only the roughest outline of the argument. It is like a route map with just the main towns shown. It does not give any of the reasoning that leads from one to the next.

'Mapping' is a good word to use, because it suggests another very useful way of representing the steps in an argument. If you enquire how to get from one place to another, people will often give you a *string* of directions: for example, 'Go up to the traffic lights and turn right. Stay on that road through a couple of bends, past the big hotel on the left. Take the third exit from the roundabout and the immediate fork to the left . . .' It can all be very confusing; and it is very easy to miss a turning or take the wrong one, after which you quickly lose any sense of where you are.

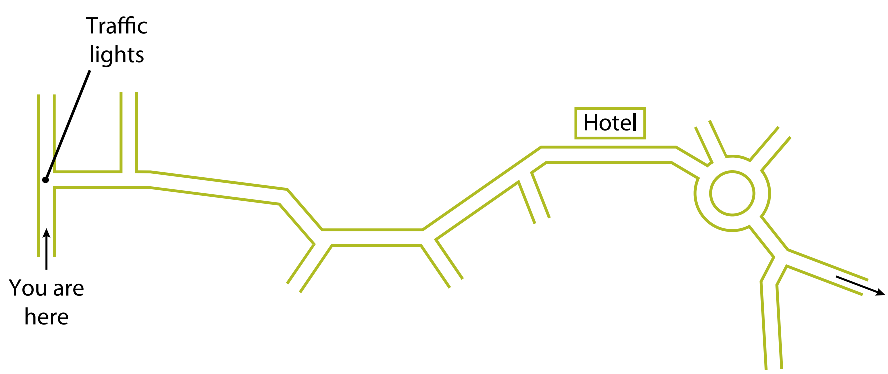
A simple map like the one below is much more helpful: it gives you an overall picture of how the journey looks, how the roads connect, how they relate to each other and the surroundings, and so on.

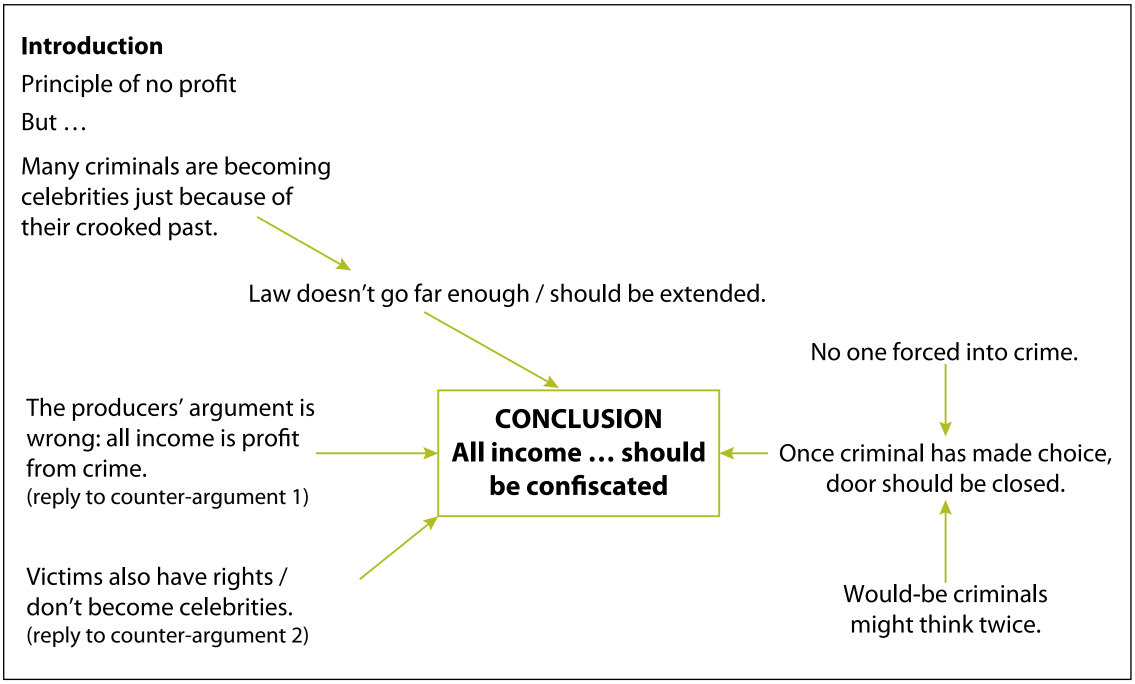
Activity

Try building up a more detailed map of the argument 'Time to get tough', showing how, in your view, the different parts of the reasoning lead to the conclusion.

Commentary

Notice that the task is to represent *your* view of the way the argument is structured. This does not mean that any analysis of the passage is as good as any other, but it does mean that there is some room for interpretation by the reader. A *suggested* map of the argument follows. Don't worry if you have taken a slightly different route to the conclusion, or summarised the claims a bit differently. So long as you have correctly understood the direction of the argument and its final conclusion, then the exercise has served its purpose.





Summary

- Longer arguments can be analysed in broadly the same way as shorter ones.
- Longer arguments may have sub-arguments as part of their reasoning.
- A very common line of reasoning is to set up a counter-argument and then knock it down.

End-of-chapter assignment

Using some of the methods discussed in this chapter, as well as those you studied in Chapters 2.4 and 2.5, map out the structure of the following argument.

SAY NO TO CHEATS

The governing bodies who control international sport are right to prohibit the use of performance-enhancing drugs and to operate their policy of zero tolerance against athletes who break the rules. There is more than enough medical evidence to establish that many of the substances that sports stars are tempted to use to increase their strength and stamina are extremely harmful to their health. Permitting their use, or turning a blind eye to it, can have tragic long-term consequences, as many former athletes have discovered to their cost.

Young people are natural risk-takers and are often reckless about their own futures. That, coupled with the huge rewards that can be won by reaching the top in their chosen sport, will often drive them to disregard medical advice and think only of the gold medal, or the big sponsorship deal, or the glory of competing for their country. Those who regulate the sports have a duty of care over these men and women. To stand by whilst they harm themselves would be grossly irresponsible.

But there is another reason why the use of drugs in sport cannot be tolerated. The purpose of sport is to discover who is the best. The only way to achieve that is to start with a level playing field and for every competitor to have an equal chance of winning. You can't say who is best if some competitors are cheating by stealing an advantage. Therefore, if drugs can be driven out of sport, we will once again know who the real champions are.

It is sometimes argued that drugs give no more of an advantage than other perfectly legitimate practices, such as following special diets and taking dietary supplements, which can also boost an athlete's performance. So can the latest hi-tech equipment and clothing, computerised training programmes, physio- and psychotherapies, and so on. Is that not cheating?

No. There is all the difference in the world between eating certain foods and taking drugs because drugs, unlike foods, are banned substances. Any athlete who wants to can take advantage of a special diet or the latest equipment and training techniques. But only those who are willing to break the rules can benefit from taking drugs. Anyway, if you start saying that drug-taking is fine because it is no different from energy-giving food you would end up having to allow athletes to run races with jet engines strapped to their backs.

One more thing: if the top athletes get away with taking drugs, the young people for whom they are role models are far more likely to do the same. For their sake too, the pressure on the cheats must never be relaxed.

4.9 Critical evaluation

In the last chapter you worked on mapping out the structure of two arguments: one with an accompanying commentary, and one on your own in the end-of-chapter assignment. In this chapter you will be looking at the same two arguments from the point of view of their strengths and weaknesses, success or failure. This is critical evaluation.

A: Time to get tough

Read through the whole argument on page 178 again to remind yourself of its conclusion and supporting reasons. If necessary, also look again at the analysis of its structure on page 181. Once you have it clear in your mind you can move on to the next range of questions: Is it a good argument? Does it work? Does the reasoning succeed in supporting the conclusion?

It is now that the work you did on analysing and mapping the argument really starts to pay off. It has split the argument up into a number of manageable bits that you can consider one by one. It has also put the different parts of the passage in their place, so that you know exactly what their functions are. So, for example, we can pass over the first paragraph because it is mostly introductory, and move straight to where the argument really begins, in paragraph 2.

Paragraph 2 draws the intermediate conclusion that the law that convicted criminals should not profit from their crimes doesn't go far enough and should apply to ex-criminal celebrities (as well as former fraudsters, bank robbers etc.).

Activity

What reasons are given in paragraph 2 for this conclusion? Are they convincing?

Commentary

The reasons given are that these celebrities often make big money *and* that they would not do so if they had not been criminals in the past. Provided you accept that both statements are true, then they do give support to the suggestion that the law needs extending, which paves the way for the main conclusion (in paragraph 5) that such income should be confiscated. For if it is a fact that some people do profit from having been law-breakers – and for no other reason than being law-breakers – then the principle referred to in the introduction is (arguably) being broken.

The big question is whether the reasons *are* both acceptable, especially the second. The first claim is fairly obviously acceptable because it is a known fact that ex-convicts who become presenters, film stars and so on make big money. It could easily be checked and figures produced to support it if anyone doubted its truth. But what grounds has the author got for the second reason, that these celebrities 'would never have had such careers if it weren't for their crooked past'? Certainly none that are stated. It is an *unsupported claim*, which the author is expecting the reader to take on trust.

Assumption

If you cast your mind back to Chapter 2.9 you will recall that many, if not all, natural-language arguments rest on implicit *assumptions* as well as on stated *reasons*. The conclusion that the author draws in paragraph 2 rests on certain such assumptions: for example, that ex-criminal celebrities do not have talents that could have made them famous or successful if they had not been criminals. Unless you assume this you cannot accept the conclusion. But since the reader has no more reason to accept than to reject the assumption, it is a potential weakness in the argument.

Flaw

It could even be said that the need to make this assumption is a flaw, or reasoning error, if you consider it to be an *unwarranted* assumption. Recall, from Chapter 2.10, that a common flaw in reasoning is the assumption that because two things are both true, one is therefore the cause of the other. Does the author make that mistake here? Is he saying that because a celebrity was once a criminal, that must be the cause of their rise to fame and consequent wealth?

If you think that is what he is saying, then it would be right to identify this as a flaw in the argument. If an argument depends on an unwarranted assumption, then it is fair to say it is flawed, or that it is unsound, or that there is a 'hole in the argument'.

But the author is no fool, and is obviously aware of the potential weakness in paragraph 2. That is probably why, in the next paragraph, he 'anticipates' a counter-argument that challenges his assumption(s). The purpose behind this is not to admit to a weakness, but to block the challenge that threatens to expose it. The challenge is that celebrity wealth does not come *directly* from crime, but from 'redirected talent'. The author's response is firstly that the producers and others who make this challenge take a cut of the profits and therefore '*would* say something like that'; and secondly that gangsters need *no* talent: their criminal reputations are enough. And he concludes that the income from becoming a celebrity is therefore still profit from crime, whether it is direct or indirect. It is a strong and uncompromising response.

Activity

How successful do you think the author's reply is in paragraph 3? Does it meet the objection or not – and why?

Commentary

The response does not sweep away the objections; and it doesn't give any good reason to warrant the author's assumptions. We'll take the second part of the response first. This is simply that an ex-convict does not need any talent. But, even if it is true, the fact that someone needs no talent to become a celebrity does not mean that he or she *has* no talent – say, as comedian, or actor, or poet. This remains a mere assumption, and one that is easily contested, for there clearly have been ex-criminals who have won acclaim for other achievements besides crime.

The first part of the reply is no better. In fact it is no more than an insinuation. The author wants us to believe that the producers and others are all motivated by profit, and would therefore say whatever was needed to protect their 'cut'. It doesn't answer the actual claim that ex-convicts may have talents as well as notoriety. There is also a fresh assumption here, namely that the only people who claim that ex-convicts have talents are producers or others who have a vested interest. In reality there may be many people, with *no* vested interest, who would also agree with the counter-argument.

Attacking the person

This line of argument is a very common kind of fallacy, which needs to be guarded against. It has its own Latin name, *argumentum ad hominem*, meaning an argument directed 'at the person' (literally the man), rather than at the reasoning. What makes it a fallacy is that the argument could be perfectly sound and effective, even if the person who is making it is supposedly unreliable or wicked or deceitful or stupid, or has a vested interest, or anything else that the opponent wants to say to attack their reputation. If the people who have succeeded in becoming celebrities *do* also have talent, then the counter-argument is a strong one, whether or not some of the people who say so have selfish reasons for wanting it to be true. You cannot make the argument go away just by discrediting those who may use it. Yet it is surprising how often this strategy is used.

What you can legitimately say is that if the only support for some point of view comes from an obviously unreliable source and from no other, then we ought to treat it with some suspicion. But that is a very different matter from saying, as the author does in this case, that because certain people 'would say that, wouldn't they!', the *substance* of what they say must be false.

Activity

Another counter-argument and response follow in the fourth paragraph. Critically evaluate the reasoning in this paragraph, identifying any assumptions and/or flaws that it contains.

Commentary

You probably picked up straight away that there was another *ad hominem* argument here. The claim that a concern for the rights of ex-convicts is 'typical of . . . woolly-minded liberals' is obviously directed at the person rather than their argument. However, the author does go on to say *why* such concerns are misplaced, and here the argument is much stronger. Thus if you ignore the *ad hominem* part of the paragraph you are still left with two or three reasons that do respond to the objection, and (if true) also support the author's own argument. These are the claims that:

- victims also have rights, one of which is the right not to see those who hurt them enjoying wealth and celebrity
- victims don't get the same chances (of celebrity) as ex-convicts.

These are powerfully *persuasive* points. You can easily imagine how frustrating and insulting it would be for someone who had been attacked or robbed to later watch the person who had done this hosting a television show, or seeing his bestselling autobiography serialised in the newspapers or made into a successful film. The victim might be forgiven for thinking, 'Some of that fame has been got at *my* expense. The criminal gets the money and I get nothing. What is more, I am not a celebrity because no one is really interested in my injuries or losses, only in his wickedness.'

But, persuasive as it may be, is this reasoning *sound*? Are there any assumptions hidden behind the strong language? Arguably, yes. For a start you would have to assume that there really is a 'right' of the kind the author claims for the victim. People have rights not to be harmed by others, but those rights are dealt with by the courts when they hand out their sentences. Once such sentences have been served, is there really a continuing right for the victim never to see the criminal doing well? *Arguably*, no – as we shall see when we look at further argument in the next unit.

What the author is asking us to accept in this paragraph is that allowing criminals to exercise their rights to a fresh start is *unfair* to their former victims. But this requires another major assumption. It is the assumption that if victims and criminals both have rights, the victim's rights should come first. Without this assumption there are no grounds for the conclusion; for if, as the counter-argument claims, an ex-convict has the same rights as anyone else, then it is hard to see how the author can claim that the victim should have some *special* right over the criminal. This is a potential weakness in the argument, and it is one we will return to in Chapter 4.10.

Conclusion

So we come to the last paragraph, which consists of the conclusion and a further sub-argument. It has two strands. One is that people freely choose to become criminals; and that if they make that choice they should be barred from future ('respectable') wealth. The other is that if people thinking of becoming criminals know they will be effectively outlawed in this way they may have second thoughts about turning to crime at all.

Activity

As you did with the earlier steps in the argument, critically evaluate the reasoning in the last paragraph.

Commentary

This is possibly the strongest part of the argument. It places the responsibility for becoming a criminal firmly on the individual, and suggests, reasonably enough, that if that individual then faces having his wealth restricted, he has no one to blame but himself. Opponents of the argument cannot say that the criminal has not been warned. The argument is strengthened further by the claim that this may also deter people from crime, which is probably the best argument there is for punishment of any sort.

But here, too, there are certain questionable assumptions. One is that young people tempted by crime would even *think* about becoming legally rich and famous, far into the future. And if they did, would they care that they would be prevented from doing so? Probably not. Another is the assumption that people do all freely choose their lives; that none is ever drawn into bad ways by their upbringing, or the influence of others, or through knowing no better. Without the assumption that there is truly free choice, it would be harsh to say no one should ever be given a second chance.

Power of persuasion: rhetoric

If you read the 'Time to get tough' text casually, and uncritically, it is easy to be impressed by the argument. Your first reaction might be: yes, many criminals *do* profit from the fact that they have done wrong and become well known because of it. And this does not seem right or fair. But, as we have seen, the argument is not necessarily as sound or as conclusive as it may at first seem: there are a number of hidden assumptions and even flaws in the reasoning, when you come to consider it critically.

Part of the persuasiveness of this argument comes from the language the author uses to press his case. Look at two of the phrases used in paragraph 2: 'glitzy new careers' and 'crooked past'. Both help to build up a picture of something both cheap and nasty. In the next paragraph we are told that a 'notorious gangster needs no talent', reinforcing the negative impression that is being created of the convict-turned-celebrity.

We call this expressive ingredient of the text *rhetoric*, to distinguish it from the plain reasoning, the underlying argument. Authors use rhetorical devices of various kinds to embellish their arguments, to make them more forceful. There is nothing wrong with this: it is not a misuse, or some kind of cheating, to express an argument in a forceful way, provided there is an argument to embellish. When rhetoric is misused is when there is nothing else but strong words, and there are no substantial grounds underlying it. Don't make the mistake of picking out a colourful phrase and labelling it as a flaw just because it is highly rhetorical. *Do*, however, be on guard against authors who employ *empty* rhetoric: colourful language to camouflage weak or non-existent argument. (Journalists, politicians, and some lawyers are among the worst offenders!)

Of course, the impression that the author's language creates might be the *right* impression, or at least one that you can sympathise with. Many of the celebrities that the author has in mind may well be thoroughly unpleasant, untalented people; and the celebrity they gain may be shallow, 'glitzy', and the rewards undeserved. But that should not blind you to the fact that well-chosen language can heavily influence the way you respond to an argument; that there is always a danger that the *reasoning* can take second place to emotions or sympathies. And if that happens you are not responding in a fully critical way.

We also saw, in paragraph 4, how potential opponents of the argument are dismissed as 'woolly-minded'. According to the author they are 'endlessly ready to defend the rights of thugs and murderers without a thought for their victims'. And we are presented with the image of these same thugs and murderers 'strutting about enjoying . . . a mega-buck income'. The language leaves us in no doubt which side the author is on. But more than that, the author wants to manoeuvre us into a kind of trap, where the choice seems to be between defending the bad guys or supporting their innocent victims.

A critical approach reveals that this argument is strongly biased when it comes to describing the different groups of people involved. There is no concession that there may be some ex-convicts who have genuinely turned their backs on crime, who have real talent as actors or writers, and who do what they can to put right the harm they have caused. Does the author include such people in the same category as those whom he describes as 'strutting about' in their 'glitzy new careers'? The fact is we don't know, because he has conveniently – and no doubt deliberately – left them out of the picture.

Decision time

So, do we rate this as a good argument or a poor one, overall? That final verdict is left to you. You will probably agree that it is quite a persuasive argument, but that it has weaknesses as well as strengths; and that it makes some claims and assumptions that are, at the very least, questionable. Whether or not these are enough to make you reject the argument, you must decide. You will have the chance to do so in the end-of-chapter assignments.

Be careful, however, that in making this decision you are not just saying whether you agree or disagree with the author's opinion or his conclusions. You could quite reasonably think that the conclusion is right but that the argument is poor. Alternatively, you might think it is a strong and compelling argument, but, for reasons of your own, disagree with its conclusion. This is the most difficult position for a critical thinker to be in. If you really find the argument compelling, and you do not dispute its premises, then rationally you should accept its conclusion, even if this means changing a previously held view. If you still reject the conclusion, you need to be able to say where the argument fails – and that can be quite hard to do if it is a persuasive argument.

B: Say no to cheats

We turn now to the argument you analysed for the assignment at the end of Chapter 4.8: 'Say no to cheats'. It contains a very common line of argument that occupies the first two paragraphs. It takes the following form: 'Such-and-such is harmful, or could be harmful. Therefore it should be prohibited.' This line of reasoning is often referred to as the argument from harm, and is an important *ethical* argument.

Activity

Reread paragraphs 1 and 2 of the passage on page 182, and remind yourself of the reasons given there to support the main conclusion. In arguing for the main conclusion, what underlying *assumption* is also made? Do you think it is a warranted assumption?

Commentary

The argument in the first two paragraphs is as follows:

R1 Medical evidence and past experience suggest that performance-enhancing drugs (PED) are harmful.
R2 Young athletes are reckless.
R3 To stand by while they harm themselves would be irresponsible.

IC The governing bodies have a 'duty of care' for athletes.

C They are right to prohibit PED.

This seems a reasonable argument. If you accept the truth of the premises, and there is no obvious reason not to, then a strict ban on PED would seem like a sensible policy to follow. But 'sensible' does not necessarily mean 'right', and that brings us to the big assumption that the argument makes: that athletes don't have the right to make these choices for themselves; or that the authorities *do* have the right to make the choices for them, just on the grounds of the dangers PED may pose to their health.

The argument from harm (or risk or danger) to the need for prohibition is often underpinned by this kind of assumption: that those in charge have the right to tell grown men and women what they may or may not do to their own bodies. Is it a warranted assumption? In general, no. Of course, authorities do on occasions impose rules for our own good or safety. Many countries prohibit the riding of motorcycles without a crash helmet, or driving of cars without a safety belt. But there are many other dangerous activities which we are not prevented from doing (such as mountaineering and skydiving) on the grounds that although they are dangerous, we nevertheless have the right to do them if we want. Usually a prohibition needs other arguments beside the argument from *self-harm*, for example that the harm extends to others as well. For example, the strongest argument for banning smoking in public places is that non-smokers as well as smokers are affected. If the argument were only that smoking harms the smoker, it would not have anything like the force that it does have.

So the argument contained in the first two paragraphs alone looks a bit wobbly after all, not from what it states but from what it assumes. However, the author was probably well aware of this because his argument does not end there. It goes on to say (paragraph 3): 'But there is another reason . . . (for not tolerating PED)'.

The argument from fairness

The second main strand of the reasoning is the argument that it is unfair, in fact cheating, to take PED, and that they should be prohibited for that reason as well as the health risks. Paragraph 3 concludes that if drugs can be driven out of sport we will (once again) be able to identify the 'real champions'.

There is another assumption lurking here: that there are not some other ways, besides PED, of gaining unfair advantages. To meet that possible objection, the author sets out, and responds to, a counter-argument that there are indeed some practices that are perfectly legitimate but are cheating of a sort. The author's response is that PED are in a different class, precisely because they are prohibited.

Activity

Give your evaluation of the author's response to the counter-argument in paragraph 5. Is the reasoning sound, or can you see any flaws in it?

Commentary

There are in fact three serious flaws that need to be looked at very carefully. These are known as the 'straw man', the 'slippery slope' and 'begging the question'. Two of them relate to the last sentence of paragraph 5: 'Anyway, if you start saying that drug-taking is fine because it is no different from energy-giving food you would end up having to allow athletes to run races with jet engines strapped to their backs.'

A straw man

A 'straw man' argument is one in which the opposing argument has deliberately been made weak, to the point where no one would be likely to make or support it. It gets its strange name from the custom of making human figures out of straw for target practice, for example to shoot arrows at.

This is what the author does here. Whether or not you knew the name 'straw man', you should have noticed that in the counter-argument there is no suggestion that drug-taking is 'fine', or that it is no different from eating food. The counter-argument is much more subtle than that: it merely points out that there is a difficulty in distinguishing between permitted ways of getting an advantage and prohibited ones. That does not mean that anyone raising the objection thinks PED should be permitted, only that the problem is not as simple as it seems.

Thus the author is arguing against an opponent who doesn't really exist. It *looks* as though he has scored a point, but it doesn't count because it is such a cheap point. You will often find this flaw in arguments that you read. It can be persuasive if you fail to spot it. And, if it's done deliberately, it is *cheating*!

A slippery slope

Even if there were no 'straw man' fault in the argument, there is another flaw in the same sentence. It has a curious name, too: it's often called a 'slippery slope'. This comes from the idea that once you are on a slippery slope you can't stop yourself going all the way to the bottom. In this case, if you say that some PED are very like some food supplements, then, according to the author, there is nothing to stop you saying that anything athletes do to gain an advantage is all right.

This is obviously nonsense. The difference between special diets or training techniques and the use of certain drugs is really quite narrow. Even the experts have some difficulty drawing a line between, say, a 'food supplement' and an actual drug. This is why the counter-argument has to be taken seriously even if you are in favour of prohibiting PED. The idea that athletes could use jet-propulsion is in a completely different league, and it is perfectly possible to argue for one without having to go to the other extreme.

Begging the question

The third flaw relates to the second sentence in the paragraph: the claim that PED are different from other ways of improving performance *because they are banned*, and that that is what makes it cheating to use them. But the main conclusion is that drug-taking *should* be banned. You cannot validly say that something should be banned just because it is bad, and bad because it is banned! This is what is known as 'begging the question'. You can see why it is called begging the question with the argument simplified as follows:

> It is right to ban PED (conclusion).
> *Why?*
> Because using PED is cheating.
> *Why is it cheating?*
> Because PED are banned.

Another way to describe this flaw is to point out that it contains *circular* reasoning, or a circular argument. The author is arguing *for* the ban on PED *from* the ban on PED. Many of the flaws you find in arguments are due to circular reasoning or question-begging. Sometimes the circularity is obvious, as it is in this argument. In others it is much more carefully disguised, and you have to be vigilant to spot it.

The argument as a whole

We have found a number of weaknesses, flaws and questionable assumptions in the argument for prohibiting performance-enhancing drugs. That does not mean that we have to reject the argument as a whole, and it certainly doesn't mean we have to reject its conclusion. Most people find the practice of taking PED totally unacceptable and are in full agreement with its prohibition. Most people also consider it to be cheating and believe that it harms the health of athletes.

But the converse is also true. Just because we agree with the author's main conclusion of an argument does not mean we have to approve of the reasoning. As critical thinkers we need to be able to evaluate an argument objectively whether we agree with it or not. In fact, agreeing with the author can often make the job of evaluation more difficult because we are likely to be making the same assumptions and wanting the same outcome.

Summary

- A critical evaluation means deciding whether the claims and assumptions made in an argument are warranted.
- It means identifying any flaws in the reasoning.
- It means assessing the strength of the support that the reasons, if true, give to the conclusion.
- It means distinguishing between the rhetoric and the reasoning in the text.

End-of-chapter assignments

1 Look at the following response to the argument 'Time to get tough', and critically evaluate the reasoning it employs.

> You call people like me woolly-minded liberals, but look what you are arguing for: denying anyone who has committed a crime a chance to earn a living, however hard they may try to go straight and start afresh. As well as being inhumane, that will have the opposite effect from what you want. You'll just end up with streets full of ex-cons who can't get work and are driven back to violent crime, and even more victims to feel sorry for.

2 Consider the following short argument, on a very different topic. Is it sound? If not, identify what is wrong with it.

> The dinosaurs obviously became extinct because of a single catastrophic event such as a large meteorite or dramatic upheaval in the climate. This would mean that they did not undergo a gradual disappearance lasting many centuries or millennia, but that they were wiped out practically overnight. The fact that they died out so quickly also means that there could only have been one cause of their extinction, not many as was once assumed; and that whatever the cause was, it was immense and final.

3 Choose one of the two arguments studied in the chapter. Summarise the critical comments that were made, and respond to them with your own observations. Finally, give an overall evaluation of the argument, saying how successfully or unsuccessfully it supports its conclusion(s).

Answers and comments are on page 325.

4.10 Responding with further argument

Evaluating an argument means deciding whether or not the claims made in it are acceptable, and whether or not they support the conclusion. Further argument goes a bit further: it is your opportunity to put some of your own ideas on the table, either supporting or challenging the author's conclusions.

It has to be said straight away that further argument is not *any* argument: it must relate directly to the text you are working on. It is not a chance just to set off on some line of your own that happens to be on a related topic. You would get no credit in an exam if you read the article 'Time to get tough' – which featured in the last two units – and then wrote about prison reform, or the abolition or reintroduction of the death penalty. There may be issues that connect these topics to the argument about profiting from crime, but they are not central issues. Your further argument must be for or against the conclusion. Otherwise it is just a digression.

Evaluation often leads very naturally into further argument, and it is sometimes difficult to say where one ends and the other begins. For example, here is part of a student's response to the third paragraph of 'Time to get tough':

> [1] The author says that notorious gangsters don't need any talent to attract an audience, and that their reputations are enough. This may be true, but it doesn't mean that notorious gangsters don't ever have some talent. They may be very talented. People often think of a gangster being a stupid person, who just uses violence to get their way, but there are gangsters who have got where they are by their intelligence. It takes brains and imagination to plan a big crime and get away with it. It takes brains to be a television presenter. So you can't say that because someone has been a criminal they haven't got the ability to be a celebrity. I read a book by a reformed drug addict who had stolen to buy drugs, and it was brilliant, as good as any other writer could do. It wouldn't have been published and sold in the bookshops if he was stupid and couldn't write. Therefore this statement by the author is misleading.

Is this extract from the student's essay evaluation or further argument, or both? Plainly it is both. It is a critical evaluation because it exposes a weakness, a questionable assumption, in the author's reasoning. However, it does much more than just *say* there is a weakness. It highlights it by bringing in fresh claims and counter-examples that challenge the author's assumption that a person cannot be a criminal *and* be talented. The student uses her own reasons for concluding that the author's claim is misleading. She even draws on her own (reading) experience to illustrate the point she is making. This clearly marks it as further argument and not just evaluation.

Of course it is not a *decisive* further argument. It doesn't completely undermine the author's case: it merely kicks away one of the supporting planks. To this extent we can say it damages the argument rather than destroys it: it seriously weakens it, but not fatally.

Counter-example

Counter-examples – i.e. examples that challenge a claim – are very powerful weapons for attacking arguments. As we saw in the above extract, just one example of an ex-criminal who arguably does have talent challenges one of the author's main premises.

Activity

Look again at paragraph 4 of 'Time to get tough' (if you don't already know it by heart!) and find a claim that could be challenged with a counter-example. If you know of a real-life counter-example, raise it. If not, suggest a possible one. Then develop the counter-example into a short further argument.

Commentary

An obvious target is the last sentence of the paragraph: the claim that victims don't get the chance to become celebrities. It is highly vulnerable to counter-examples and, whether you were able to think of an actual one or not, it is clearly not far-fetched to suggest that a victim of, say, a high-profile kidnapping or hostage-taking could become famous as a result, and gain financially from telling their story.

Such an example could be developed as follows:

> A number of victims of crime have themselves become celebrities and made big profits from publishing their stories or appearing in the media. Is this fair? There are many other people who have suffered from accidents or misfortune who have never been heard of. If you are going to ban some groups of people from celebrity income, simply because other people have not had the same opportunities (like the author does), then you would have to ban everyone from making income from their pasts – criminals and victims alike. Otherwise how would you decide who deserved their celebrity status and who did not?

New lines of argument

But further argument does not have to begin from a particular point of evaluation. Provided you do not wander off the central issues, you can launch your own argument from the passage as a whole. You may, for example, feel that the author has missed out an important consideration that has an impact on his conclusions. Raising it would be a legitimate form of further argument.

For example, there is no discussion in the article about the motives criminals have for becoming celebrities. Nor is there any mention of the consequences. The author seems to assume that the motives are always selfish, on the part of either the criminal or the producers etc. who take a cut; and that nothing, apart from satisfying greed, comes of it. Here are three pieces of further argument, adapted from student responses, which take a completely different line:

[2] Criminals are selfish people. They take what is not theirs and what others have worked hard to get. They disobey laws. They evade taxes. No one is going to tell me that when and if they decide to go straight and become big showbiz personalities they suddenly change into decent, law-abiding citizens. All they are in it for is themselves, and they will do whatever is necessary to get as much as they can. Leopards don't change their spots. Cheats and thieves don't become honest, they just find other ways to cheat.

[3] Some criminals grow up while they are in prison and come out looking for legal jobs, and some go into acting or writing to make a living. The parts they play in films and the books they write will usually be about criminals or about prison, and they have the experience to make this realistic and true to life. This has a very useful purpose because it lets other people know what it is like to be a criminal or a prisoner. It is not

glamorous or romantic like it often is in fiction, it's ugly and dangerous.

[4] Young people admire celebrities and want to be like them. If you let big-time gangsters and murderers become celebrities you give young people a very bad example to follow. Criminals become role models. Also you give them the idea they can be rich and famous by being wicked and violent.

Activity

What point is being made in each of these lines of further argument? Do they support the argument in the article, or do they challenge it?

Commentary

These were all examples of relevant and perceptive further argument. Whether you agree with what they say or not, they make a valuable contribution to the debate.

Argument [2] supports the author's conclusion far more than it challenges it, though it takes a quite different line of approach. It would make a good response to any suggestion that criminals can turn over a new leaf or put crime behind them. It implies that criminal celebrities will go on being dishonest if it suits them. As you might expect, this student went on to conclude that, given their records, they do not deserve to keep the money they make.

The next extract [3] introduces the idea that there can be good consequences from criminals becoming actors and writers. This is not an angle that is covered by the author, but it is a relevant point to consider. Experiences of life in the criminal world and in prison do add to public awareness. If this is a good thing – and the student claims that it is – then allowing criminals to become writers, actors and so on does have a useful purpose. It would follow that there is some justification for rewarding them, which of course challenges rather than supports the author's conclusion.

The third piece also considers the consequences of allowing criminals to become role models. It obviously supports the argument.

Rights – and wrongs

Probably the most important part of the argument in 'Time to get tough' is the issue of people's *rights*. As observed when we were evaluating the argument, the author clearly assumes – and wants us to assume – that ex-convicts don't have the same rights as other people, especially their victims, because they have chosen a life of crime. Opposed to this is the view that once the criminal has served their prison sentence, then their debt to society has been paid in full, and they come out with all their human rights restored. As we know, the author tries to rubbish this view as 'woolly-minded' thinking. But that doesn't stop you from developing it more sympathetically in your own argument. For example:

[5] It is the job of courts to punish criminals who are caught. Unless their crime is bad enough for a life sentence, they only lose their human rights while the sentence lasts. When they are released they become ordinary citizens again, and should have the same rights as all other citizens, especially if they have learned from their mistakes and are trying to 'go straight'. This is not woolly-minded at all. What is woolly-minded is using our feelings of sympathy for the victims as an argument for punishing ex-convicts for the rest of their lives. That's unjust. As for the victims' right, yes, they do have the right to see the person who has harmed them punished. But the courts decide how much, not the victims, or the media.

Balancing 'for' and 'against'

Of course you may not disagree with the author's reasoning in the way the last critic does. Instead you may agree with the author that the law as it stands gives too little consideration to the victims' feelings. You might argue that whereas a convict gets a limited sentence to serve, the victim may carry the injuries or scars for a lifetime. Where that is the case, doesn't it add insult to injury if the criminal later makes a lot of money by telling or selling the story?

But there is another possible response that we have to consider before we finish this discussion. Sometimes, not infrequently, we hear arguments for both sides of some difficult issue and we are impressed by both of them – or alternatively by neither of them. For example, you may feel, after evaluating and thinking carefully about this argument, that those who champion the victim and those who champion the ex-criminal both have a point, and that whichever way you decide you will benefit one at the expense of the other. In other words, if you stand by the rights of one group, you affect the rights of another group.

That very often happens in real life, and it makes it difficult, or even impossible, for those who have to make decisions to do the 'right thing' by everyone. There is not always a clear choice.

Concluding that there is a balance between equally strong arguments – or equally weak ones – is a perfectly acceptable position to take. It should not be used as a cowardly way of avoiding an uncomfortable decision; but if your critical reasoning leads you to that conclusion, then you have no choice but to declare a 'draw'.

The next and final example demonstrates how further argument can lead to a balanced or neutral position:

[6] It is obviously not much of a punishment for a vicious criminal to come from prison and make a million dollars from a film about the crime, none of which is given to the victims who suffered from what he did to them. But equally it is not very just if someone has completed their sentence and is then punished again by having doors closed on certain careers. It might even drive them back into crime, instead of going straight, which would create other victims. It all depends on whose side you look at it from.

I think talking about 'rights' is the wrong way to approach this problem. We should think about what is best for society rather than about individual people: criminals or victims. Perhaps if we were all less interested in wealth and celebrity, the problem wouldn't arise in the first place, meaning that we are all a bit to blame.

Summary

- Further argument can arise out of evaluation, or it can be a new line of reasoning altogether.
- Further arguments can be raised in support of the author's conclusion(s), or in opposition to them.
- Sometimes further argument leads to a balanced or neutral conclusion.

End-of-chapter assignment

'Where performance-enhancing drugs in sport are concerned, zero-tolerance is the only policy that should be considered.'

Write your own argument to support or challenge this claim.

4.11 A self-assessment

This final chapter in the unit brings together a range of the critical skills you have been using. It consists of an activity in three parts, and is based on a standard exam question type. There is one difference: the passage to which the questions relate is from an authentic published source. For that reason the activity is not only good examination practice; it is also a sample of how to read critically and perceptively in a real-life situation.

Most of the time, when you encounter a news story or magazine article, you respond to it with casual interest, but little more than that. That's fine, if you are reading for entertainment or just gathering information. But there are other times when you need to engage with a text more actively, on a deeper level. This applies if the text is on a subject you are studying at school or college; or if you have to respond to it in a discussion or debate; or if it relates to your work. There are other occasions, too, when there is no particular external reason for you to engage with it critically, but the article just 'grabs' you, and you want more from it than you would get from skimming through it once.

The document you will be working on (page 196) was published in an edition of *Whale and Dolphin*, the magazine of Whale and Dolphin Conservation (WDC). It has some interesting connections with the material you worked on in Chapter 4.6, but it makes a very different point. The natural features of authentic texts make the task of critical assessment more interesting, and more realistic, but at the same time more challenging. As with any text, you should read or scan the passage once to get the gist of it. Note the kind of text it is – its *genre* – and its source. These factors may influence how you interpret and evaluate it later. If it is an argument, note its conclusion and the kind of reasons or premises that are offered. Then answer the following questions, rereading the text as necessary. (Although these are examination-style questions, which would normally have a time limit, there is no time restriction here. Think about the text and questions *in depth*, and apply all of the concepts and critical methods you have been studying in Units 2 and 4.)

Activity

a Show that you understand the structure of the argument. You should identify the main conclusion and the reasoning given to support it.

b Critically evaluate the argument. You should identify any assumptions, flaws and weaknesses and assess their effect on the strength of the reasoning.

c 'Animals that show high levels of intelligence deserve to be treated like humans.'

Write your own argument to support or challenge this claim.

Commentary

The purpose of this commentary is to guide you in assessing your own responses to the questions: not just what you wrote, but how you went about it. Remember that even before you were given the three questions, you were asked to read the passage once through to get

WALK THIS WAY!

Studies show the art of tail-walking is spreading amongst Adelaide's Port River dolphins.

WDC studies in Australia have revealed a growing number of dolphins in the wild are learning to walk on water. WDC's Dr Mike Bossley has been observing Adelaide's Port River dolphins for the past 24 years and has previously documented spectacular tail-walking in two adult female dolphins, Billie and Wave.

Now it seems that tail-walking is spreading through the Port River dolphin community. Up to six dolphins, including young infants, have been seen mastering the technique – furiously paddling their tail fluke, forcing their body out and across the water.

According to Dr Bossley, the dolphins seem to walk on water for fun, as it has no other obvious benefit. The behaviour seems to be cultural, although unusually it is not linked to any practical use such as foraging for food. Tail-walking is rare in the wild and more commonly seen among captive dolphins trained to perform tricks.

Billie is thought to have learnt the trick during a brief period when she was held captive in a dolphinarium, before being released back into the wild. It appears that she has passed this trick onto others in the pod who now practise many times each day.

WDC dolphin photographers Marianna Boorman and Barbara Saberton and have recently documented Wave's calf, Tallula, also attempting to tail-walk. 'As far as we are aware, tail-walking has no practical function and is performed as some form of recreation, like human dancing or gymnastics,' says Dr Bossley.

Adelaide's dolphins are not performing operas, or composing symphonies as far as we know. But tail-walking in dolphins adds more evidence to the contention that dolphins are very intelligent and so similar to humans that they are worthy of a special ethical status: that of 'non-human persons'.

Whale and Dolphin: magazine of Whale and Dolphin Conservation (WDC)

some general view of what it is about. This is different from the directed reading that you need to do in order to answer the questions, but just as important.

Context and genre

When you first engage with a new text, one of the first questions to ask is: What is the *context*? There is less chance of misunderstanding a passage if you know something of its background and the purpose behind it. In this case we already know the source, as it is acknowledged at the end of the article. There is no named author, but we are told that the piece appeared in the magazine belonging to Whale and Dolphin Conservation. This tells you something about the *genre* to which the text belongs. ('Genre', remember, means a kind or type, or sometimes a style.) From its name alone it is a safe bet that the magazine is principally concerned with conservation issues regarding marine animals.

Its contents will be broadly scientific. But it will probably have an agenda, or a 'philosophy', which will influence the kind of articles it contains and the kind of messages they will send to the reader. We can also assume fairly safely that the readers who subscribe to the magazine will be sympathetic to arguments that champion whales and dolphins and which argue for their welfare and even their 'rights'. You may well have similar sympathies; many of us do. Dolphins are lovable, playful and seemingly intelligent creatures; and it is not difficult to see why people might think that they deserve the 'special ethical status' to which the writer refers.

These contextual details are important when you move from analysing the article to evaluating the reasoning in it. In order to think critically about this passage, you must guard against being influenced by emotions or sympathies, and be aware of any bias in the author's treatment of the evidence. Obviously, the author is motivated by the wish to protect and champion the cause of dolphins. There is nothing wrong with this. 'Bias' should not necessarily be an accusation. It is not a hidden agenda. But if there is an agenda, hidden or open, it should be recognised as part of the context, and taken into account.

The questions

You may have noticed that the three questions correspond to the three core components of critical thinking: (a) analysis; (b) evaluation; and (c) presenting further reasoning of your own (see Chapter 1.2). These are also the assessment objectives for practically every critical thinking examination syllabus, including the Cambridge Thinking Skills AS Level. This activity addresses all three. We'll discuss them in turn.

(a) Analysis

The bulk of the text is informative and descriptive, and it is only towards the end that the author's purpose becomes really evident. However, once the reader gets to the last paragraph it is clear where the argument is leading. There he states the 'contention' that dolphins are similar to humans – so similar, in fact, that they deserve to be considered as 'non-human persons', and he concludes that tail-walking adds to the evidence that supports this contention. The conclusion is thus quite complex. You could identify it in full by simply quoting the last sentence. If you paraphrased and abbreviated it you may have said something like this:

> C Tail-walking supports the view that dolphins are so intelligent they deserve the ethical status of 'non-human persons'.

There is an alternative way to analyse this sentence, however. You could say that tail-walking adds to the evidence for special status *because* it shows how intelligent dolphins are, and how similar to humans. In other words the first part of the long last sentence is now an intermediate conclusion; the second half the main conclusion. This is a deeper analysis; also a more structured one. But either interpretation captures the author's purpose. (Note that the conclusion is not that dolphins deserve 'person' status. That would be far too strong, and if you were to interpret the conclusion that way, and then criticise it for *being* too strong, you would have committed a classic 'straw man' fallacy.)

Now we move to the body of the argument. Paragraphs 1 and 2 provide the factual (evidential/observational) base, and one of the main premises, namely that dolphins have been seen 'walking on water'. The photograph could be included in the evidence, as could the first sentence of paragraph 5.

However, the claim being made is not just that the dolphins are walking on water but that they are *learning* to do it; being taught. This is not just assumed. It is inferred from the fact that the practice is observed to have spread from Billie and Wave to several other dolphins. There is a further point in support of this inference, in that Billie was once in captivity and is thought to have learned the trick there. As stated in paragraph 3, tail-walking is rare in the wild but more common in captivity; and in paragraph 4 we learn: 'It appears that [Billie] has passed this trick onto others in the pod'. What we have therefore is a sub-argument supporting the major premise in paragraph 1 that dolphins 'are learning to walk on water'.

The second major premise is that – according to Dr Bossley – the dolphins seem to be performing the trick for fun. The reasoning for this claim is that there is no other obvious benefit, such as foraging for food. Dr Bossley is quoted as inferring from this that it is recreational, 'like human dancing or gymnastics'.

The reasoning to these two intermediate conclusions is untidy, in the sense that they are mixed up together. That is how it often is in ordinary-language arguments. In a more standard argument you would find the two sub-arguments separated from each other. Your job, therefore, was to identify and extract the underlying argument. You could have done this either descriptively, as above, or in standard form, for example:

R1 Tail-walking (TW) has been observed to be spreading among Port River dolphins.
R2 TW is rare in the wild, but more common in captivity.
R3 One of the dolphins is thought to have learned TW while in captivity.

IC1 A growing number of dolphins seem to be learning to walk on water.
R4 TW seems to have no practical purpose.

IC2 It seems to be for fun (like human dancing, gymnastics).

IC3 TW is evidence of intelligence and similarity to humans.
C TW is evidence that dolphins deserve status of 'non-human persons'.

(IC3 and C could be one main conclusion.)

As pointed out in earlier chapters, there are often alternative ways of interpreting many natural-language arguments, and your analysis may have differed in some ways from the one above. This is not a problem, provided you have correctly identified the conclusion, and the main reasons. Also, the *depth* of analysis that you give may depend on how long you have to do it in. Here, with unlimited time, we can thoroughly dissect the reasoning, and examine its structure in detail. In an exam, where you may have no more than half an hour to answer all three of the questions, you will need to pare your analysis down to the key points.

The key points are the conclusion, obviously, and the two main premises that tail-walking appears to be learned (IC1) and that it appears to be fun (IC2). These are the backbone of the argument. If these three elements are not identified in your analysis of the text, read the passage again with the above comments in mind. Whilst some arguments leave themselves open to more than one interpretation, in this passage it is difficult to see any other obvious direction for the argument.

A final point: some of you may have noted in your analysis that the evidence that is cited is not 'direct' evidence (see Chapter 4.3, page 145). The observations and inferences are attributed to Dr Mike Bossley. However, the conclusion is the author's. We will see the significance of this shortly when we turn to evaluating the argument.

(b) Evaluation

Once you have identified the conclusion and the main strands of reasoning, it is very much clearer what evaluative points apply. The basic critical questions are:

- whether the reasons (evidence, observations) really do justify the conclusions
- if so, whether the reasons are credible.

The order in which you deal with these questions is a matter of preference. As a general rule it makes good sense to take them in the above order. If the conclusion does not follow from the reasons, it really doesn't matter whether the claims are true or not, since the argument is unsound either way; whereas even when the premises are true and/ or acceptable we still have to check that they support the conclusion (or conclusions).

However, on this occasion, there is so little work to do on the premises that it is as well to answer the second question first. Yes, the evidence is credible. We can't be 100 per cent sure that the photograph isn't a fake, or that Dr Bossley hasn't made up the whole story. But we can be sure that this is very unlikely, and that the purely factual claims are plausible. Dolphins do learn this trick in captivity, and some get returned to the wild where it would be no great surprise if other dolphins copied them. The claims are also verifiable: they could easily be checked, so a reputable magazine would be unlikely to invent them. It would do the WDC cause no good to be found to have made false or unsubstantiated claims.

As noted above, the bulk of the claims, and inferences, are attributed to Dr Bossley. It is therefore relevant to ask whether he is a reliable source (see Chapter 4.4). Again the answer is a pretty confident yes. With 24 years of experience observing dolphins, Dr Bossley almost certainly has had ample opportunity and expertise to make the observations and draw informed inferences from them.

So we come to the reasoning itself. We know, thanks to our analysis, that it consists of two sub-arguments leading to the main premises that tail-walking is apparently learned, and apparently performed for fun. Why 'apparently'? Because in the text the claims are routinely qualified by words such as 'seems' or 'appears'. So we have an inference from increasing numbers of dolphins being *seen* to walk on water since the arrival of Billie and Wave (which are observed facts), to their *seeming* to have learned it from each other. That is a reasonable claim. And we have the inference from its having no obvious practical purpose to its *seeming* to be purely recreational, like dancing etc. The recreational part seems reasonable, too, though the comparison with dancing is questionable.

These inferences are defensible. There is evidence that the dolphins *appear* to be having fun and learning tricks from each other. (It is plausible, too, since dolphins appear to be having fun a lot of the time anyway.) The problem with the argument arises when the author wants to say that tail-walking supports the contention about special ethical status. Let's assume, for the sake of argument, that dolphins do teach and learn and practise skills, that their behaviour is cultural, and that they do certain things that are no different from dancing or gymnastics. There might, on these counts, be some grounds for giving dolphins a special ethical status, more like that of persons.

But Dr Bossley does not claim anything as strong as this. His claims are cautious and qualified: '. . . dolphins *seem* to walk on water for fun'; it '*appears* that [Billie] has passed this trick on to others in the pod'; '"*As far as we are aware*, tail-walking is . . . like dancing"'. And so on. Dr Bossley is reported as quite rightly presenting these ideas as speculation, not as fact. The only hard fact that is documented is that dolphins have been seen tail-walking. It is the author of the article who takes it to be evidence of intelligence on a near-human scale. But the evidence, so-called, is too weak to support the much stronger and more controversial 'contention'.

Clarification

Another point you may have made was that the term 'ethical status of non-human persons' needs explaining and defining, rather than just throwing in at the end. 'Ethical status' in the case of humans is familiar enough. It brings with it certain entitlements: not to be killed or subjected to cruelty, or denied freedom or justice before the law, and so on. But what can be meant by the status of '*non*-human persons'? On one reading it is almost contradictory: What is a person if it is not human? On another reading, it could just mean an animal anyway. If the status of non-human persons differs from that of humans *and* from that of other animals, we need to know what it is. No conclusion can be fully justified unless it is clear exactly what is being argued for. (There is an important lesson here for question (c) when we come to it.)

Assumptions

As with many arguments, the problem with the reasoning in this passage can be put down to a major *assumption* that the author makes. In drawing the conclusion, the author assumes that an animal's behaviour is a reliable indicator of its intelligence, and/or of its thoughts or feelings. Perhaps this is a *reasonable* assumption. But it is an assumption nonetheless: there is no independent support for it.

This assumption is not explicitly stated in the text. However, it is *implicit*, meaning that even though the author doesn't state it, it must be true for the conclusion to follow from the premises. We can put this to the test by seeing what effect it would have on the argument if we *denied* the assumption. If it were false that observed behaviour can tell us anything about inner processes or human-like feelings, then the observation of tail-walking becomes worthless as evidence that dolphins are intelligent, or that they are performing the act for 'fun', or 'teaching' each other. They may *appear* to be, as stated; but if appearances count for nothing, these observations are not evidence at all.

On the other hand, if you consider that the assumption is warranted, and that their behaviour is a reliable indicator of what dolphins experience, then you may feel that this argument does have some strength. One important point in its favour is that the conclusion itself is not overstated. It does not declare that tail-walking is some kind of proof that dolphins are like people. It just says that it adds more evidence for such a view. However, by saying that it adds more evidence, there is a new assumption that some such evidence already exists. If it doesn't, then on its own the evidence of tail-walking looks even weaker.

Flaws

You were also asked whether any flaws or fallacies can be found in the reasoning. There are several possible candidates, but we will concentrate for now on just one part of the document, namely the last two paragraphs. Here we have Dr Bossley's claim that: '"As far as we are aware, tail-walking has no practical function and is performed as some form of recreation, like human dancing or gymnastics."' Firstly, this carries the implication that if something has no practical purpose, then it has to be recreational, which could be seen as an example of *restricting the options* (see Chapter 4.7). There may be other possible explanations for the behaviour besides these two. However, if you think that all acts are either functional or recreational, then it is legitimate to imply this.

But there is arguably a more serious fault here. It is using Dr Bossley's claim as evidence for the contention that dolphins are 'so similar to humans that they are worthy of a special ethical status . . .'. 'Recreations', especially activities such as dancing and gymnastics, are distinctively human. We don't know if animals do anything that resembles our sporting or artistic pastimes; so it is a major assumption to suppose dolphins do, especially in the context of arguing for similarities between dolphins and humans. This lays the argument open to the charge of *begging the question*. How can we justify the claim that tail-walking is 'like human dancing' without assuming that there is something human about dolphin behaviour? But that is the very issue that the argument is about. (Note that 'question' in this context should be understood as what is at issue, and not as an ordinary question. Unfortunately people often use the term 'beg the question' to mean ask, or prompt or raise a question. But that is not its traditional or technical meaning.)

There is one more classic fallacy that could be mentioned. It is a common one, and one you may have identified without giving it a name. It is the fallacy of claiming that because there is no evidence for something, it is (or is probably) false; or, conversely if there is no evidence against something, it is (or is probably) true. It is known by the Latin *argumentum ad ignorantiam*, meaning argument from ignorance, or appeal to ignorance: 'ignorance' meaning absence of knowledge or evidence. (It doesn't imply stupidity!) Is this applicable here? Is Dr Bossley saying that because we aren't 'aware' (i.e. don't know) of any practical function for tail-walking, it must have no practical function, and therefore be recreational instead? If so, then this is a fairly clear case of *argumentum ad ignorantiam*. There could be functions of tail-walking that no one is aware of.

The principle of charity revisited

The evaluation so far has been heavily critical. Has it been *unfair* in the process? Not if the above interpretation of the reasoning is deemed to be fair. So long as 'Walk this way!' is understood as a definite argument, giving reasons for the conclusion that dolphins are sufficiently like humans to deserve special status, then it is fair to take serious issue with it. There is insufficient evidence to infer anything about the extent to which dolphin intelligence or motivation resembles that of humans. As we have seen, the author relies upon what *appears* to be the case to infer what *is* the case; and that is always a dangerous step. A robot that is programmed to make the sound of laughter may look as if it is amused by something, but no one would say it really found it funny. And as we have seen, there are at least three charges of fallacies which could be levelled at the text.

If you gave the argument more credit than this, that does not necessarily mean that you are wrong. It may just mean that you interpreted it more *charitably*. The principle of charity was introduced in Chapter 2.7 (page 52). Its role in assessing arguments is a very important one. The maxim is that if there are two or more interpretations of a text which the author could *plausibly* have intended, we should settle on the most favourable one, not the least. An interpretation obviously has to correspond to what the author has actually said: we can't just add new evidence, or change the premises to help the author out. But if on one genuine interpretation the author's case is strong, and on other(s) weak, then a fair-minded reader will aim his or her evaluations at the strong one.

For instance, you might argue that there is no question-begging because Dr Bossley is still talking only about an *appearance* of dancing and recreation. He is saying that dolphins *look like* they are having a good time (in the way humans do when *they* dance). Of course, that weakens the premise, but it acquits it of the fallacy. You could defend the argument against the other two fallacy claims in a similar way. For example: Dr Bossley is not inferring that tail-walking follows from the lack of evidence but just that *as far as he can tell* it is just a bit of fun. Overall, too, you might want to say that the author's final conclusion is quite moderate: merely that these observations, however they are explained, add something to the case for treating dolphins more like we would treat ourselves; and that the reasoning is up to supporting that claim.

There is another more radical interpretation which we must always consider if we apply the principle of charity fully. It is that the article is not a serious, or hard-line, or *literal* argument at all. On reading 'Walk this way!' you may have felt that, whilst it was expressed in the *style* of an argument, the author was really just using it to explore an interesting idea; to try out a hypothesis. You might say it was a *quasi*-argument. That way you would interpret the text less as a full-blown argument, and more as a thought-provoking discussion, perhaps deliberately going too far just to liven up the animal rights debate.

There is a danger when applying critical thinking to real-life texts in assuming that any contentious piece of writing or speech must be understood as an all-out argument. There are other ways of making a case, and a quasi-argument may be one of them. But there is a danger as well in applying the principle of charity too liberally. You should not use it to let every author 'off the hook'. If, after careful and critical assessment, you really think that the author is in the business of arguing for a conclusion, and persuading the reader that it is right, then you must judge it accordingly, even if that means rejecting it.

(c) Further argument

The commentary for this part of the activity will inevitably be lighter than for the first two. That is because it is your turn to produce the argument. The authors of this book cannot anticipate what your argument will be. We can, however, give some guidelines for you to use in assessing yourself. The guidelines take the form of questions, and provide a checklist of advice for answering questions of this type.

(i) What did you take the task, or instruction, to be?

Note that you were asked to produce an argument to support or challenge the quotation. You were not asked to discuss the *topic* in an even-handed way, without reaching any particular conclusion of your own. You were asked to argue for the statement or against it: to take sides. Did you do that? If not, you missed the point of the question.

(ii) What did you make of the statement: 'Animals that show high levels of intelligence deserve to be treated like humans'?

Some statements may allow us say: 'This is neither right nor wrong,' and to give a balance of arguments for each side. But this statement doesn't really leave that option. Either highly intelligent animals deserve to be treated like humans, or they don't. Even if you wanted to dilute it by adding words to the effect of '. . . . in some ways but not others', or 'to some extent', that would be a challenge to the statement as it stands.

We saw in the previous commentary, under the subheading 'Clarification', that you need to state very plainly what your conclusion is before you set out to defend it. You need to do this for yourself as well as for your readers. Whether you were supporting or challenging the quotation, you should have made it clear how you understood it: for example, what 'high levels' would include, and what 'treated like humans' means.

(iii) Did your reasoning really support your conclusion?

Stating your own conclusion clearly and explicitly is important. You can start by stating it, or leave it until the end. Or you can repeat it in more than one place, for emphasis. But merely stating it is not the end of the matter. The reasoning that you give for the conclusion really must support it. It is very easy, partway through your response, to waver, or give way to doubts that you haven't really got such a good case after all. The solution is to plan thoroughly what you are going to say – and why – before you start to write. For instance: 'I support the statement because: R1, R2, R3 . . .' Each of these should be a substantial reason, or item of evidence. If you don't have at least two or three effective reasons in mind before you begin, you may regret the line you have chosen.

(iv) Did you develop some of your reasons?

More important than having lots of separate reasons is the development you give to your reasons – to some of them at least. A major premise in your argument may need evidence to support it – in other words, a sub-argument. Development may also take the form of explaining or clarifying. If your argument is just a list of reasons, plainly stated, then think about ways in which you could have enriched and reinforced each step.

(v) Did you anticipate objections and opposing arguments before you started, and deal with some in your response?

One important and effective way to develop your reasoning is to anticipate and counter what the other side in the debate might say. For instance, suppose one of the steps in your argument was that more intelligent animals are more likely to feel pain in the way humans do, so we should spare them pain as we would humans. One objection an opponent may make – and some do – is that we have no evidence of what animal pain is like, or even that animals are conscious of pain at all; so treating them like humans would be futile and costly. You can develop your own point by anticipating this objection, and then responding critically to it. For example, you could reply that just because we cannot know that animals are conscious of pain, we can't just dismiss the possibility because of that. That would constitute the so-called 'argument from ignorance': that *lack* of evidence for some claim is grounds for denying it.

This is not itself a line of reasoning that you should have included in your answer. It is an example of the kind of structure that you can build into your own arguments, to develop and strengthen your own premises. By showing that your observations are not only positive reasons for your conclusion, but also that they are resistant to counter-claims and counter-arguments, your case is strengthened and shown to be more thoughtful, and more critical.

Summary

- In this final chapter you have had the opportunity to apply the three core components of critical thinking. These are:
  - analysing and interpreting texts (including considerations of context, genre, source, etc.)
  - evaluating an argument
  - presenting further argument of your own.

End-of-chapter assignments

1 Explain briefly why it may be relevant to the evaluation of the argument in 'Walk this way!' to know its source.

2 To what extent would you say the author of 'Walk this way!' argues scientifically?

3 Which of the following sentences expresses an assumption that is implicit in paragraph 3 of the argument? (Give reasons for your answer.)

   A Acts that have no benefit must be done for fun if they are done at all.
   B Foraging for food is not a cultural activity.
   C It is wrong to train captive dolphins to perform tricks.
   D Captive dolphins must enjoy performing tricks.

4 Briefly explain the meaning of the word 'anthropomorphic', with the help of a dictionary if you wish. How might the concept of anthropomorphism be used to challenge the argument in the WDC article?

5 **Examination practice**
Answer the three questions again with a time limit of 10–15 minutes each. If you wish you may also revise your earlier answers now that you have studied the commentary:

   a Show that you understand the structure of the argument. You should identify the main conclusion and the reasoning given to support it.
   b Critically evaluate the argument. You should identify any assumptions, flaws and weaknesses and assess their effect on the strength of the reasoning.
   c 'Animals that show high levels of intelligence deserve to be treated like humans.'

   Write your own argument to support or challenge this claim.

Answers and comments are on pages 325–26.

5.1 Combining skills – using imagination

The next four chapters deal with more advanced problems. In some cases these are just harder or longer examples based on the skills you have already learned. In other cases, slightly more advanced use of mathematics is required. This does not go beyond algebra and probability at relatively simple levels but, if you are not confident with this, you can first look at Chapter 6.1, which may help you in using these mathematical techniques. The problems may involve the use of several different skills in one question, require extra stages of intermediate result or require more imagination in developing methods of solution. The examples in this unit, some of which are longer and harder than those you are likely to encounter in AS Level thinking skills tests, will help you to improve your skills and make the standard questions seem easier. They will be particularly useful for those candidates taking higher-level tests, including A2 Level and some university admissions tests. The end-of-chapter assignments include a question from an A Level Thinking Skills paper and show the progressive nature of such questions, where either additional material is introduced or the conditions of the question are changed. Further examples showing the nature and difficulty of actual A Level questions can be found in past papers.

The problem below is an example of one requiring imagination; although data extraction and processing skills are needed, the main difficulty is in finding a method by which to solve the problem.

Activity

Grunfling is an activity held in Bolandia, where competitors have to contort their faces into the most extreme shapes. Several Bolandian villages have a grunfling competition each year. Each village puts up a champion grunfler who demonstrates his or her skills, then the villages vote one by one. (They are not allowed to vote for their own grunfler.) Each village awards 8 votes to their favourite, 4 to the second, 2 to the third and 1 to the fourth. Clearly, tactical voting is important, so the order of voting is changed every year. This year, the villages vote in order from most northerly to most southerly. The results before the last two villages have voted are shown (in voting order). Who still stands a chance of winning?

Fartown	6
Waterton	5
Blackport	6
Longwood	24
Gigglesford	12
White Stones	9
Martinsville	24
South Peak	4
Riverton	13
Runcastle	17

Commentary

This is mainly a data-extraction type question. Such questions are normally quite straightforward but this one includes a large amount of information to digest, and a method of solving it also needs to be found.

There are three important things (the first skill is to identify these):

1. The scoring system, which means that with two villages left to vote, the maximum extra votes that any one village can score is 16.
2. The fact that a village cannot vote for itself, which means that Riverton and Runcastle can only receive a maximum of 8 more votes.
3. Some villages might score no more, so any village that can pass the mark of 24 can still win.

Given these three things, the method becomes much clearer. The appropriate maximum available must be added to each team and the result compared with 24. The allocation of the lesser votes is unimportant, as they could go to villages who have no hope anyway.

Adding 16 votes to each of the first eight villages, we see that four of them can exceed 24: Longwood, Gigglesford, White Stones and Martinsville. Adding 8 votes to each of the last two, we see that Riverton cannot reach 24 but Runcastle can reach 25. So five teams can still win. Runcastle would be best advised not to vote for Longwood or Martinsville!

You may see that this question required no new skills, and the mathematics was limited to simple addition and counting. The difficulty in this question was in using the information correctly and seeing how best to proceed.

The next activity gives an example in which the main problem is in identifying a method of proceeding. The information in this case is much simpler.

Activity

A survey of Bolandian petrol prices showed the average to be 82.5¢ per litre. Filling stations in the province of Dorland made up 5% of the survey and the Dorland average was 86¢ per litre.

On average, how much more expensive is petrol in Dorland than in the rest of the country?

Commentary

This problem is not, in principle, any harder than those we have encountered earlier. It is mathematically slightly more complex and a clear idea of the meaning of an average must be retained.

We can quickly note that 5% is $\frac{1}{20}$ of the total. One easy way to proceed is to assume that there were 20 filling stations in the survey, one of which was in Dorland.

The sum of the prices at all Bolandian filling stations must have been 20 × 82.5¢ = 1650¢. The price in the Dorland filling station was 86¢. Therefore the sum of the prices in the remaining 19 was 1650¢ – 86¢ = 1564¢. The average in the rest of the country was

$$\frac{1564}{19} = 82\frac{6}{19}$$

or about 82.3¢. So Dorland prices are, on average, 3.7¢ more expensive than in the rest of the country.

Since all the numbers are just over 80¢, we could make life easier by subtracting 80¢ from everything, leaving smaller numbers to work with. As long as we remember to add the 80¢ back on at the end, this will still give the right answer. For example, if we wanted the average of 82¢ and 86¢, we could say this was (82¢ + 86¢) ÷ 2 = 84¢. It would be much easier to note that the average of 2¢ and 6¢ is 4¢, then add this back on to the 80¢. In the example above the calculations reduce to:

$20 \times 2.5¢ = 50¢$
$50¢ - 6¢ = 44¢$
$\frac{44}{19}¢ = 2\frac{6}{19}¢$

Once again, experience and a lot of practice is the way to become efficient at solving the harder problems. The more different types of problem you see, the more you will be able to build on your skills and combine skills you have previously learned into techniques for solving new types of problem.

The activity below uses simple probability, something we have encountered very little so far. Once again, Chapter 6.1 gives some help if you are not familiar with the mathematics. An alternative way of answering the problem using permutations is also shown below. The question itself is probably harder and longer than anything you will encounter in a thinking skills examination.

Activity

My local supermarket has a promotional offer. It gives a coloured token with every spend over \$50. There are three colours: red, blue and yellow. When you qualify for a token, you take a random one from a large bag which contains equal numbers of each colour. When you have collected one of each, you get a \$20 rebate from your next shopping bill.

In order to maximise her chances, Helga makes sure she spends just over \$50 each time she shops and plans on shopping four times in the two weeks the promotion will run. She is sure she will then have one of each.

What is her percentage chance of getting a full set (to the nearest 1%)?

A 2% **B** 10% **C** 11%
D 44% **E** 100%

Commentary

As noted in the introduction, there are two ways of approaching this problem. Using probability, we can say that it doesn't matter what colour she gets on her first visit. The chances of her getting a different colour on the second visit are ⅔.

There are a lot of dead alleys here, so we need to concentrate on the routes which lead to success. These are (where 'different' means a colour she hasn't had before):

1. Any – different – different – repeat
2. Any – repeat – different – different
3. Any – different – repeat – different

There must be two 'differents' and the repeat can be anywhere in the sequence. We can now look at the probability of these three winning combinations:

1. $1 \times \frac{2}{3} \times \frac{1}{3} \times 1 = \frac{2}{9}$
2. $1 \times \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{2}{27}$
3. $1 \times \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{4}{27}$

Adding these I get $\frac{(6+2+4)}{27} = \frac{12}{27} = 44\%$ (to the nearest 1%). D is the correct answer.

We now look at an alternative way of solving this problem using permutations. In total there are 3^4 (= 243) orders in which she can get her four tokens. However, all of these will include at least one repeat so we must be careful. Of these, any combination including ABC (e.g. CABA) will do. All of these combinations will have one repeat, so we can list the winning combinations.

Listing those with two reds (Rs) we have:

RRBY RRYB RBRY RYRB RBYR RYBR BRRY
YRRB BRYR YRBR BYRR YBRR

This is 12 in total.
(For those familiar with permutations, this is $4! \div 2!1!1!$ Here the exclamation mark means 'factorial' and means multiplying together all the integers up to the one shown, so $4! = 1 \times 2 \times 3 \times 4$.)

There will be the same number with two Bs and with two Ys, making 36 which fulfil the requirement.

We now need to list the losing combinations. These must fall into three categories:

4 of the same colour: 3 combinations (RRRR etc.)
3 of one and 1 of another: 24 combinations (RRRB, RRBR etc. – check yourself that this is right)
2 of one and 2 of another: 18 combinations (RRBB, RBRB, RBBR, BRBR, BRRB, BBRR and the same with the two other pairs of colours)

Thus we have 36 which win and 45 which lose; 81 in total, so $^{36}/_{81}$ win or 44%.

The two methods of solving this were similar in difficulty, but the permutations method took a lot more care in not missing any options. As noted in Unit 3 this is typical of many problems, both in examinations and in the real world: there is often more than one way of solving a problem and it is necessary to keep an open mind, especially if the method you are trying is not working or is taking too long.

Summary

- We have looked at more difficult problem-solving questions that require a combination of skills to solve them.
- Longer questions can use several different skills and progressively introduce additional complexity.
- The value of experience has been emphasised in recognising the skills needed for a question and applying them in an appropriate manner.
- We have seen the importance of recognising the important elements in a question and simplifying it by concentrating on these.
- We have seen how imagination may be required to come up with methods of solution for types of problem that you may not have previously seen.
- Problems can sometimes have more than one method of solution, so it is important to keep the mind open for alternatives and to choose a method which is effective.

End-of-chapter assignments

1 *Study the information below and answer the questions. Show your working*.

The driving licences issued in Great Britain up until 1 April 1999 did not have a photograph, but there were features to help the police to check if a licence they were shown was likely to be a valid licence for a particular driver.

Jeremy noticed that the six-digit number (shown in bold) on his driving licence might be somehow associated with his date of birth: SMITH **704309** J99RX. He was born on 30 April 1979.

Iain's number is 806210, and he was born on 21 June 1980. Between them they thought they understood how the digits were selected and arranged, and correctly predicted Fred's six-digit number, knowing that he was born on 17 March 1981.

a **(i)** What was Fred's number? [1]

(ii) Although they could make this prediction knowing the numbers and dates of birth of both Iain and Jeremy, they could not be sure how the numbers were constructed by just looking at the number and date of birth of **only one** of them. Why not? [1]

(iii) Give an example of a date of birth which would have been sufficient on its own to make this prediction with confidence. [1]

Emma pointed out that it must be a more complicated system than Jeremy thought, as her number is 662126, and her (female) friend Jocelyn has 752232.

Jeremy, knowing that Emma's birthday is 12 December, correctly suggested that this is because a specific number was added to one of the digits for females.

b **(i)** How much is added, and to which digit? [2]

(ii) What is Jocelyn's date of birth? [1]

c Although never implemented, the authorities considered identifying people who had been born outside Great Britain by using a similar system to that which identifies gender.
Give an example of how this could have been done, within the six digits, without losing any of the existing information. [1]

d Sometimes people tried to use the driving licence of one of their parents. Given that a police officer can estimate a person's age to within ten years, what is the chance that the deception would be noticed from looking at the person and the number on the driving licence? [1]

e Using a random number for making a fake licence for a male, what is the probability that it would fail to give a valid month and date (ignoring the year)? [2]

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2 Fastrack runs a non-stop express service between Aaland and Matsberg, which takes 40 minutes. Stagebus offers a stopping service between the same two towns, serving three intermediate villages an equal distance apart.

The Stagebus stops for 5 minutes at each village and takes 1 hour 15 minutes from leaving Matsberg to arriving at Aaland.

The first Fastrack bus in the morning leaves Aaland at 8 a.m., and the first Stagebus leaves Matsberg at 7.45 a.m.

At what time do they pass each other (to the nearest minute)?

A 8.10 **B** 8.16 **C** 8.20
D 8.24 **E** 8.26

3 There are four teams in the netball league on the island of Naldia. In a season, they play each other once. Three points are awarded for a win, one for a draw and none for a loss. At the end of the season, the points were as follows:

Dunrovia	6
Arbadia	4
Brindling	4
Crittle	2

a How many matches were drawn?

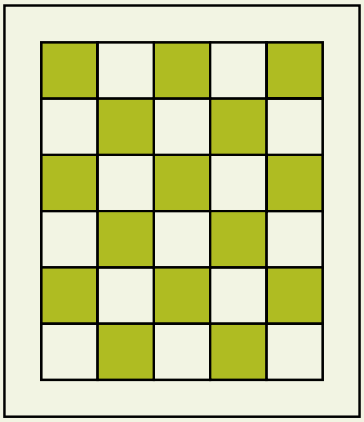
b What was the result of the match between Dunrovia and Crittle?

c If Brindling beat Dunrovia, can you determine the results of all the matches?

4 Andy, Benita and Chico went out for a meal together. When the bill came, they thought they would divide it equally between them. However, Chico admitted to having chosen more expensive dishes and noticed that his total was $3 more than the amount he would have paid if they split it equally.

If Andy and Benita's bills were $12 individually, how much was Chico's?

5 Fatima is making a quilt. The overall size is 1.7 m × 2.0 m. It will have a pattern of 6 × 5 patchwork squares in the middle and an equal border all the way around as shown.



What size should the patchwork squares be?

6 Bill, Harry and Fred run a gardening business. Bill pays all the annual fixed costs (insurance, telephone line rental, etc.) by instalments, which amount to $400 per week. Harry buys the materials for any job they do. Fred collects the payment for jobs. They split all profits evenly and settle up after every job is completed. They have just done a landscaping and re-fencing job for Mrs Keane that took the three of them exactly two weeks. Materials cost $1400 and Mrs Keane paid Fred $4900.

How much does Fred owe Bill and Harry?

Answers and comments are on pages 326–27.

5.2 Developing models

In Chapter 3.11 we looked at how models can be used by governments, industry and so on to carry out 'what if?' analyses and look at how changes to an environment can affect various factors. In this chapter, modelling is taken further. Questions may involve the application of more complex models or the development of a model for a given situation. In the longer, multiple question items which may be encountered in A Level examinations or some admissions tests, the individual questions usually increase in complexity, either by asking for a deeper analysis or by introducing new situations or conditions.

The activities in this section are progressive, starting with the application of a model which is provided and progressing through simple linear models to the development of a non-linear model. The final activity is harder than those that would be encountered in an A Level examination and will be useful for those intending to take university admissions tests or those wishing to prepare themselves better by tackling harder questions. Candidates will find a range of questions of appropriate difficulty in past papers.

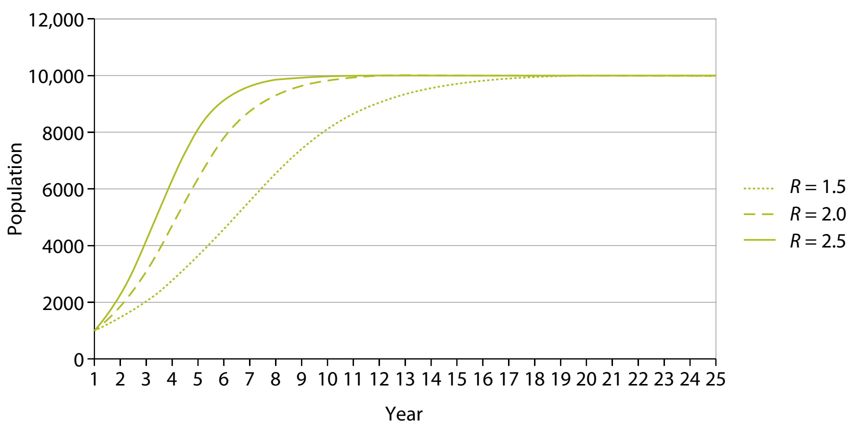
The first example shows how models can be useful in real situations.

The equation below is a very simplistic mathematical model for a fish farm. It is called the Beverton–Holt model and considers the effect of various reproduction rates and the maximum capacity of the farm.

$$n_{t+1} = \frac{Rn_t}{(1 + n_t(R-1)/K)}$$

For the non-mathematical, this equation may look rather frightening, but don't worry, we are not going to do any hard algebra and the examples you will encounter in examinations will use much simpler mathematics.

In the equation, R is the reproduction rate ($R = 1.5$ means the population, if unlimited, would rise by 50% every year – this allows for both births and deaths). K is the maximum capacity of the fish farm. n_t is the population in the current year and n_{t+1} the population in the next year. We can try putting some numbers into this equation and looking at what happens. We assume that the initial population (n_0) is 1000 fish and the maximum capacity is 10,000 (beyond this the fish would die of starvation or overcrowding), then look at how the population increases year by year for three different values of R. The results are shown in the graph below:



We can see that the maximum capacity is reached in 10–15 years and the time to reach maximum capacity reduces as the reproduction rate increases. This is exactly what we would expect.

Now what happens if we do some fishing? We will look at the case where $R = 1.5$ and we remove various amounts of fish each year, from year 5 on (we can do this by simply subtracting f fish from the stock for each year in the calculation we carried out based on the Beverton–Holt equation). The results are shown below for annual removal rates of 500, 550 and 600 fish.

We now begin to see how useful such models can be. The population is very sensitive to the amount of fishing: 500 per year is sustainable; 550 leads to a catastrophic drop in the stocks after 12 years. Although this model is not totally realistic, it gives an insight into how models can be of commercial value. For those who are comfortable with the equation and spreadsheets, it is easy to play with the parameters of this model and carry out exactly the sort of 'what if?' analysis we mentioned before.

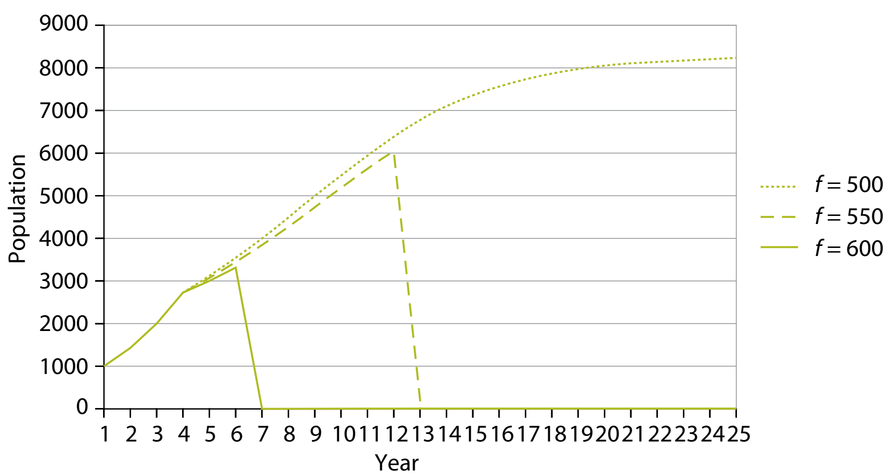
As an additional activity you might consider alternative fishing strategies: for example, waiting longer before starting to fish or taking a percentage of the population rather than a fixed number of fish.

The modelling problems described in Chapter 3.11 involved choosing the correct model of a given situation. More advanced modelling questions such as may be encountered in A2 Level examinations (e.g. Cambridge Thinking Skills Paper 3 or BMAT Paper 1) can require the solver to use a model to draw conclusions or actually to develop a mathematical model for a given situation and make inferences from the model derived. Some of the problems we have already seen are in this category, but the model is so simple that you are usually unaware that you are using it. For example, the activity in Chapter 3.5 about Petra's electricity involved recognising that the bill, made up of a fixed monthly charge and an amount per unit, could be represented by:

cost = fixed charge + u × units used

where u is the charge per unit. This equation is a simple mathematical model.

The following is an example that leads to a model which requires only relatively simple mathematics.



Activity

Perfect Pots is a company making decorative plant pots. Its overheads (rent on premises, insurance, etc.) are $15,000 per year. There are four administrative staff (manager, accountant, sales director and secretary) earning a total of $85,000 per year. The pots are made by a number of skilled workers; each can produce up to 5000 pots a year and earns $20,000 per year. Materials, power and so on cost $1000 per 10,000 pots.

How will the company's profits vary with the number of pots made and sold and the selling price of the pots (assuming the company only makes pots to supply orders)?

Commentary

The model depends on the number of workers, and it must be remembered that each one cannot produce more than 5000 pots per year.

The mathematics of this model are quite simple, depending only on multiplication, addition and subtraction. If the number of workers is n, the number of pots produced and sold is m and the selling price per pot is p, the profit can be calculated as follows:

$$\text{Income} = mp$$

$$\text{Expenditure} = 100{,}000 + 20{,}000n + \frac{1000m}{10{,}000}$$

$$\text{Profit} = mp - 100{,}000 - 20{,}000n - \frac{m}{10}$$

The table on the next page shows how this varies, assuming the number of workers employed is controlled by the number of pots produced.

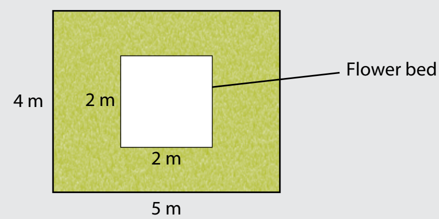
This type of model is useful to the accountant and sales director in producing sales targets. This also leads to the type of 'what if?' analysis that is commonly used in economics. Note that there are points at which selling extra pots means employing an extra worker, which can lead to a fall in profits.

The next activity requires the creation of a relatively simple model and is a good introduction to modelling.

Activity

Greenfinger Garden Services (GGS) offer a range of garden maintenance tasks, including lawn maintenance. They charge one rate per square metre for mowing lawns and a different rate per linear metre for trimming the edges.

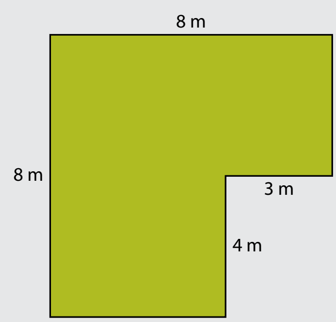
1 Germaine has a rectangular lawn 5 m by 4 m. GGS have been charging him $42 for mowing and trimming the edges of his lawn. He now wants to put a 2 m by 2 m flower bed in the middle of his lawn, and has asked GGS how much the lawn maintenance charge will be when he has done this. The new lawn is shown below:



GGS have quoted $51 for mowing and trimming the edges on his new lawn. By how much has the edge length been increased on the new lawn?

2 In order to run their business efficiently, GGS need a general method for calculating the amount they will charge for other sizes or shapes of lawn. Develop a formula or general rule for calculating the charge for any shaped lawn (assuming it has right-angled corners).

3 They have been asked to quote for a new job. They have measured the lawn and it is shown in the diagram below. What would be the charge for this lawn?



		Selling price per pot ($)			
		10	12	14	16
Annual production	Workers employed	Profit ($)			
1000	1	–110,100	–108,100	–106,100	–104,100
2000	1	–100,200	–96,200	–92,200	–88,200
3000	1	–90,300	–84,300	–78,300	–72,300
4000	1	–80,400	–72,400	–64,400	–56,400
5000	1	–70,500	–60,500	–50,500	–40,500
6000	2	–80,600	–68,600	–56,600	–44,600
7000	2	–70,700	–56,700	–42,700	–28,700
8000	2	–60,800	–44,800	–28,800	–12,800
9000	2	–50,900	–32,900	–14,900	3100
10,000	2	–41,000	–21,000	–1000	19,000
11,000	3	–51,100	–29,100	–7100	14,900
12,000	3	–41,200	–17,200	6800	30,800
13,000	3	–31,300	–5,300	20,700	46,700
14,000	3	–21,400	6600	34,600	62,600
15,000	3	–11,500	18,500	48,500	78,500
16,000	4	–21,600	10,400	42,400	74,400
17,000	4	–11,700	22,300	56,300	90,300
18,000	4	–1800	34,200	70,200	106,200
19,000	4	8100	46,100	84,100	122,100
20,000	4	18,000	58,000	98,000	138,000

Commentary

This is quite a simple and common model. Those familiar with simultaneous equations will probably recognise the style of the question.

1. This question changes the original conditions and simple length/area calculations are required. Before the modification, the lawn was quite simple with an area of 5 m × 4 m = 20 m^2 and an edge length of 2 × (5 m + 4 m) = 18 m. After modification, the lawn area has been reduced by 2 m × 2 m = 4 m^2 and the edge length increased by 2 × (2 m + 2 m) = 8 m. This answers the first question. The new area is 16 m^2 and the new edge length is 26 m.
2. Candidates are now asked to develop a general model for the rates to be charged in GGS's business. We know that there are two prices; we will say that the cost per square metre for mowing is *a* and the cost per linear metre for edge trimming is *e*.

   We can construct two simple equations:

Before modification:	$20a + 18e = 42$
After modification:	$16a + 26e = 51$

   These can be solved using standard methods, or by trial and improvement, to give:

   $a = 0.75$ and $e = 1.50$

   These are the prices of the two services in dollars: \$0.75 per square metre for mowing and \$1.50 per linear metre for trimming edges.

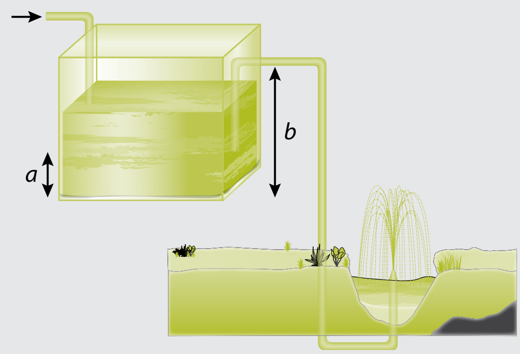
   This answers the second question: the cost formula is $0.75A + 1.50L$, where A is the lawn area and L is the edge length. We can alternatively express this in words, as \$0.75 per square metre of lawn plus \$1.50 per metre of edge.
3. The third question introduces a further new lawn and askes for the cost to be calculated, so the model which was developed above must now be applied to the new situation. The area of the lawn shown (calculated as the surrounding rectangle minus the cut-out) is 8 m × 8 m – 4 m × 3 m = 64 m^2 – 12 m^2 = 52 m^2. The length of the edge is the same as the surrounding rectangle, so is 2 × (8 m + 8 m) = 32 m. The GGS cost for maintenance would be:

   $$52 \times 0.75 + 32 \times 1.5 = 39 + 48 = \$87$$

The question above led to a linear model in that all the prices depended only on sums of the prices multiplied by the relevant areas or lengths. The activity below is non-linear so is a little more complicated mathematically but can be handled by similar methods and should not be beyond those with a grasp of simple algebra.

Activity

A garden water feature is shown below.



The base of the tank is a 20 cm × 25 cm rectangle and it is fed at a constant rate of 10 litres per minute. The tank fills until the depth is *b*, as shown in the diagram, then water flows to the fountain until the depth falls below *a*, when the exit flow stops. The cycle is then repeated. When the tank is emptying (with water running in at the same time), it loses ¼ of the height of water every minute. (You can assume that, during each minute, the fall in height is linear.)

1. Starting from completely empty, how long does it take until the tank starts discharging?
2. When the pipe to the fountain starts discharging, how much water flows out in the first minute?
3. Sketch a graph showing the flow out of the tank against time for a = 20 cm and b = 80 cm. Show more than one cycle. Assume the tank is empty at time 0.

Commentary

This question is progressive, like most longer modelling questions. Starting with a relatively simply calculation, the candidate is expected to develop and apply the model in increasingly difficult ways. This question is harder than those candidates would expect to meet in an A Level examination.

There are some relatively simple calculations which are necessary first in order to calculate the cycle times. It is important to take care to work in consistent units – in this case centimetres, square centimetres and litres (1000 cubic centimetres) are convenient.

1. The tank has a cross-sectional area of $20 \times 25 = 500$ square centimetres and fills at 10 litres per minute. 1 litre is 1000 cubic centimetres, so the tank fills at $\frac{10{,}000}{500}$ cm per minute or 20 cm per minute. Thus it will fill to height b in $b/20$ minutes (with b in cm).
2. We are told that in the first minute, the tank will lose ¼ of its height, so from b to $0.75b$. The volume it will lose is $\frac{0.25b \times 500}{1000}$ litres (the factor of 1000 changes cubic cm to litres), or $0.125b$ litres (again with b in cm). However, in this time there is an inflow of 10 litres, so the actual outflow will be $0.125b + 10$ litres. When doing a calculation like this, it is very easy to get carried away with the difficult bits and forget something simple – in this case that the drop in level allowed for the inflow occurring. It is always important to read carefully both the information given and the question, to be sure of exactly what is required.
3. We know that there is no flow from time zero to time $b/20$ (as calculated in question 1). This is $80/20$ minutes or 4 minutes. We can now calculate the height at the end of each minute. At 4 minutes, the height is 80 cm. At 5 minutes, it is 60 cm (it has lost ¼ of its height). The remainder of the calculations down to a height of 20 cm are shown below. (These can easily be done with a calculator.)

Time (min)	0	4	5	6	7	8	9
Height (cm)	0	80	60	45	33.75	25.31	18.98

The flow will stop when the height drops to 20 cm. Assuming the drop in height is linear over the last minute, it will reach 20 cm from 25.31 cm in:

$$\frac{(25.31 - 20.00)}{(25.31 - 18.98)}$$

$$= \frac{5.31}{6.33}$$

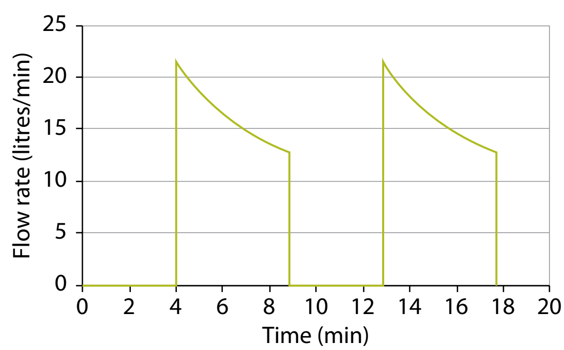
$$= 0.84 \text{ minutes}$$

We could now calculate the outflow of water through the fountain during each minute. For example from 5 to 6 minutes, there is a drop in height of 15 cm, meaning a loss in volume in the tank of $\frac{15 \times 500}{1000}$ = 7.5 litres. However, in this time 10 litres has flowed in, so the flow rate (the volume flowing out per unit time) to the fountain has been 17.5 litres per minute. Repeating these calculations for each minute would allow us to produce a graph of sorts. However, you may be able to see that we could not draw a point showing the flow rate at exactly 4 minutes, only the approximate flow rate at 4.5 minutes. The next step, to produce a more accurate graph, requires some clear thinking (but no particularly difficult mathematics) and is the sort of step which might help improve marks in an A Level examination question.

The height at the end of each minute is ¾ that at the start of the minute. We are told that we may approximate linearly during each minute, so the height after half a minute is $\frac{(1.00+0.75)}{2}$ = 0.875 times that at the start of the minute. In the example above, where the loss in volume from minute 5 to minute 6 was 7.5 litres, we can say that the flow rate at the start of the minute was $\frac{7.5}{0.875}$ = 8.57 litres per minute and the flow rate at the start of minute 6 was ¾ of this or 6.43 litres per minute. To this we must always add the 10 litres per minute flowing in, so the actual values at the start of these two minutes would be 18.57 and 16.43 litres per minute.

We can now repeat these calculations for each minute (remembering that during the periods from when the height has fallen to 20 cm to when it recovers to 80 cm the flow rate is zero). We may also calculate the flow rate just before the flow stops by using the method from the paragraph above, but remembering that the flow stops at 8.84 minutes. The results are shown in the table and graph below. This graph is what was required in the original question.

Time (minutes)	Flow rate (litres per minute)
0	0
4	0
4	21.43
5	18.57
6	16.43
7	14.82
8	13.62
8.84	12.86
8.84	0
12.84	0
12.84	21.43
13.84	18.57
14.84	16.43
15.84	14.82
16.84	13.62
17.68	12.86
17.68	0



Summary

- We have seen how models can be developed for a variety of situations that allow the prediction of what will happen if certain variables change.
- A model can be used to carry out 'what if?' analyses.
- When answering more complex questions, it is important to remember what is required and to reread the material and question if necessary to check that all the important information has been incorporated.
- In order to get better marks in higher-level examinations, it may be necessary to do extra work and to think carefully to improve the quality of the answer.

End-of-chapter assignments

(A calculator or computer spreadsheet will be useful for these assignments.)

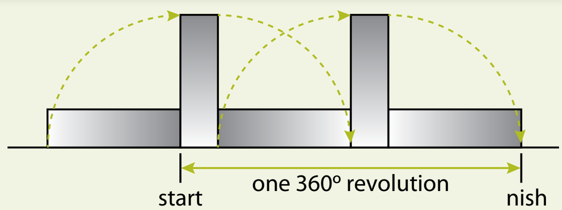
1 Duane and Mervin are going to town, a distance of 12 km. They have only one bike between them, so they decide that one should ride a certain distance while the other walks. The cyclist will then leave the bike by the side of the track for the walker to pick up when he arrives, and continue on foot. The walker will then ride the same distance; and they will repeat the process until they get to town. Duane rides at 15 km/h and walks at 6 km/h. Mervin rides at 20 km/h and walks at 4 km/h.

How long does it take until they both reach town if they use the best strategy?

If you have time, you may consider what would happen if they cycle different distances: can the time for them both to arrive in town be improved?

2 *Study the information below and answer the questions. Show your working.*

Large blocks of stone can be moved by 'rolling' them. The diagram below shows how a single stone can be moved in this way.



It is quicker to lower a tall stone than to raise it. From experience, a stonemason knows that he can turn a stone through 90° in (b/h) minutes, where b is the length of the face that is flat on the ground, and h is the vertical height of the stone as he is about to turn it.

For example, a block that has two of its dimensions as 5 metres and 2 metres can be turned 180° in ($5/2$) minutes + ($2/5$) minutes = 2 minutes 54 seconds.

The stonemason wishes to move large blocks of stone, in order to then cut them into manageable pieces for tombstones. He is considering how to move them most quickly.

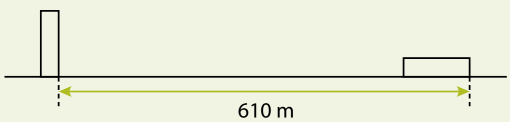
In order to move a block, he chooses the initial orientation, and then rolls it in the same direction for the whole journey.

He will only consider blocks that are cuboid in shape and have dimensions that are whole numbers of metres.

a Consider a block with dimensions 2 m × 2 m × 6 m. Calculate the minimum possible **time** that it would take to roll the block through 360°. [1]

b Consider a block with dimensions 1 m × 4 m × 6 m. Calculate all the different possible **distances** that the block could travel in one 360° revolution, according to the different initial orientations. [1]

c If a 24 m^3 block is to travel at least 610 m, what is the smallest possible number of 90° turns that will be needed?



[3]

d What dimensions for a 24 m^3 block will allow for the smallest possible time to move it 610 m? State the time it will take, to the nearest minute. [4]

He decides that he needs 61 m^3 of stone for the next season. He can only move a maximum of 24 m^3 at a time. It takes 5 minutes for him to return the 610 m distance without a stone.

e Show that it is possible for him to move **exactly** 61 m^3 of stone in less than 500 minutes. [4]

f He realises that by transporting more than 61 m^3 of stone in total, he can reduce the overall amount of time. However, he does not want to move any more than 70 m^3 or there will be too much waste.

What set of block sizes should he move to minimise his total time? [2]

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3 (**Harder task**) A motor race consists of 60 laps of 5 km each. Some of the specifications for the Marlin team car are as follows:

fuel consumption: 1 litre/km
fuel tank capacity: 160 litres
refuel rate: 15 litres/second
pit stop time: 10 seconds plus time to refuel
average speed (no fuel): 75 seconds/lap
speed with fuel: 0.12 seconds slower/lap
for each 5 litres of fuel carried

It may be seen that the car cannot carry enough fuel to complete the race without a pit stop. However, the car goes more slowly the more fuel it carries. The fuel gauge is very accurate, so it can effectively be run down to zero before refuelling. (Hint: in order to calculate the average lap time for each section you may use the average fuel load. Assume the race is broken into equal distances between pit stops.)

How many pit stops should the car make to complete the race in the fastest possible time – 1, 2 or 3?

4 A shop sells three types of nuts:

Brazil nuts: 80¢/kg
walnuts: 70¢/kg
hazelnuts: 40¢/kg

The shopkeeper makes 50% profit on each type of nut. She wishes to sell mixed nuts at 60¢ per kg. What proportion should the mix of the three nuts be if she is to make 50% profit on the mixed nuts? Is there one answer or a range of answers? If so, which contains the most even mix of nuts?
Can you generalise the result?

Answers and comments are on pages 327–30.

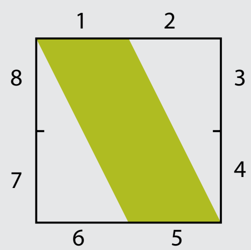
5.3 Carrying out investigations

An investigation is a problem where a set of information is given and the student is asked to consider various scenarios, either to find which is the best, or just to consider the results of various options. Investigations are closely related to modelling, in that a model may be developed to help with the investigation. Alternatively, a set of rules may be formulated which determine a set of possibilities. Some investigations can be quite open-ended, meaning some students will be able to take problems further, extract more detail, illustrate the results better and so on.

The example below is investigative: you have to consider various options and their effect on the result. It uses the skills of spatial reasoning and searching. Investigation questions can use any of the skill types covered in Unit 3 or any combinations of them.

Activity

A company making decorative wall tiles is introducing a new range and their designer has decided to base them on a 2 × 2 grid system so they can be fitted together in various combinations. She has decided to make all possible tiles that can be created by choosing half of one edge (starting from a corner) and joining this to any other half-edge on the tile, filling in the enclosed area with colour. The tile below is one example.



The eight half-edges of the tiles are numbered 1 to 8 on the diagram above and any one may be joined to any other. In this case, half-edge 1 is joined to half-edge 5.

The entire set of tiles consists of all combinations excluding any which may be rotated into each other; so, for example, the two options shown below (1 connected to 5 and 3 connected to 7) are the same tile.

Reflections which result in different tiles are included in the set. The set includes half-edge 1 joined to itself, which will be a blank white tile.

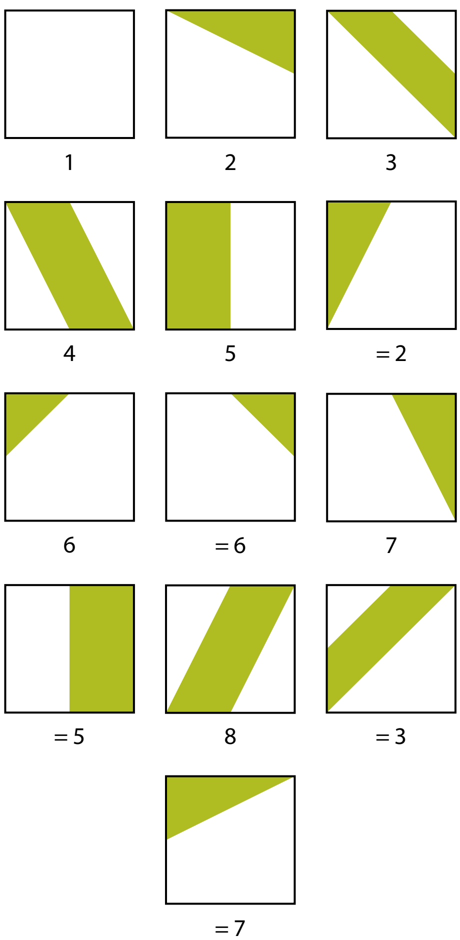
How many different tiles are there in the set?

Make a symmetrical 4 × 4 pattern of tiles which includes at least one of each and with colours matching at all joining edges.

Commentary

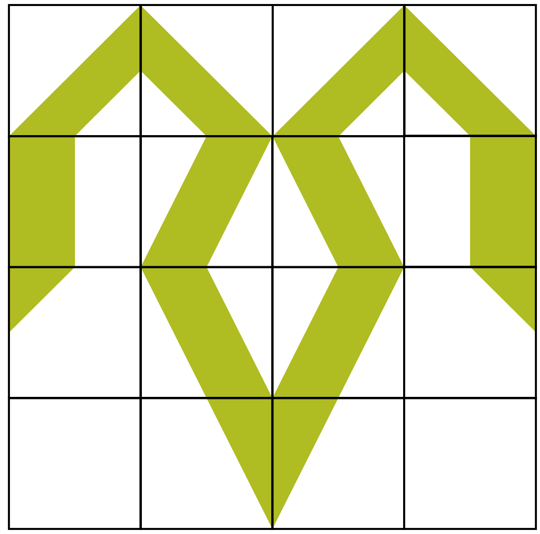
It is possible to count the maximum number of tiles that there could be. Half-edge 1 could be connected to itself or any of the other seven half-edges. (Note that the blank tile is equivalent to connecting a half-edge to itself or the adjacent half-edge on the same side of the square.) Half-edge 2 could be connected to any other than half-edge 1 and itself (both of which we have already counted). Thus, we must investigate 7 + 6 possibilities. Beyond this, looking at connections from half-edges 3, 4 etc. all will produce rotations of those already found.

The full set of 13 possibilities is illustrated below. The numbers below the tiles give a successive count of new ones. Where it says, for example, '= 2', this means that the tile is equivalent (i.e. can be rotated into) tile 2.



There are eight distinguishably different tiles in total.

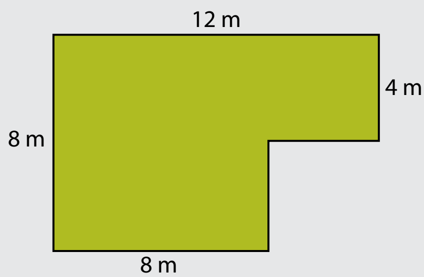
One symmetrical 4 × 4 pattern is shown below. There are many others. Remember that the colours must match at the edges.



As an additional activity, you might look at all similar tiles which join two one-third-edges instead of half-edges, an example of which is shown below. There are not so many of these to make the problem too long, but it will take a little more care to identify all the different ones. To start with, how many options do you need to look at?

Activity

I mow my lawn (as shown in the diagram) using a push-along mower. My speed when mowing is 1 m/s. My mower cuts a strip 0.5 m wide. When I reach the edge, I must turn the mower around. If I turn it through 90° it takes me 5 seconds; if I turn it through 180° it takes me 8 seconds. Every 30 m, I need to empty the grass box, which takes 1 minute. Each time I start a stretch, I must start 1 m into the lawn (as I don't want to stand in the flower beds), but I can mow right to the edge in front of me. I only mow in straight lines.



There are various strategies I can use. I can do it all side-to-side or top to bottom (in both cases remembering to cover any bits I may miss by starting 1 m inside the edges). Alternatively, I can go right round the outside, then do the next strip in, and so on until I get to the centre.

How long will it take me using the best strategy?

Commentary

This is a realistic problem and requires both data-processing and a search (of possible strategies). In cases like this, it is not always possible to be absolutely sure that you have found the optimum – but the investigative process will often make the best strategy clear.

We consider only one possibility here; you should go on to look at others for yourself.

The overall area of the lawn (calculated as the surrounding rectangle minus the cut-out) is 96 m^2 – 16 m^2 = 80 m^2. This means that, regardless of the strategy, I will need to empty the grass box six times (once every 30 m for a mower 0.5 m wide). This takes 6 minutes.

Using the side-to-side strategy: If I start in the bottom-left corner, each strip on the short section will be 7 m long (starting 1 m inside the lawn). Since the mower cuts a strip 0.5 m wide, the lawn width of 4 m for this section requires 8 strips – making 56 m in total (56 seconds). I will make 8 × 180° turns taking 64 seconds (the last turn makes me ready to do the long section). So this section of the lawn takes 56 seconds + 64 seconds = 2 minutes.

The long section will take 11 m × 8 strips = 88 m (88 seconds) and 7 × 180° turns (56 seconds). The total time for this section is 88 seconds + 56 seconds = 2 minutes 24 seconds.

I must now consider the bits I left by starting inside the edge. The left-hand edge is easy, as I am now at the top-left corner. To do this, I do a 90° turn and mow the 7 m back to the start, which takes 5 + 7 = 12 seconds. The mown strip was only 0.5 m wide, so I must do it again, 0.5 m in from the edge, involving another 180° turn and 7 m mowing: 8 + 7 = 15 seconds. The total is 27 seconds.

The bits I missed on the right-hand edges are more complicated. There are two 4 m sections. It is most efficient to mow these when I get there. When I get to the bottom-right (after the first strip) I do a 90° turn (5 seconds), mow 3 m, make another 180° turn (8 seconds) and mow 3 m back. I then need to turn 90° (5 seconds) to be ready for the next strip. (Note that this saved me one 180° turn in the first section). This takes me 5 + 3 + 8 + 3 + 5 – 8 = 16 seconds (the –8 is for the time saved on the first turn).

The top-right 4 m strip will take exactly the same time (if done after the first long strip): 16 seconds.

The total time taken is:

short section: 2 minutes
long section: 2 minutes 24 seconds
left edge: 27 seconds
right edges: $2 \times 16 = 32$ seconds
emptying grass box: 6 minutes
Total: 11 minutes 23 seconds

You should now be able to convince yourself (without doing much more work) whether the up-and-down method would be better or worse. This leaves only the round-and-round, or spiral, method to investigate – you can do this for yourself.

This exercise was surprisingly complicated: it required quite a lot of calculation and needed great care, both in deciding the order of actions and arithmetically. This is typical of investigative problems, in real life as well as in examinations.

Summary

- We have seen that investigations are closely related to models.
- In an investigation, we are not required to develop a mathematical model, although one may be used as part of the investigation.
- An investigation will usually require a search to be made of possibilities, which may sometimes lead to the identification of a maximum or minimum.
- Investigations, like models, can be open-ended. In this case it is important to concentrate on lines which lead to the required answer rather than following all possible paths.

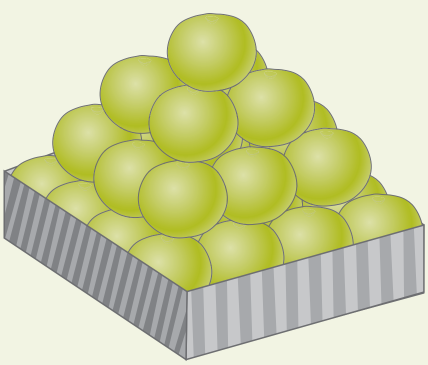
End-of-chapter assignments

1 Coins in most of the world's currencies are based on a decimal system, the individual coins (below \$1) being, for example, 1¢, 2¢, 5¢, 10¢, 20¢ and 50¢ (some may also include a 25¢ coin). Consider a single transaction to buy one item.

a Starting from a purchase worth 1¢, up to what amount can such a transaction be carried out using only one or two coins? This could involve the purchaser paying the exact amount with two coins, or the purchaser offering one coin and receiving one as change (for example, an item costing 3¢ can be purchased by offering a 5¢ coin and receiving a 2¢ coin in change).

b Can you develop an alternative coin system which uses relatively few coins but can make a big range of values using only one or two coins? For example, consider a coin system starting with 1¢, 3¢, 5¢ and so on. This investigation is potentially open-ended, but practicality will limit the area of search (note that a system starting with 2¢, 5¢, 9¢ could not even do a transaction for 1¢ using two coins).

2 A fruit-seller displays his oranges in square boxes which take a whole number of oranges on each side. The bottom layer fills the box and higher layers are placed by putting oranges in the 'dimples' in the layer below until no more layers can be made. This is shown below left for a box of 16 (4×4) oranges on the bottom layer. Clearly a 2×2 box would contain 5 oranges (4 on the bottom and 1 above). How many oranges would a 5×5 box contain? Can you generalise for any square box?



What would happen with a rectangular box? Start with a box containing 4×5 oranges on the bottom layer. Can you develop rules which would allow you to calculate the number of oranges stacked in any rectangular box?

3 Milly is running a game at her school fête to raise money for the school. Her idea is to get people to throw two dice. The players pay \$1 per game and they win \$2 if the two numbers they throw differ by more than 2.

If 200 people play the game, how much money will she expect to raise?

She is worried that people may be able to calculate the odds for this game easily, and that this may discourage them from playing. What alternative criteria could she consider for a win? What about, for example, the product of the numbers on the two dice or the two values written as a two-digit number (e.g. 2 and 5 become 25)? In each case you think of, work out the criterion for a win to ensure that she makes a similar profit to that calculated above. Look at these and any other possibilities you may think of, calculating the odds of winning for different rules. You could also play the game as a class activity and see whether the experimental odds match the calculated value.

Answers and comments are on page 330.

5.4 Data analysis and inference

We saw in Chapter 3.8 that problems involving making inferences from data or suggesting reasons for the nature of the data may appear in either the critical thinking or the problem-solving sections of thinking skills examinations. In this type of question for thinking skills examinations at AS Level and those using short questions, the nature of the data is usually presented explicitly and little analysis is required. This chapter deals with longer questions which may appear in Cambridge A2 examinations, BMAT Paper 1 and AQA Unit 2.

Data analysis may be carried out for a number of reasons and using a wide variety of methods. Some data is collected to investigate a hypothesis or to make decisions on a course of action (for example, will reducing a speed limit reduce road accidents?). Other data is collected as routine and analysis may be much more open-ended, to try to discover patterns and trends.

Examination questions normally use several of the skills introduced in Unit 3. Data selection and processing are obvious, but searching and suggesting hypotheses for variation are also central to this analysis. This type of question does not cover statistical significance finding, but the search for patterns in complex data is an important part of problem solving. The following introductory example uses relatively simple data to illustrate some of the techniques used.

Activity

The table below shows class sizes in publicly funded elementary schools.

	Percentage of pupils by class size			Average class size
Year	<19 pupils	19–25 pupils	>25 pupils	
2006	12.7	50.7	36.6	23.6
2007	15.3	58.9	25.8	22.8
2008	15.5	62.6	21.9	22.6
2009	16.1	62.9	21.0	22.5
2010	21.6	53.6	24.8	22.4
2011	20.2	56.7	23.2	22.5

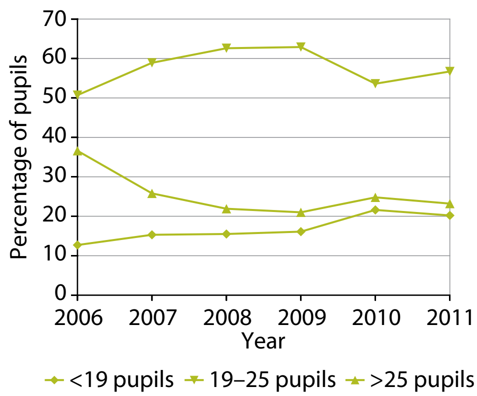
Summary Statistics for Schools in Scotland 2011

1 Draw a graph showing how the percentage of pupils in each class size has varied over the period shown. Express what is shown by this graph in a few short sentences.
2 The table shows the percentage of pupils in classes of the size shown. If we assume that the average class size of classes with less than 19 pupils is 10 and the average for classes over 25 is 30, what are the percentages of actual classes for the three sizes in 2011?
3 The average class sizes have remained constant over the period shown but there have been significant changes in the proportions of pupils in the various sizes of class. How is this possible?

Commentary

This exercise asks three quite clear questions which can be answered by analysing the data in the table in an appropriate manner.

1 The graph is shown below.



This shows the percentage of pupils in larger classes to have fallen over the period shown, and the percentage of pupils in smaller classes to have risen. This also means that (unless the total number of pupils has fallen dramatically) the number of pupils in smaller classes must have risen at the expense of the number of pupils in larger classes.

2 Taking the 2011 figures, we can assume 1000 children in total (it is easier to work with numbers rather than percentages). We then have 202 children in classes averaging 10 each, or 20.2 classes; 567 children in classes averaging 22 each, or 25.8 classes; and 232 children in classes averaging 30 each, or 7.7 classes. This is a total of 20.2 + 25.8 + 7.7 = 53.7 classes. The percentages are 37.6% of classes with under 19 pupils; 48.0% of classes with 19–25 pupils; and 14.3% of classes with over 25 pupils.

3 Using the calculations for question 2, we have a total of 20.2 + 25.8 + 7.7 = 53.7 classes for 1000 pupils, or an average class size of 18.6 children. This is lower than the quoted average in the table, presumably because the estimated sizes of classes in each category in question 2 were wrong. Similar calculations for 2006 give percentages for each class size as follows:

<19: 26.5%
19–25: 48.1%
>25: 25.4%

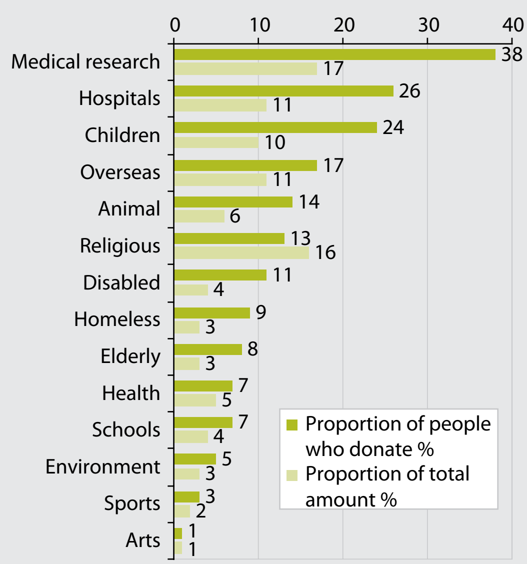
with an average class size of 20.9.

This shows that the number of classes in the middle size range has stayed relatively constant, whilst the number of larger classes has shrunk and the number of smaller classes increased, leaving the overall average relatively constant.

Longer questions at A Level can involve analysing quite complex data and determining what conclusions may be drawn from it. The activity below is of this type.

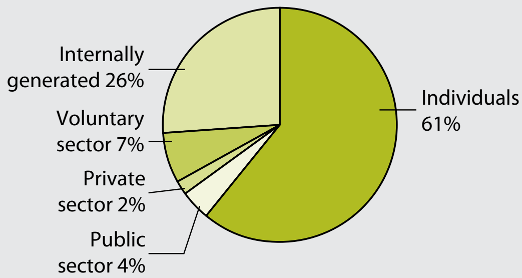
Activity

The graph shows which types of charities in the UK benefit from donations from individual members of the general public. The total amount donated to charities by individuals was estimated to be £11 billion.



UK Giving 2011, NCVO

The sources of all income for medical charities are shown in the pie chart.



Answer the following questions and give brief explanations of your answers.

1 For which type of charity do individuals donate the largest average amount?
2 For which type of charity do individuals donate the smallest average amount?
3 What is the estimated total income of medical research charities?
4 It has been stated elsewhere that 6% of charities receive 90% of the total income, yet medical research, the largest beneficiary, accounts for 17% of donations. Explain this.

Commentary

1 The bar chart shows the percentage of people who donate and the percentage of total donations. Thus the type of charity with the highest proportion of donations relative to the proportion of people contributing will get the highest average donation and vice versa. On this basis, the charity type receiving the highest average contribution is religious organisations (the only one for which the percentage of total donations exceeds the percentage of people contributing).
2 The charity type with the lowest average donation is homeless (3 : 1 ratio). The only other charity type approaching this is disabled (approximately 2.75 : 1).
3 Individual donations to medical research charities amount to 17% of £11 billion, or £1.87 billion. The pie chart shows that individuals contribute 61% of all donations to medical research charities, so the total donations must amount to £ $\frac{1.87}{0.61}$ billion = £3.07 billion.
4 This is explained by the fact that a small number of charities receive very large incomes: there will be a large number of medical research charities, some of which will be very small so will not contribute to the 6%, that receive 90% of the income. The 6% will be made up of a small number of charities in the top few categories. In the lower parts of the chart, there will be a huge number of very specialised charities receiving very small incomes. The numbers on the chart do not relate to the numbers of charities, only to the proportions.

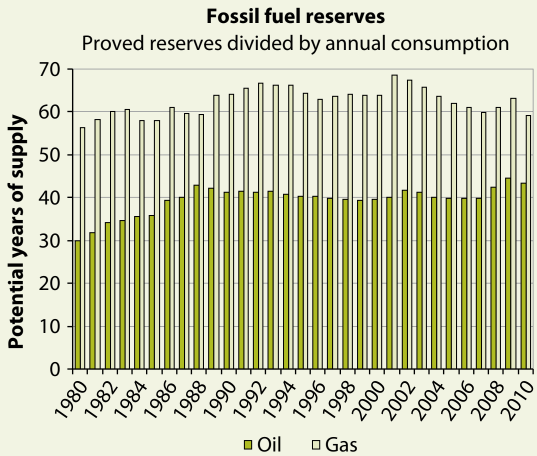
One final repeated warning: correlation does not always mean cause and effect. Sometimes two variables appear to correlate, but one does not lead to the other. The correlation may be coincidental, a statistical fluke, or both observations may be caused by a third factor. One classic example is that there is a close correlation between ice cream sales and deaths due to drowning. It would be ridiculous to say that either of these is a cause of the other. In fact, they both increase during hot weather. Many similar examples can be found.

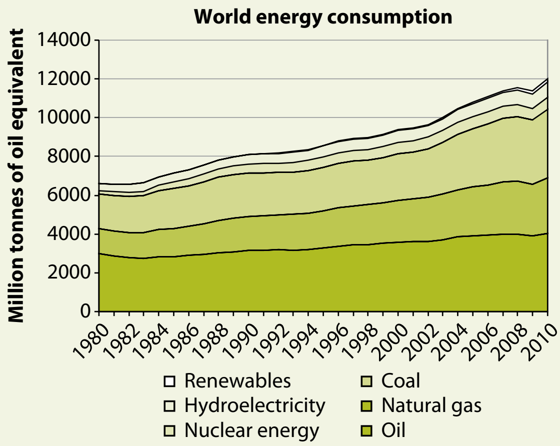
Summary

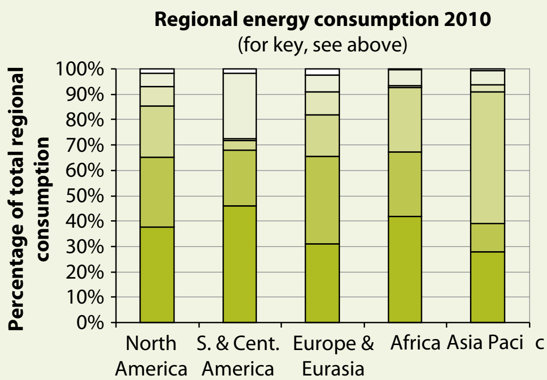
- Complex data sets have been introduced which require a range of skills to analyse.
- We have seen that it is necessary to process data – grouping, averaging and so on – and sometimes to graph data in order to identify patterns and trends which may be used to draw conclusions.
- We have seen that extended examples where more data is supplied can require analysis that may lead to a range of conclusions.

End-of-chapter assignments

1 The graphs show estimates of world fossil fuel reserves, world energy consumption and regional energy consumption by fuel source.







BP Statistical Review of World Energy, June 2011

a The first graph shows the proved gas and oil reserves divided by the actual consumption for each year; the second graph shows actual energy consumption over time; and the third graph shows the percentage consumption of various fuels in different regions of the world. Are the following statements true or false, or can they not be confirmed? Give a brief reason for your answer in each case.

A The world's oil supplies will run out in about 40 years, and the gas supplies in about 60 years.

B There are about 50% more proved gas reserves than oil reserves.

C Over recent years, new discoveries of oil and gas have just about matched consumption.

D Oil and gas reserves are being discovered at an increasing rate.

E Energy consumption is increasing, whilst the known available reserves are fixed.

b Known oil reserves (expressed as potential years of consumption) rose during the 1980s and have been roughly constant since. During the 1980s, world consumption of oil rose by a much smaller amount than the known reserves. Consider what might have caused the reserves graph to behave as it has from 1980 to 2011.

c A comment on this report from the website of the Green Supply Chain stated:

> The report certainly offers some causes for alarm, starting with oil development versus demand.
>
> Global oil production increased by 1.8 million barrels per day or 2.2% in 2010, but did not match the rapid 3.1% growth in consumption, hence leading

to a sharp rise in prices, reaching levels second only to those seen in 2008.

Of greater long-term concern, proven oil reserves worldwide grew only 0.5% in 2010, to 1368 billion barrels . . . So, consumption growth of 3.1% was six times the growth of reserve identification, spelling long-term trouble for prices.

Given that reserves are finite and world energy consumption is rising, what would be the implications of higher prices and less use of fossil fuels on world energy reserves and consumption?

2 In the group stages of a European football tournament, teams were in groups of four in which each team played all the others, making six games in total. The top two teams in each group after this stage went through to the quarter-finals. Teams were awarded three points for a win, one for a draw and none for losing. After four matches in Group 1, the situation was as follows. (Some data is missing from the table.)

Team	Played	Points	W	D	L	Goals for	Goals against
Greece	2	4				3	2
Spain	2	4				2	1
Portugal	2	3				3	2
Russia	2	0				0	3

The remaining two games are Spain vs Portugal and Greece vs Russia.

a Can you reconstruct the missing data (games won, drawn and lost for each team)? How much further can this be taken: can the results of individual games be established; can the scores be deduced?

b Which teams can still qualify for the next stages and what results of the final two matches will be needed to send each possible pair of teams through? (Note: in the event of a draw on points between two teams, the results of the match between those two teams will decide who goes through; if this was a draw, the difference between goals scored and goals conceded decides and, if this is equal, the team with the most goals scored will go through. If all else fails, qualification will be decided by the drawing of lots.)

3 The exercise below is open-ended, in that no specific questions are asked. This is quite typical of real-world problem-solving in relation to scientific work. The experimenter should come to the results with an open mind and squeeze as much information from them as possible (without claiming too much where the results are not entirely clear).

An experiment was carried out to study the growth of doba-berries using a range of amounts of water and fertiliser. 30 beds were laid out, each with an area of 1 square metre. They were each watered daily with amounts of water from 5 to 30 litres. At the start of the experiment amounts of fertiliser from 0 g to 25 g were applied to each plot. When the crop was ripe, the yield from each square metre was measured. The results are shown in the following table. The results for a fertiliser application of 10 g are missing because of a problem with the beds.

Analyse this data and draw conclusions on the effect of water and fertiliser on the crop. Also consider how the two factors may interact with each other. A full statistical analysis is not required; the conclusions may be drawn by averaging and graphing the data in various ways.

Crop yield: kg/m²		Water input: litres/m²/day					
		5	10	15	20	25	30
Fertiliser: g/m²	0	3.55	4.58	5.76	5.36	4.04	2.04
	5	4.54	5.83	7.16	7.54	6.82	4.73
	15	5.21	7.73	9.22	9.32	9.06	8.89
	20	4.85	6.89	8.95	10.27	10.40	9.38
	25	3.97	6.42	9.04	9.62	10.83	10.32

Answers and comments are on pages 331–33.

6.1 Using other mathematical methods

Some types of question may be answered in a more straightforward manner by using mathematical techniques of a slightly higher level than those required so far. In particular, simple algebra can be used to give a clear statement of the problem, which can then be solved by standard mathematical methods. Other areas where some mathematical knowledge can help are those such as probability, permutations and combinations, and the use of highest common factors and lowest common multiples. Although these techniques are beyond the elementary methods we have used so far, they are dealt with in the early stages of secondary education, and most candidates for thinking skills examinations will have some knowledge and skill in these areas. Probability is covered in Chapter 6.3.

Percentages

Most people have a grasp of simple percentages: if a candidate gets 33% of the vote in an election it is quite easy to understand that this means about $\frac{1}{3}$ of voters voted for them. Things become a little more complicated when we try to multiply or divide percentages or deal with percentages over 100. There are, however, very easy ways to tackle these to make them easier to understand. In the example above, suppose only 60% of those eligible to vote actually voted in the election. What percentage of the total number eligible did the candidate get? Once again, most people will be able to handle this, but it is easier to move away from percentages to do it. Multiplying 33 by 60 does not help a lot; we need to understand that 33% is $\frac{1}{3}$ and 60% is $\frac{3}{5}$, then multiply the proportions together: $\frac{1}{3} \times \frac{3}{5} = \frac{1}{5}$ or 20% is the answer. If a town's population is now 120% of what it was 10 years ago, when it was 50,000, the population is now $1.2 \times 50{,}000$, or 60,000. Once again we had to move from percentages to ratios to do the calculation.

In many cases where problems involve percentages the best way to proceed is to use real numbers rather than percentages. In the first example above, if 100 people were eligible to vote, 60 actually voted. Of these 33% or 33 out of 100 voted for the candidate, so $60 \times \frac{33}{100}$, or 20 voted for them. This may seem unnecessary in this simple case, but the value of this approach becomes clearer in the example below.

Activity

A blood test is carried out to screen suspects of a crime. 2% of the population of Bolandia possess 'Factor AX' which is identified by the test. However, the test is not perfect and 5% of those not having Factor AX are found positive by the test (these are called false positives). Furthermore, in 10% of those with Factor AX, the test fails to identify them as having it (false negatives).

A suspect for a crime was tested and found positive for Factor AX. A lawyer for the defence asked what the chances were that somebody testing positive in the test actually had Factor AX.

Commentary

Although a fictitious situation, this is similar to many real problems which medical and legal professionals have to deal with on a regular basis, for example in cancer diagnosis. The answer is much less obvious than it seems and many people will glance at the results and give an answer of 95%, which is 100 minus the percentage of false positives.

Let us now take the approach of putting in real numbers. In this case we will start with a very large number (as some of the percentages are quite small). Say the population of Bolandia is 10,000. Then 2%, or 200, of these have Factor AX. Of these 200, 180 are found positive by the test (i.e. found to have Factor AX) and 20 are found negative. Of the 9800 without Factor AX, 5%, or 490, are found positive and 9310 are found negative. The table below shows the results.

	Found positive	Found negative	Total
With Factor AX	180	20	200
Without Factor AX	490	9310	9800
Total	670	9330	10,000

We can now answer the question: 670 people are diagnosed positive. Of these, 180 have Factor AX. $\frac{180}{670}$ is 0.27 or 27%. This is the required answer, the percentage chance that a person found positive in the test has Factor AX. Working this out directly from the percentages would be very difficult.

Algebra

Consider the problem below. This is similar to one we encountered earlier. It can be solved using intuition or trial and error, but the algebraic method illustrated is quicker. Use of such techniques can be a particular help when working on thinking skills questions under time pressure.

Activity

A ferry travels at 20 km/hour downstream but only 15 km/hour upstream. Its journey between two towns takes 5 hours longer going up than coming down. How far apart are the two towns?

Before looking at the algebraic solution below, you may like to consider alternative ways of solving the question.

Commentary

If the distance between the two towns is x km, we have:

Time upstream = $\frac{x}{15}$ hours

Time downstream = $\frac{x}{20}$ hours

Thus, since the difference between these times is 5 hours:

$$\frac{x}{15} - \frac{x}{20} = 5$$

Multiplying both sides by 60:

$$4x - 3x = 300$$

So x, the distance between the towns, is 300 km. Put this answer back into the question to check that it is right.

This was a very simple example and hardly needed the formality of a mathematical solution. However, similar methods can be used for more complex questions to reduce them to equations that can be solved quite easily. Try the problem below.

Activity

Kara has just left the house of her friend Betsy after visiting, to walk home. 7 minutes after Kara leaves, Betsy realises that Kara has left her phone behind. She chases Kara on her bicycle. Kara is walking at 1.5 m/s; Betsy rides her bike at 5 m/s.

How far has Kara walked when Betsy catches her?

Commentary

Once again, there is more than one way of answering this question, but algebra can make it much more straightforward. If Kara has walked x metres when Betsy catches her, the time taken in seconds from Kara leaving Betsy's house is $\frac{x}{1.5}$. The time for Betsy to cycle this distance is $\frac{x}{5}$. We know that Kara takes 7 minutes (420 seconds) longer than Betsy, so:

$$\frac{x}{1.5} - \frac{x}{15} = 420$$

Multiplying both sides by 15:

$10x - 3x = 420 \times 15 = 6300$, so
$x = 900$ metres

900 metres takes Kara 600 seconds and takes Betsy 180 seconds – a difference of 420 seconds or 7 minutes as required. We could also calculate that it takes Betsy 3 minutes to catch Kara.

Lowest common factors and multiples

Another example follows of a problem that can be solved using a simple mathematical technique.

Activity

From a boat at sea, I can see two lighthouses. The Sandy Head lighthouse flashes every 6 seconds. The Dogwin lighthouse flashes every 8 seconds. They have just flashed together. When will they flash together again?

Commentary

There is a straightforward way of solving this with little mathematics; just list when the flashes happen:

Sandy Head: 6, 12, 18, 24, 30 seconds later
Dogwin: 8, 16, 24 seconds later

So they coincide at 24 seconds. Those with a little more mathematical knowledge will spot that this is an example of a lowest common multiple (LCM) problem. The answer is the LCM of 6 and 8. The prime factors of 6 are 2 and 3; the prime factors of 8 are 2, 2 and 2. One of the 2s is common to both so the LCM is $2 \times 2 \times 2 \times 3 = 24$, the same answer as before.

In this case there is little to choose between the two methods, but if the counting method gave no coincidence for 30 or 40 values, the LCM method would be much faster. There is another lighthouse example in the end-of-chapter assignments, but with a twist. Problem-solving question-setters often use such twists to take problems out of the straightforwardly mathematical so that candidates must use their ingenuity rather than just knowledge. Even so, using the mathematics you do know can often reduce the time necessary for a question.

Permutations and combinations

Another area where a little mathematics can help is in problems involving permutations and combinations. Here is another simple example.

Activity

Three married couples and three single people meet for a dinner. Everybody shakes hands with everybody else, except that nobody shakes hands with the person to whom they are married.

How many handshakes are there?

Commentary

Without the twist of the married couples, this would be very straightforward – the answer is $\frac{9 \times 8}{2} = 36$. You have to divide by 2 because the '9×8' calculation counts A shaking hands with B and B shaking hands with A. The married couples can be taken care of easily, because they would represent three of the handshakes, so the total is 33.

The alternative way to do this is to count: AB, AC, AD . . . AI, BC, BD, etc. This is very time-consuming.

Summary

- This chapter has shown how knowledge of a few relatively simple mathematical techniques can make the solution of some problem-solving questions quicker and more reliable.
- Percentage calculations can be simplified by replacing the percentages with real numbers.
- The use of algebra, lowest common factors and multiples, and permutations and combinations can aid the finding of methods of solution and shorten the work required for some problems.

End-of-chapter assignments

1 Rita has a small shop. 40% of the money she receives from selling cornflakes is profit. Next week she is having a sale and is selling cornflakes at three packets for the price of two. What percentage profit will she make on cornflakes sold under this offer?

2 At my local baker's, the price of bread rolls is 25¢ and I went with exactly the right money to buy the number I needed. When I got there, I found they had an offer giving 5¢ off all rolls if you bought eight or more. Consequently, I found I could buy three more for exactly the same money. How many was I originally going to buy?

3 From my boat at sea I can see three lighthouses, which flash with different patterns:

- Lighthouse A flashes 1 second on, 2 seconds off, 1 second on, 1 second off, then repeats.
- Lighthouse B flashes 1 second on, 3 seconds off, 1 second on, 2 seconds off, 1 second on, 3 seconds off, then repeats.
- Lighthouse C flashes 2 seconds on, 1 second off, 1 second on, 2 seconds off, then repeats.

They have all just started their cycles at the same time. When do they next all go on at the same time?

4 Four friends have a photograph taken with them all throwing their graduation hats in the air. Afterwards they pick up the hats and find they all have the wrong hat. How many different combinations of picking up the hats are there? In how many of these combinations do they all have the wrong hat?

Answers and comments are on pages 333–34.

6.2 Graphical methods of solution

It can often be useful to draw a simple picture when trying to analyse a problem. This can take the form of a map, a diagram or a sketched graph. Some examples where such pictures can help are given below.

Activity

Map

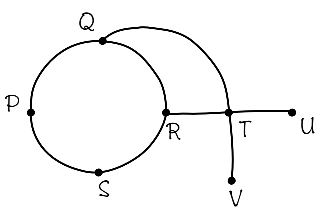
The town of Perros is connected to Queenston then to Ramwich and finally Sandsend and back to Perros by a circular bus service. Ramwich has a bus service to Upperhouse via Tempsfield. Queenstown has a bus service to Ventham via Tempsfield.

Orla is visiting the area and wants to look at all these towns starting and finishing at Perros. What is the smallest number of stages (i.e. journeys from one town the next) she can do the journey in?

A 7 **B** 8 **C** 9 **D** 10

Commentary

It would be very difficult to answer this question without some sort of picture. Our sketch of the towns and bus services only has to be quite rough and is shown below.



This now becomes a straightforward problem. In order to achieve the minimum number of stages, the shortcut between Q and T must be taken either on the way out or on the way back (but not both as we need to visit R). It is possible to go either way round, but both will result in the same number of stages. One minimum route is:

P-Q-T-U-T-V-T-R-S-P

The answer is C, 9 stages.

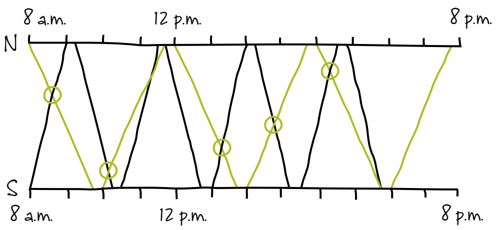
Activity

Graph

Two buses run services between Southbay and Norhill. One is an express service which completes a one-way journey in one hour. The other is a stopping service which takes 1 hour 45 minutes. The express service starts at Southbay at 8 a.m. and the stopping service starts at Norhill at 8 a.m. When each bus reaches its destination, it waits for 15 minutes before setting off again. This continues throughout the day. The last journey of the day is the last to finish before 8 p.m., each bus stopping at the town where it started.

How many times do the drivers pass each other in opposite directions on the road during the day?

Commentary



The black line shows the express service bus which starts from Southbay at 8 a.m. This takes one hour to reach Norhill, where it stops for 15 minutes; the next line shows the return journey and so on through the day.

Similarly, the coloured line shows the stopping service, starting at Norhill at 8 a.m. and taking 1 hour 45 minutes to reach Southbay, where it waits for 15 minutes before starting the return journey.

The intersections, shown by circles, indicate where the buses pass in opposite directions: five times in total. There is also one point where the fast bus overtakes the slower one and various positions when they are at either Southbay or Norhill at the same time.

This question would have taken a very long time to solve without the diagram as the crossing points would have had to be inferred from a timetable.

Venn and Carroll diagrams

Venn diagrams were introduced in Chapter 3.5. The problems considered there were relatively simple and could have been solved without the diagrams, just by using a bit of clear thinking.

In this chapter we are going to look at problems that are more complicated and, although they could be solved without the use of diagrams, the diagram makes the solution much more straightforward.

Taking a problem of a similar nature to that which was used to introduce Venn diagrams, the extension to one more category makes analysis of the problem much more complex, as shown below.

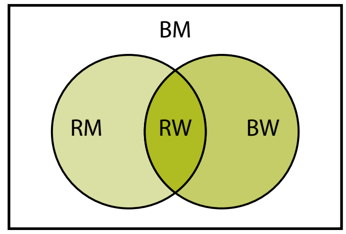
Activity

Elections have just been held in the town of Bicton. There were two parties, the Reds and the Blues. Turnout to vote was 70%. The Reds got 60% of the vote and the Blues the remaining 40%. An exit poll showed that 30% of women voting voted Red, whilst 70% voted Blue. (There are equal numbers of men and women registered to vote and the percentage turnout was the same for men and women.)

What proportion of men in the total electorate voted Blue?

Commentary

A Venn diagram for this problem is shown below. The rectangle represents all those who voted. We do not need to consider the non-voters as the exit poll does not categorise whether non-voters can be defined as Blue, Red, Men or Women. We just need to remember that only 70% of the electorate voted.



The left circle represents the Red voters and the right circle represents Women voters. R represents Red, B represents Blue, W represents Women and M represents Men.

We know that the Red vote was 60% of those who voted, so the areas:

RM + RW = 0.6 × 0.7 = 0.42, i.e. 42% of the electorate, and
BM + BW = 0.4 × 0.7 = 0.28, i.e. 28% of the electorate

We know that 50% of the electorate were women; 70% of these voted; of these, 30% voted Red and 70% voted Blue, so:

RW = 0.5 × 0.7 × 0.3
= 0.105, i.e. 10.5% of the electorate, and
BW = 0.5 × 0.7 × 0.7
= 0.245, i.e. 24.5% of the electorate

We can now calculate the proportion of the electorate in each area of the diagram:

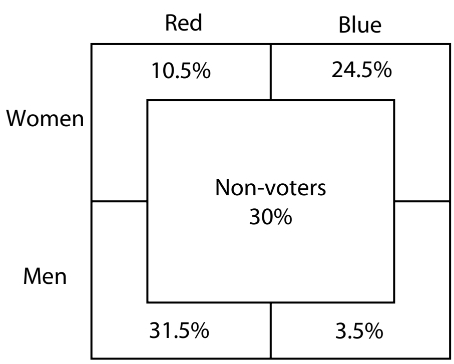
RW = 10.5%, BW = 24.5%, RM = 31.5% and BM = 3.5%

We can check that this is correct as these add up to 70% – the turnout, and both men and women add to 35% – equal numbers.
The proportion of women voting Red is 10.5/(10.5 + 24.5) = 30% and the proportion of Red voters is (10.5 + 31.5)/70 = 60%.

The area BM indicates that 3.5% of the electorate were men who voted Blue. Since half the electorate are men, we can now answer the original question: 7% of men voted Blue.

This question can also be solved using a Carroll diagram (originally devised by Lewis Carroll, author of *Alice's Adventures in Wonderland*), which is really just a table representing the areas shown in the Venn diagram. Some people may find Carroll diagrams easier to understand. Venn and Carroll diagrams become more complicated when there are more categories of things involved, but a problem involving more than three categories is unlikely to appear in a thinking skills examination. A Carroll diagram for two categories is just a 2 × 2 table (it has four areas, just like the Venn diagram). You might like to revisit the Venn diagram activity in Chapter 3.5 using a Carroll diagram.

The Carroll diagram for three categories may be drawn with an inner rectangle expressing one level of the third category (e.g. non-voters) and, for the problem above, would appear as shown:



The inner rectangle is not subdivided as it represents the non-voters. In this case (and, in fact, in many cases) the Carroll diagram is easier to understand than the Venn diagram and the various subdivisions and sums may be more easily seen and totalled.

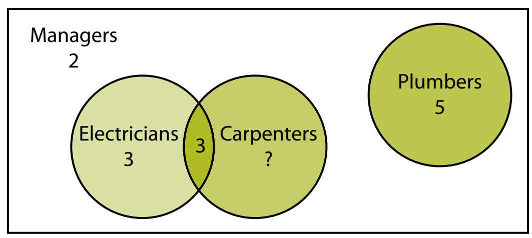
Activity

A general household repairs business has 15 workers. Two are managers and do not have specialised skills. Five are plumbers and do not do other jobs. There are six electricians and a number of carpenters. Of these, three can work as either electricians or carpenters.

How many are carpenters but not electricians?

Commentary

The Venn diagram for this problem is shown here.



As none of the plumbers are either electricians or carpenters, their area does not intersect with the other two. The entire outer box represents the 15 workers. The '2' shown on the diagram outside the circles represents the two managers who do not fit any of the other categories. The 5 plumbers are shown in their circle. The intersection between electricians and carpenters represents the 3 which fall into both categories. As there are 6 electricians, there must be 3 who are not also carpenters. We now have 13 accounted for so the remainder, 2, must be carpenters but not electricians.

Summary

- In this unit we have seen how various diagrams may be used to represent and solve problems in categorisation, logic and searching.
- We have looked at using sketched maps and graphs to clarify and simplify quite complicated problems.
- More advanced Venn and Carroll diagrams have been introduced for problems involving three levels of categorisation.

End-of-chapter assignments

1 Winston is organising a dinner to raise money for his football team. The hall he has hired is a square room measuring 15 metres by 15 metres. The tables are rectangular. Each one measures 2 metres by 80 centimetres and can seat up to eight people, as indicated in this diagram:

To fit as many people as possible into the hall, Winston plans to put the tables together, end to end, to create parallel rows. He can use as many tables as he can fit in, but he has to make sure there is a gap of at least 1.5 metres between the edge of any table and the edge of the room, and also a gap of at least 1.5 metres between rows of tables.

What is the maximum number of people that could sit down to eat at Winston's dinner?

A 190 **B** 192 **C** 228 **D** 240 **E** 288

2 Draw a Venn diagram for three categories to sort the numbers from 1 to 39 according to whether they are even, multiples of three or square numbers. Write each number in the appropriate part of the diagram.

3 The island of Nonga has two ferry ports: Waigura and Nooli. All ferries from Waigura go to Dulais on a neighbouring island. Some ferries from Nooli also go to Dulais. Some of the ferries that serve Dulais are fast hydrofoil services; those going elsewhere are slow steamboats.

Which of the following statements can safely be concluded from the information given above?

A No hydrofoils go to Dulais from Nooli.
B All hydrofoils going to Dulais leave from Waigura.
C Some hydrofoils from Nooli go to places other than Dulais.
D Some steamboats from Waigura go to Dulais.
E All hydrofoils from Waigura go to Dulais.

4 **(Harder task)** Anna and Bella both go to the gym on the same three days each week. The gym is open from 8 a.m. to 10 p.m. and either may arrive, quite randomly, any time between 8 a.m. and 3 p.m. Anna stays for one hour and Bella for ¾ hour. Over a long period, what is the percentage of times they will coincide at the gym?

Answers and comments are on pages 334–35.

6.3 Probability, tree diagrams and decision trees

Simple probability

Questions involving probability can occur at all levels of thinking skills examinations. In AS Level examinations, these are usually restricted to simple probability (e.g. the chances of a 6 coming up in a single throw of a die) or direct combinations of two probabilities (e.g. the chances of the numbers on two dice adding to 7). In the latter case, we need to distinguish between the combinations being dependent on each other or independent. The sum of the numbers on two dice is an example of an independent combination – one die is not affected by the other, and each can randomly show any number from 1 to 6.

An example of a dependent combination, where one operation depends on the results of another, is the drawing of coloured balls from a bag without replacement.

Activity

A bag contains four red balls and three blue balls. If two balls are removed from the bag, what are the chances of drawing one red and one blue ball?

Commentary

We must look at all the possibilities. The chances of drawing a red ball first are $\frac{4}{7}$. The chances of then drawing a blue ball are $\frac{3}{6}$ (not $\frac{3}{7}$ as we have already taken one ball out). We can then multiply the probabilities together to get the overall chance of this combination: $\frac{4}{7} \times \frac{3}{6} = \frac{2}{7}$.

However, we might get a blue ball first with probability $\frac{3}{7}$. The chances of then drawing a red ball second are $\frac{4}{6}$, so the overall probability is $\frac{4}{6} \times \frac{3}{7} = \frac{2}{7}$ as before. The overall probability of drawing red/blue in either order is the sum of these, i.e. $\frac{4}{7}$.

This problem could also have been solved using a tree diagram (see the next page), although in this case it would have required more calculation.

The activity below is a probability problem with a slight twist which takes it beyond being a simple mathematical calculation.

Activity

At a village fair there is a game of chance that involves throwing two dice. The dice are normal, numbered 1 to 6. One is red and one is blue. The number on the red die is multiplied by 10 and added to the number on the blue die to give a two-digit number. (So, if red is 2 and blue is 4, your score is 24.) You win a prize if you score more than 42.

What are the chances of winning?

Commentary

There are 36 (6 × 6) possible throws in all. If the red die shows 1, 2 or 3, whatever the blue die shows, you lose (18 of the throws). If the red die shows 5 or 6, whatever the blue die shows, you win (12 of the throws). This leaves 6 possible throws with the red die showing 4:

you lose with 2 of these (blue 1 and 2) and you win with 4 (blue 3, 4, 5 or 6).

So the number of ways of winning is 12 + 4 = 16 out of 36. (The number of ways of losing is 18 + 2 = 20 out of 36.) So the probability of winning is $^{16}/_{36} = ^{4}/_{9}$.

The examples below are more complex and are more likely to relate to Advanced Level and university entrance examinations.

Tree diagrams

Tree diagrams can be of help especially in probability problems that are not absolutely straightforward. They enable probabilities for every combination of events to be evaluated, and allow probabilities to be divided between all possible circumstances. They also give the advantage that, as all probabilities are calculated, we can check that the sum of them is 1.

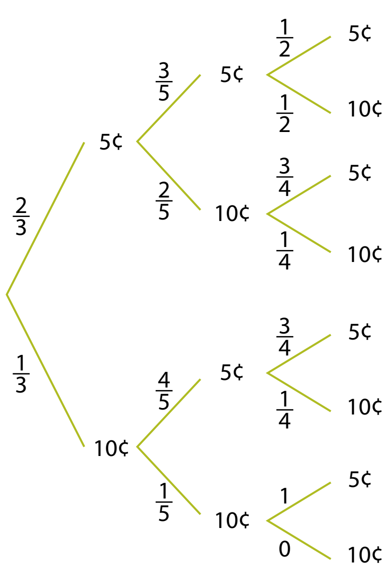
Activity

I have six coins in my pocket: four of 5¢ and two of 10¢. If I take three coins out of my pocket at random, what are the chances of the total being 20¢?

Commentary

A way of solving this problem using a tree diagram is shown below. At each stage (i.e. as each coin is drawn from the pocket) the branches of the tree lead to the possibilities – in this case only the withdrawal of a 5¢ or 10¢ coin – and the numbers beside the branches show the probability of each outcome. After three coins are withdrawn, the totals of all possible combinations of coin value may be calculated (by adding coin values along the branches) and the probability of that combination obtained (by multiplying the probabilities along the branches). After making the calculations, you may check whether you are correct as the sum of the probabilities should be 1.

The problem may now be solved. Reading from the top, combinations 2, 3 and 5 lead to a sum of 20¢. The sum of the probabilities for these three combinations is $^{1}/_{5} + ^{1}/_{5} + ^{1}/_{5} = ^{3}/_{5}$, i.e. a 60% chance.



15¢	$\frac{2}{3} \times \frac{3}{5} \times \frac{1}{2} = \frac{1}{5}$
20¢	$\frac{2}{3} \times \frac{3}{5} \times \frac{1}{2} = \frac{1}{5}$
20¢	$\frac{2}{3} \times \frac{2}{5} \times \frac{3}{4} = \frac{1}{5}$
25¢	$\frac{2}{3} \times \frac{2}{5} \times \frac{1}{4} = \frac{1}{15}$
20¢	$\frac{1}{3} \times \frac{4}{5} \times \frac{3}{4} = \frac{1}{5}$
25¢	$\frac{1}{3} \times \frac{4}{5} \times \frac{1}{4} = \frac{1}{15}$
25¢	$\frac{1}{3} \times \frac{1}{5} \times \frac{1}{1} = \frac{1}{15}$
30¢	$\frac{1}{3} \times \frac{1}{5} \times \frac{0}{1} = 0$

Decision trees

The decision tree is an extension of the probability tree diagram which is used in commerce and industry to help make strategic and financial decisions. In this case such things as costs or times are recorded on the different branches of the tree and used to estimate the average cost or time for each strategy. This will become clearer with an example; this is a very simple situation chosen to illustrate the method.

> Mary has $5000 to invest and can leave it for two years. She has a choice between a fixed rate investment at 5% interest per year, or a variable rate scheme which may rise and fall. The variable rate scheme pays 6% in the first year, but may be different in the second year. She has looked at the financial press, and the opinion of the experts is that interest rates have a 20% chance of rising to 8%, a 20% chance of rising to 6% and a 60% chance of falling to 3%. Which investment should she choose?

A decision tree diagram for this situation is shown below.

This decision tree, like most real ones, has two types of branch. The first branch shown here is a *choice*: whether to take the fixed or variable rate investment. In the upper branch we have three different *probabilities*. These are things that cannot be controlled. It is conventional to show choices as squares, probabilities (or chances) as circles, and end points as triangles.

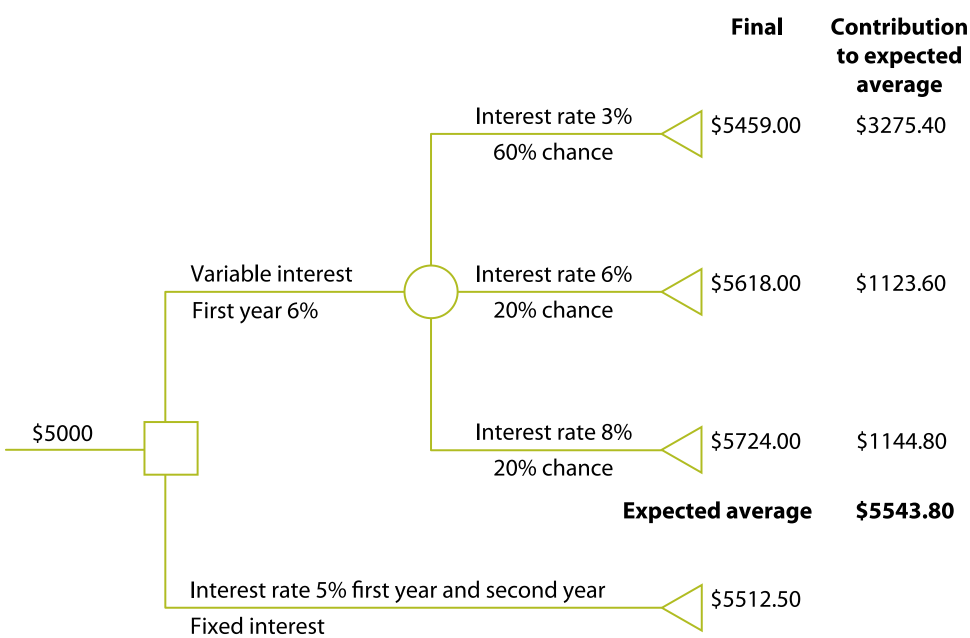
In this case, the lower branch results in interest of $250 (5% of $5000) in the first year, and $262.50 (5% of $5250) in the second year, making a grand total of $5512.50.

The method of calculation of the figures in the upper branch is as follows:

Mary earns $300 in the first year (6% interest), giving her $5300 at the start of the second year.

In the second year:

- there is a 60% chance of rates being 3% and her earning $159 interest
- there is a 20% chance of rates being 6% and her earning $318 interest
- there is a 20% chance of rates being 8% and her earning $424 interest.



In order to combine these, we calculate her expected average interest. This average is calculated as if she made a large number of investments over a period of time with the probabilities shown above: 60% of the time she would earn $459 interest, and so on. Thus the expected average amount she has at the end of two years (remembering to add the first and second years together) is:

60% of $5459 + 20% of $5618 + 20% of $5724
= $3275.40 + $1123.60 + $1144.80
= $5543.80

This is a better option than the fixed interest rate at $5512.50, but she would stand a 60% risk of only having $5459.

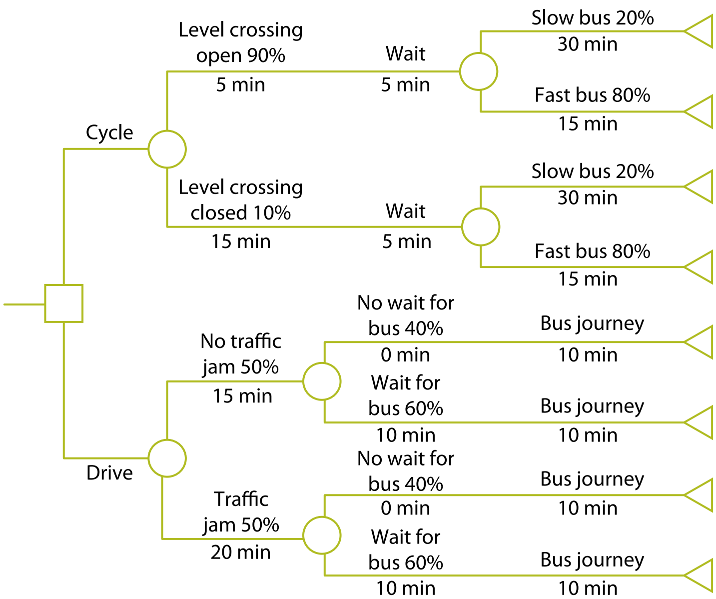
In the following activity some of the probabilities may seem quite arbitrary and approximate, and the situation is rather simplified, but real problems can often be analysed usefully in this way. This is also much more difficult in that it involves a comparison of two probability trees with extra added factors.

Activity

There are two ways I can go to work, both of which involve a two-part journey. I can cycle to the bus stop; this takes me 5 minutes normally, or 15 minutes if a level crossing for trains is closed on the way, which happens on 10% of occasions. A bus takes on average 5 minutes to come. I catch the first bus, which may be a slow bus which takes 30 minutes or a fast bus which takes 15 minutes. I get the slow bus 20% of the time.

Alternatively, I can drive to the Park and Ride car park. Driving usually takes me 15 minutes, but about half the time there is a traffic jam and it takes 20 minutes. When I get to the Park and Ride, I sometimes get the bus straight away, but 60% of the time I have to wait 10 minutes for the next one. The bus takes 10 minutes to get me to work.

1 What is my shortest time to get to work?
2 On average, what is my best option for getting to work and how long will it take me?
3 What are the chances of the first journey option taking 40 minutes or more?



Overall time	Probability	Average time
40 min	18%	7.2 min
25 min	72%	18 min
50 min	2%	1 min
35 min	8%	2.8 min
Overall average time 29.0 min		
25 min	20%	5 min
35 min	30%	10.5 min
30 min	20%	6 min
40 min	30%	12 min
Overall average time 33.5 min		

Commentary

1 Answering the first question does not require any probability analysis. The first route, at the quickest, takes 5 minutes (cycle) + 0 minutes (bus to come) + 15 minutes (fast bus) = 20 minutes.

The second route takes 15 minutes (drive) + 0 minutes (wait for bus) + 10 minutes = 25 minutes.

The shortest time is 20 minutes.

2 In order to answer the second question, we must construct a decision tree as before. This time, however, on every branch of the tree, we multiply the overall probability (converting the percentage probabilities to proportions, i.e. 90% becomes 0.9) by the overall time. We then sum these values to find the average time (this is also known as the expected value).

This can be explained as follows. For example, say there is a 30% chance of a journey taking 20 minutes and a 70% chance of the journey taking 40 minutes. If we look at 10 journeys, 3 of them will take 20 minutes and 7 of them will take 40 minutes. The total time for these 10 journeys is $(3 \times 20) + (7 \times 40) = 60 + 280 = 340$ minutes, so the average time is $\frac{340}{10} = 34$ minutes. This is equivalent to multiplying each journey time by the probability of that time: $(0.3 \times 20) + (0.7 \times 40) = 6 + 28 = 34$ minutes.

The averages are shown on the decision tree. The cycle/bus option takes an average of 29 minutes and the drive/bus option an average of 33.5 minutes, so the former is better. However, there is a small chance (2%) of the first option taking 50 minutes.

3 In order to calculate this, we look at the branches where the total time is 40 minutes or more and add the probabilities. These are 18% (for 40 minutes) and 2% (for 50 minutes), a total of 20%.

Decision making is considered further in Chapter 7.5 (page 283), showing how decision trees may be used to aid processes in critical thinking.

Summary

- We have looked at the use of probability in problem solving. The concepts of dependent and independent joint probabilities have been introduced.
- We have considered how more complex probabilities can be analysed using tree diagrams.
- The extension of tree diagrams to decision trees has been described and it has been shown how these might be used to help with decision making in commerce and industry.

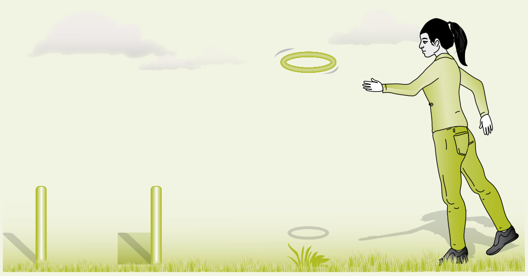
End-of-chapter assignments

1 My drawer contains eight blue socks and six black socks. If I take four socks out at random, what are the chances that they will make up two matching pairs?

2 My wife has sent me to the bank with her cash card. I do not know the four-digit number I have to enter into the machine to withdraw money. I know the first two digits are the two digits of her month of birth, in the right order. The last two digits are the date in the month of her birthday. There are no zeros and I have forgotten my wife's birthday.

What are the chances of my getting it right first time? What are the chances of my getting it right in the three attempts I am allowed?

3 A fairground game involves taking three throws to get a ring over two poles in the ground at different distances from the throwing position. Throws must be taken alternately at the two poles, but you may start with either one. You win a prize if your ring lands over a pole in two successive throws out of the three.



Clearly, it is easier to throw the ring over the nearer pole than the farther one. Is it better to make your attempts in the order 'near, far, near' or 'far, near, far', or doesn't it matter?

4 Metco make components for small electrical equipment. One production line makes 500,000 switches each year. They currently use a manual inspection system with one quality control operative. 1% of the production is faulty and the operative finds and rejects 90% of these. Metco sell the switches for \$2 each but any faulty ones which are delivered cost the company \$25 in replacement and compensation costs. The quality control operative costs \$40,000 per year to employ.

Metco's management are considering installing an automatic quality control system. This will mean the quality control operative will be made redundant, for which they will have to make a single payment of \$10,000. The manufacturers of the new system claim that it will pick up 99% of faulty switches, but the production manager is sceptical about this. He estimates that there is only a 20% chance of it being this good and an 80% chance that it will only pick up 95% of the faulty switches. The new equipment will cost \$180,000 which will be written off over four years ($\frac{1}{4}$ each year). Other manufacturing costs will not be affected by the quality control system used.

By calculating the average income and costs per year for a four-year period, investigate the economics of the old and new systems, considering which is most likely to be the best to use.

Answers and comments are on pages 336–39.

6.4 Have you solved it?

This chapter considers how you may check and be sure that your answer to a problem-solving question is correct. In real life, there might be several possible answers, or even no answer to a problem. (Can you fit a square peg into a round hole?) However, in examinations, especially those with multiple-choice answers, there must be a correct answer. One of the options in some cases might, of course, be that the task cannot be done. This means that, when you have an answer, you must have a way of being sure that it is correct.

Different problems need to be checked in different ways. Sometimes it is possible to put the answer back into the question and see if it 'fits'. This is probably the easiest way. For example, look at the question in Chapter 3.5 (page 94) about Amy and David passing on the road. We concluded that they would pass at 10.40 a.m. We can now go back and see where they both were at 10.40 a.m. Amy left at 8 a.m. so by 10.40, at 120 km/hr, she had travelled 320 km. Similarly, David, leaving at 10 a.m., had travelled 80 km. The total is 400 km, which is the distance between their two houses, so the answer is correct.

Similarly, the first problem in Chapter 3.3 (page 86), with the table showing participation in exercise, could be checked by putting the correct figure into the table and seeing whether all the rows and columns added up correctly.

You can go back and check the answers for a lot of the activities and examples in Unit 3 of the book by using the 'put it back in' method.

However, the second problem in Chapter 3.3, involving a graph showing temperatures, cannot be checked in this way. We are simply being asked to extract the right value from the graph and there is no way of putting this back in to see whether it is right. In cases like this, the answer simply has to be checked carefully. What exactly was the question asking? Is this what we answered? Is the numerical value of the answer about what we would expect?

The same applies to questions requiring a search. 'Putting the answer back' will tell you whether your answer fits the criteria asked for in the question but will not tell you whether it is the lowest (or largest) possible answer. If the search is not too large, you can sometimes check, if you are looking for the lowest answer, that all smaller answers will not work. This can be time-consuming and impractical if the search is large. It is often better to check your method and be sure that it will come up with the correct answer.

Approximation, or a feel for the magnitude of results, is a skill that can be refined through practising this type of question. This is particularly valuable when questions depend on getting the decimal point in the right place. A minimum temperature of 10°C might be acceptable when 100°C would not.

The end-of-chapter assignment considers several problems that may have a variety of ways of checking. It is always preferable to use a different method for checking the problem from that which you originally used to solve it. If you simply repeat your original calculation, it is possible that any mistake you made in the first instance you will make again.

Checking the answers of questions involving searches (see Chapter 3.6) can be more difficult. There can often be more than one way of searching but, if you have done the question efficiently, any other way may be time-consuming. It is often more important to ensure that your method of searching is 'cast-iron' and will not produce an incorrect answer unless you make a slip.

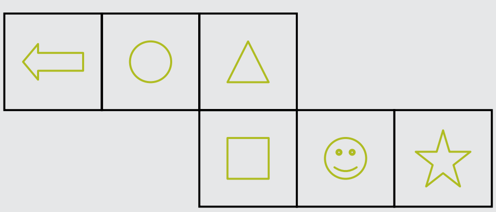
Multiple-choice questions

In questions which require numerical answers, it is usually best to work through the question to the answer and then check that it is among the list of options. Guessing can be dangerous. However, there are aspects of answering some particular types of multiple-choice questions that can help in getting the correct answer. One is elimination. This is especially useful in answering certain types of questions where the answers form part of the question, for example those involving spatial reasoning and identifying similarity between two sets of data. Even if you are guessing an answer, you can increase your chances of getting it right by eliminating one or two of the options.

It is not always necessary to check every aspect of a drawing, graph or table to be sure that it is wrong. Sometimes one needs only to check a single part – for example one plotted point on a graph – to eliminate it as a possible answer. This means that the time available for the question can be concentrated on the more likely answers and in checking that your final answer is correct. You can try this in the activity below.

Activity

The piece of card shown below, when folded up, makes a cubical die for a children's game. The sides have various symbols on them.



When the die is folded into shape, which side will be opposite the triangle?

- **A** The arrow
- **B** The circle
- **C** The square
- **D** The smiley face
- **E** The star

Commentary

We can see that both the circle and the square have an edge in common with the triangle, so neither can be opposite it when the cube is made up. Therefore both of these can be eliminated. Similarly, the smiley face has a corner in common with the triangle, so this can also be eliminated. We now only have to look at the arrow and star. There are various ways to choose, but here it is best to imagine the cube folded, whereupon we see that the star must also have a common edge with the triangle, leaving the arrow as the correct answer.

Summary

- We have seen how an answer may be checked by 'putting it back' into the question.
- This method may not work for all questions, and other ways of checking may be needed for other types of question.
- Elimination of incorrect answers can help in finding the correct solution to multiple-choice questions.

End-of-chapter assignment

Reconsider some or all of the following problems from the end-of-chapter assignments and see whether you can find ways of checking that your answer is correct. Try to use a different method from the one originally used for solving the question. Look at how you might eliminate some of the options in multiple-choice questions, where appropriate.

1. Chapter 3.2, question 2 (page 85)
2. Chapter 3.3, question 4 (page 89)
3. Chapter 3.4, question 2 (page 92)
4. Chapter 3.5, question 1 (page 97)
5. Chapter 3.7, question 3 (page 105)
6. Chapter 3.8, question 3 (page 110)
7. Chapter 3.10, question 2 (page 118)
8. Chapter 3.12, question 2 (page 124)
9. Chapter 5.1, question 2 (page 209)
10. Chapter 6.1, question 2 (page 234)

7.1 Conditions and conditionals

Conditions are familiar in everyday life. Think about the expression ‘conditions of sale’ which apply when you buy something. You buy a DVD, for example, on condition that you don’t make copies of it and sell them on to other people. The booking conditions on an airline ticket may allow a refund if you cancel up to a month before the flight, but not if you leave it any later. Another familiar example can be found in the entry requirements – another word for conditions – that colleges or universities set for admission. But although the concept is so familiar, and the word commonplace in our language, conditions can cause problems if they are not fully understood or made clear.

Let’s say you have been offered a place in a college of choice if you score 70 in the entrance exam. In other words scoring 70 is a condition of entry to the college. This might sound quite plain and straightforward. But it can be thoroughly ambiguous. For there are three ways of interpreting a condition of entry; and how you interpret it can make a lot of difference to the consequences.

Necessary and sufficient conditions

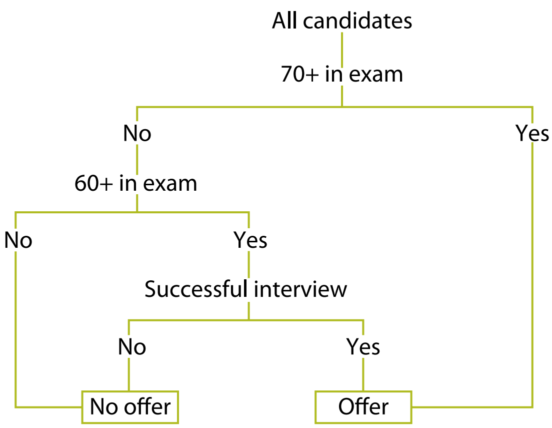
Conditions fall into two categories according to whether they are necessary or sufficient. Scoring 70, for example, could be a *necessary* condition, in which case you will not get into the college if you score 69 or less. But if it is a necessary condition *only*, then a score of 70 may not, on its own, be enough to secure you a place. The exam may be followed by an interview to choose the best students from all those who scored 70 or more. This practice is very common in circumstances where there is a lot of competition for a limited number of places. Under such a condition, therefore, a score of 70 would be *necessary*, but not *sufficient* – which could be quite a shock if you scored 80 and still got turned down!

Alternatively, scoring 70 may be a *sufficient* condition. If it is truly sufficient, and you do score 70, you are accepted, and that is the end of it. There are no other hurdles to clear. But when you say something is a sufficient condition, that doesn’t mean it is also a necessary one. For example, there may be a second chance for anyone who scored, say, 60 or more to be interviewed, and to gain a place that way, so that as well as those who automatically qualify by exam there are others who may qualify by interview. This, too, is a common practice, in circumstances where there are more places than there are strong applicants who are likely to meet the qualifying condition.

There is, of course, a third way of applying the condition, and that is to make it necessary *and* sufficient at the same time. This would mean that you get in if you score 70 or more and don’t get in if you score 69 or less. This is not such a common practice in a context like entry requirements, for the very good reason that it would allow no flexibility. If the entry conditions were both necessary *and* sufficient, a department could end up with fewer students than it would like to have, or with more than it can cater for.

Flow diagrams

One useful way to present this kind of data is in a flow diagram, or flow chart. From the following diagram you can read off the information that a score of 70 is a sufficient condition for an offer, because a Yes response leads straight to an offer. But it is not a necessary condition, because a No response can also lead to an offer. This is a fairly simple scenario, with only two paths leading to a positive outcome. In more complex situations, with several branching paths, a diagram can be a very useful aid for 'reading off' the conditions.



According to the diagram, a score of at least 60 is a necessary condition, because the No branch leads straight to a refusal. But it is not a sufficient condition because there is still another condition to be met after satisfying the 60+ condition: another branching of the tree.

Conditional statements

Conditional statements, that is statements that stipulate conditions, typically contain the word 'if', or 'if' followed shortly by 'then'. For example:

[1] If Mia scored 70 or more, then she has a place.

Note that [1] is not an argument; it is just a statement. It would be an argument if it were expressed as follows:

[2] Mia scored more than 70 and *therefore* she has a place.

The difference is that in [2] it is *asserted* that Mia did score more than the required mark, whereas in [1] it remains a possibility. In both cases, however, getting 70 or more is presented as a *sufficient condition*. It may also be a necessary condition, but the sentence doesn't tell us whether or not it is. To express necessary conditions you may need to employ other words such as 'not', 'only' or 'unless'.

Activity

Here are six more conditional statements. For each one say whether scoring 70 or more is a necessary or a sufficient condition, or both:

A You will be offered a place only if you score 70 or more.

B If you don't get 70 or more you won't be offered a place.

C You will be offered a place if and only if you score 70 or more.

D If you get 70, you are in.

E Unless you score 70, you won't get a place.

F If you score 70 you're in, but if you don't you can always re-sit the exam.

Commentary

In A and B the pass mark is a necessary condition. Look at them carefully and you will see they say the same thing. However, neither of them says whether there is any other requirement, such as an interview or a medical or even some residential condition, such as living in the country or town where the college is. All A and B assert is that 70 is the minimum requirement, which is yet another way of saying that it is *necessary* for admission.

C sets a necessary and sufficient condition. It is an abbreviation (or 'contraction') of two statements: 'You will get in if you score 70 or more' and 'You won't if you don't.' In logic such statements are called biconditionals, 'bi-' meaning 'two'. There are two conditions in one.

In D the condition is sufficient: it doesn't say whether it is necessary as well. Compare it with [1], and note that it is really just another way of expressing the same condition.

E obviously states a necessary condition but, unlike A and B, it emphasises that scoring 70 is not also a sufficient condition. F appears to do the opposite: it states a score of 70 in the exam is a sufficient condition but adds that if you get less you can re-sit. However, you may still have to get 70 or more at some time, so it is unclear whether the mark of 70 is necessary as well as sufficient. Possibly on the re-sit the required mark will be lower, so as to fill any remaining places.

The structure of conditionals

A conditional is a complex statement that is true or false *as a whole*, independently of whether the parts of it are true or not. You were introduced briefly to complex claims, including conditionals, in Chapter 2.2. The example there was the claim that:

> [3] Many parts of the world will soon be submerged if nothing is done to reverse climate change.

This statement consists of two shorter sentences (or *clauses*), connected by 'if'. Note that the order of the sentences can be reversed, bringing the *if*-clause to the front. This is the standard way to express a conditional in logic.

> [3a] *If* nothing is done about climate change *then* many parts of the world will soon be submerged.

The *if*-clause is called the 'antecedent' because logically it comes before the *then*-clause. The *then*-clause is called the 'consequent', because it follows logically from the antecedent. If the antecedent is true, then the consequent is true too. This logical relation holds whether the conditional is expressed like [3] or [3a]; and whether or not the word 'then' is included.

Conditional claims are extremely valuable tools for our thinking and reasoning. Without them we would not be able to reason *hypothetically* – that is, without knowing whether or not the antecedent was true. Another term for this is *suppositional* reasoning. In [3] what the speaker is effectively saying is:

> [3b] '*Suppose* we do nothing about climate change, this is what will happen.'

or

> [3c] Suppose we did nothing about climate change, this is what would happen.

None of the claims above means that nothing will be done about climate change. Nor does it mean that parts of the world will be submerged in the near future. The only claim that is being made is that this will happen *if* we do (or did) nothing; it is the *consequence* of doing nothing.

Hypothetical claims

Conditional statements and remarks are sometimes referred to as 'hypotheticals'. 'Hypothetical', in this context, means 'conditionally true'. Politicians are often asked hypothetical questions, particularly by journalists and media presenters, to try to get them to commit themselves to some prediction, or future course of action.
For example:

> 'Minister, what will you do if these allegations of bribery turn out to be true? Will you resign?'

To which the politician is likely to reply:

> 'I am not going to answer that question, because it is purely hypothetical. The allegations aren't true.'

If she is persistent enough, the journalist may get the minister to concede:

> [4] 'All right, I would resign if I had taken the bribe. But I haven't.'

This is not a statement that the minister will resign, only that he would under certain conditions. It is thus a hypothetical statement. Statement [3c] is also hypothetical in the sense that the speaker is not suggesting or predicting that nothing will be done. Indeed the speaker is assuming that something will be done in view of the consequences if it is not done.

Logical form

Logicians show the structure or form of complex statements by substituting letters (*p*, *q*, *r*, etc.) for the actual clauses. A conditional statement has the form:

If *p* then *q*.

Statement [1], at the beginning of the chapter, has this form. In [1] *p* stands for: 'Mia scored 70 or more', and *q* stands for: 'She (Mia) has a place'. If we wanted to say that Mia did not get 70 or more, or that she does not have a place, we could write 'not-*p*' or 'not-*q*' respectively. In Chapter 7.2 we shall explore ways in which these formal expressions can be helpful in understanding and evaluating some arguments.

Conditions and reasoning errors

We have looked in some detail at conditions and conditional (hypothetical) statements because some of the most serious weaknesses and flaws in arguments come from confusing them.

Activity

Critically evaluate the following two arguments. What role do necessary and/or sufficient conditions play in the reasoning? Are these good or bad arguments?

[5] If, as alleged, the government minister has a business interest that he has not declared, he would have certainly been forced to resign. Last night he did resign, so there must be truth in the allegation.

[6] A government minister would not resign over an allegation of undeclared interests unless there was some truth in it. The fact that he has resigned means that there is some truth.

Commentary

We'll consider the two arguments in turn, starting with [5]. There are various ways in which you could find fault with this argument. You could say, for example, that it assumes, without justification, that the minister's reason for resigning was the undeclared business interest, whereas he might have resigned for some other reason altogether. Another way to explain this is that although the discovery of undeclared interests would be *sufficient* to force the minister's resignation, it is not a *necessary* condition, since (as already observed) something else might have forced it. The underlying argument in [5] is as follows:

If the minister has undeclared interests, he would have had to resign.

He has resigned.

He must have an undeclared interest. (The allegation must be true.)

The argument in [5] is clearly unsound. [6] does not make the same error. The first premise states a necessary condition: it is equivalent to saying that a minister would resign *only if* the allegation were true; or that if a government minister resigns over such an allegation, then it must bear some truth. Therefore, since the minister has resigned, the inference can only be that there is some truth in the allegation. The reasoning in [6] is solid.

Summary

- Conditions can be divided into two kinds: necessary and sufficient.
- Conditional, or hypothetical, statements typically have the form 'If *p* then *q*'.
- Confusing necessary with sufficient conditions often results in reasoning errors.

End-of-chapter assignments

1 A tutor made the following prediction to a group of students: 'If you have not read the coursebook, you won't pass the exam.'

a Explain this prediction in terms of necessary and sufficient conditions.

b Which one of the following has to be true if the tutor's prediction was correct – and why do the others *not* have to be true?

A All those who read the book passed the exam.

B All those who had not read the book failed the exam.

C The same number of students read the book as passed the exam.

D Only those who passed the exam had read the book.

E None of those who failed the exam had read the book.

OCR

2 A car insurance company has special terms for young drivers. If the driver is 25 or over and has a clean licence (i.e. no driving convictions), then the application is approved. If the driver has a clean licence and is under 25 but is 21 or over, the application is approved only if he or she has an Advanced Driving Qualification (ADQ). Applications from drivers under 25 with no ADQ are refused. Drivers are also refused if they are under 21 (with or without an ADQ). So are any drivers who do not have a clean licence.

a Draw a flow diagram which represents the information in the above text. (It is advisable to start with: 'Clean licence – Yes or No?'

b Use your diagram, and/or the text, to say whether each of the following statements is correct or incorrect – and why:

A Being 21 or over is a necessary condition for approval.

B Possession of an ADQ and a clean licence are sufficient for approval.

C Being 25 or over is neither a necessary nor a sufficient condition for approval.

D For Jason, who is 23, passing an ADQ is a necessary but insufficient condition for approval.

E Being under 21 is a sufficient condition for refusal.

3 **(Harder task)**

It is just as well, from an evolutionary standpoint, that water freezes with its molecules bonding to form a very open lattice. This unusual structure is such that the density of water ice is less than that of liquid water, which is why ice floats unlike other solids with tighter structures such as iron. (An iron bar placed in molten iron will sink.) Where and when the oceans freeze, ice forms a layer of insulation on the surface which holds in the heat of the liquid below. Without this protection the seas would freeze solid, from the bottom up; and life as we know it, which began in water, would not exist.

Explain and assess the reasoning in the above argument. In your analysis state whether the unusual structure of water is presented as:

- a sufficient condition for life as we know it
- a necessary but insufficient condition for life as we know it
- both a necessary and sufficient condition for life as we know it
- neither a necessary nor sufficient condition for life as we know it.

Answers and comments are on page 339.

7.2 Soundness and validity: a taste of logic

A good argument is one that can be trusted. If the reasons from which it starts are true, we need to know that the conclusion will be true too, and true for the reasons given. An argument which gives that assurance, and whose reasons are warranted, can be rated as *sound*. An argument which fails on either of those counts is *unsound*. Critical evaluation of an argument basically means judging its soundness.

Validity

Obviously, if we don't accept the reasons (premises) that are given for a conclusion then we cannot trust the conclusion either. But even if we do accept all the reasons as true, we may still find, on inspection, that what is inferred from those reasons simply does not follow. Thirdly, there are many instances in which we simply don't know whether the reasons are true or not, but we still want to know that the reasoning is good, so that *if* the premises are true we can be sure that the conclusion would be true as well. An argument that gives that assurance is said to be *valid*. And it remains valid – though not sound – even if the premises are known to be false.

What we need, therefore, is a way of judging the quality of reasoning in an argument that is independent of the truth of the premises; or at least which sets aside the truth-or-falsity issue whilst judging the quality of the reasoning in isolation. The discipline which provides the methodology for this judgement is logic.

Logic

We saw in Chapter 2.10 that the word 'sound' has both a special meaning in logic and critical thinking, and a general meaning. So it can apply to boats, buildings and so on, as well as to arguments. Likewise, 'valid' and 'invalid' can be used to describe a whole range of objects, material and abstract. A rail ticket is valid for certain journeys but not for others, and is invalid if it is out of date. An argument, or form of reasoning, is invalid if its premise could be true and its conclusion false. Likewise it is valid if, whenever the premises are true, the conclusion *cannot* be false.

It is crucial to note that when logicians talk about validity they are talking about *forms* of argument, not just about individual arguments. An argument is valid or invalid by virtue of its form. Individual arguments are different from each other because they are made up from different sentences with different meanings; but countless different arguments can share the same form. In fact, if you think back to Chapter 2.5, you will remember that all arguments have the same basic or 'standard' form:

[1] $R_1, R_2, \ldots R_n / C$

or

$$\begin{array}{l} R_1 \\ R_2 \\ \ldots R_n \\ \hline C \end{array}$$

where 'R' stands for a reason or premise, and 'C' for a conclusion. The separator '/', or the horizontal line, stands for the logical relation of 'following from', and is roughly equivalent to the word 'so' or 'therefore'.

Since [1] is the form of any argument whatsoever, it is obviously not a valid form, because some standard arguments are valid and some are not. Any number of invalid arguments could be made by substituting true sentences for R_1, R_2, etc. and a false one for C. Substituting different sentences for the letters R . . . and C makes it possible to *test* arguments for validity. If you can find any examples in which the Rs are all true and C false, you know the argument is invalid, even though there may be other examples where the conclusion is true. For an argument to be valid every argument with the same form must be valid too.

Activity

Here is a short example to illustrate what is involved in testing the validity of an argument.

[2] Many insects have wings and those that do can fly. Birds also have wings, and parrots are birds, so they can fly too.

Decide for yourself whether [2] is valid, giving reasons for your evaluation. Take some time over this. It is not as simple as it looks.

Commentary

We'll begin by analysing the argument. It makes three claims, followed by the conclusion:

R1 Many insects have wings and those that do can fly.
R2 Birds have wings.
R3 Parrots are birds.

C Parrots can fly (too).

How should we evaluate this argument? We can see that the premises are all true. We can also see that the conclusion is true: parrots *can* fly. These facts may have fooled you into thinking that the argument was valid, and therefore sound as well. It isn't. Although the conclusion is true it is not *made true* by the reasons. The fact (R2) that birds have wings doesn't mean all of them can fly, and therefore the fact (R3) that parrots are birds doesn't establish that they can fly either. R1 really gives no support to the conclusion, because what is true for insects has no bearing on what is true for birds. It is irrelevant.

We can see how invalid [2] is if we substitute 'penguins' for 'parrots', because penguins are birds that cannot fly:

[3] Many insects have wings and those that do can fly. Birds also have wings, and *penguins* are birds, so *penguins* can fly too.

In [3] the premises are just as true as they were in [2], but in [3] the conclusion is false. Therefore [3] is invalid. However, [2] and [3] have precisely the same *form*, so both are invalid (and therefore unsound too).

A valid form

Here is a different argument.

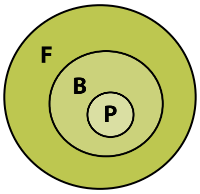
[4] All birds can fly. Penguins are birds, so penguins can fly.

Activity

What do you make of this argument? Is it valid? Is it sound?

Commentary

Again you may be surprised. This argument is valid. It is valid because *if* all birds really could fly then penguins would be able to fly (because they are birds). It is unsound because it is *not* true that all birds can fly. But that is not a fault with the reasoning, only with the first premise. The point to remember is that validity is to do with the form of the argument, not the subject matter. The validity of [4] has nothing to do with birds and things that can fly, but applies to any class of objects whatsoever. We can see why [4] is valid if we represent it in a Euler diagram; diagrams and symbols can often show the form of an argument better than words:



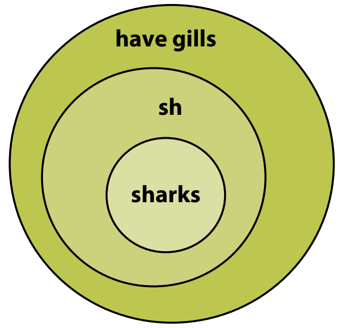
In this diagram we replace things that can *fly* with the letter F, *birds* with B and *penguins* with P. Then we forget about what these mean. What the diagram shows is that whatever Ps are, they are all Bs, because the P circle is completely enclosed by the B circle. Likewise the B circle, and the P circle with it, are completely inside the F circle. Therefore, since all Ps are Bs and all Bs are Fs, it follows that all Ps are Fs – whatever P, B and F stand for. And that is why [4] is valid.

Soundness

You should now be able to see that this form of argument will *never* give a false conclusion if its two premises are true. So if we take a valid structure, like [4], and substitute true premises, we have a sound argument and a reliable conclusion. For instance:

[5]	R1	All fish have gills.
	R2	Sharks are fish.
	C	Sharks have gills.

This argument rests on the truth of R1 and R2. Someone might object that R1 was false because whales and dolphins, which are mammals, are 'fish' in the everyday sense of the word – i.e. creatures that live and swim in the sea – but don't have gills. That would be a challenge to the soundness of the argument, but not to its validity. You could respond by stating that all *true* fish (which excludes the aquatic mammals, jellyfish and so on) have gills; and sharks are true fish. But however you resolve that dispute, [5] remains valid, as the Euler diagram confirms:



Deductive reasoning

Examples [4] and [5] are valid arguments and [5], arguably, is sound as well. To be more precise we ought to say that these are *deductively* valid arguments. That is because the above definition of validity really applies to certain types of reasoning called deduction, or deductive reasoning. Deductive arguments, so long as they are valid, are very strict, rigorous arguments in which the conclusion follows *inescapably* from the premises. But by the same token, an attempted deduction that is invalid fails completely, so that regardless of the truth of its premises, it is unsound. You cannot have a deductive argument that is 'a bit valid' or 'very nearly valid': it's all or nothing.

Here is a centuries-old example that logicians have used to illustrate deductive validity:

[6] All men are mortal. Socrates is a man. Therefore Socrates is mortal.

You may have noticed that this is very similar in form to example [4] above, and it is valid for the same reasons. It is often contrasted with the next argument, which makes all the same claims but is certainly *not* valid:

[7] All men are mortal. Socrates is mortal. Therefore Socrates is a man.

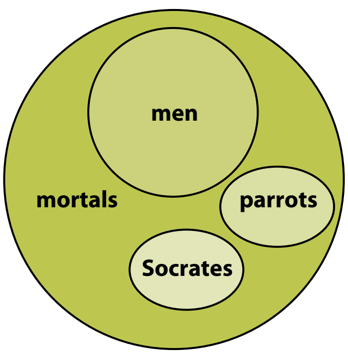
Activity

Discuss why [7] is not valid. What is wrong with it, and how is it different from [6]?

Commentary

In [6] we are told Socrates is a man and that all men are mortals. That tells us that Socrates is also mortal. In [7] we are again told that all men are mortal, and that Socrates is mortal too. But that would not tell us that Socrates is a man, if we did not already know it. There are many other classes of mortals besides men: women, children, parrots and penguins, to name just a few. Therefore the premises in [7] do not themselves establish that the Socrates referred to in the argument is a man. (If we didn't know differently 'Socrates' could be the name of a parrot.)

Here is a Euler diagram showing the invalidity of [7]:



The diagram shows that men, Socrates and parrots are all mortal, but does not establish that all individuals called 'Socrates' are men (or parrots). As it happens Socrates was a man, so the conclusion is true; but it does not follow from the reasons.

Formal logic

Because logicians are primarily concerned with different forms of argument, they are less concerned with the meanings of the sentences that express them. As we have seen, symbols, diagrams and formulas can be used to show the form of an argument. The symbols may stand for individuals, like Socrates; or classes of things, like birds. Or they can stand for whole sentences, like 'Parrots can fly,' or 'Whales are not fish.'

Another way to reach the conclusion of [5], for example, is as follows:

[8] If sharks are fish, they have gills. Sharks *are* fish, so they *do* have gills.

As we saw in the last chapter, the form of conditional sentences can be given by replacing each of the simple sentences with a letter. If the letter *f* stands for 'sharks are *fish*', and *g* for 'sharks have *gills*', [8] can be written:

If *f* then *g*
f
____
g

This form of argument is always valid, whatever sentences you substitute for *f* and *g* (or whatever symbols you use). In fact, [8] is so obviously valid that it hardly needs saying.

Here is another simple but valid argument. It has the same first premise as before, but this time the second premise is a *denial* of *g* – written 'Not-*g*' – and the conclusion is 'Not-*f*'.

[9] If *f* then *g*
Not-*g*
____
Not-*f*

The validity of [9] is not quite as obvious as [8], but it is a valid form of argument. For example:

[9a] If (*f*) whales are fish, then (*g*) whales have gills. Whales do *not* have gills (Not-*g*), so whales are *not* fish (Not-*f*).

Or in more natural language:

[9b] If whales were fish they'd have gills; but they don't, so they're not.

What both of these imply is that *anything* that is a fish would have gills. So if a whale – or anything else – doesn't have gills, it is not a fish. This is even implied by:

[9c] Whales aren't fish; they've got no gills.

Strictly speaking, of course, [9c] is not logically valid because it has a premise missing. However, in the less formal discipline of critical thinking we can interpret [9c] as a sound argument because the missing premise is so very clearly implied. By arguing from a whale's lack of gills to the conclusion that whales are not fish, there is a clear, though unstated, assumption that if whales *were* fish, they *would* have gills – or just that all fish have gills (see Chapter 2.9).

Activity

Two short arguments follow. At first glance they resemble [8] and [9] respectively. But on close inspection you will see that there are differences. The question is, are either or both of them valid?

[10] If you were bitten by a poisonous spider, you would already have a red, swollen wound. This wound is red and swollen, so obviously you were bitten by a poisonous spider.

[11] If that were a spider bite, you'd need to see a doctor. But it isn't a spider bite, so you don't need a doctor.

Commentary

This time we'll use the letters *p* and *q*, the traditional logical symbols for any claim (or 'proposition') whatsoever. We can then see that the first premise in [10] and [11] has the same form as [8] and [9], namely: 'If *p* then *q*'. But there the similarity ends. For in each case the second premise and the conclusion are reversed. [10] has the form:

If *p* then *q*
q

---

p

and [11]:

If *p* then *q*
Not-*p*

Not-*q*

These might seem minor differences, but they are enough to make both the arguments invalid. Even if the premises are true – and we'll assume they are – the conclusions don't necessarily follow. In [10] you are told that if you were bitten by a spider you would have a red, swollen wound, i.e. *q*. But we cannot assume that any red, swollen wound must be a spider bite. Other wounds can be red and swollen. So the premises of the argument can be true, and the conclusion could be false; which, by definition, makes the reasoning invalid.

Similarly in [11], there may be other reasons, besides a spider's bite, why you need to see a doctor. So the fact that if it *were* a spider bite you *would* need a doctor, doesn't mean that if it is not a spider bite, you don't. Again, the conclusion can be false, even if both premises are true.

Critical thinking and logic

The examples we have been examining in this chapter are of the sort that logic books use to define and explain validity. They are not meant to be 'real' arguments, in the sense of resembling everyday reasoning. They are contrived and artificial, and deliberately so, because that is the best way to display their form. No one in an ordinary, practical situation would go to the bother of arguing that such-and-such a person was mortal because he or she was human, *and* all humans are mortal. What [6] and [7] are for (on page 256) is to show the difference between a valid and an invalid form of argument, so as to make them easier to recognise when we are interpreting more authentic, natural arguments.

Critical thinking is directed towards real, *live* arguments that you come across in newspapers, magazines, blogs, scientific theories, political debates and so on. The purpose of analysing live arguments is to try to reveal their underlying logical form as plainly as possible, without the frills of natural language, so as to judge whether or not the reasoning is sound – and if not, *why* not. Sometimes formal logic can assist in this (though not always).

The next argument is still a made-up example, but it is expressed in a more natural style of language, and a more realistic context. Suppose someone – we'll call her Andrea – has inherited a ring with a large stone in it which she has reason to think is a diamond. What is more, she is right in her belief; but not being an expert, she has no way of knowing for sure. A friend – some friend! – offers to have it valued for her. He returns with the surprising and disappointing news that the ring is practically worthless, and that therefore the stone is not a diamond:

[12] 'Let's face it: if a stone that big was a real diamond, this ring of yours would be worth thousands of dollars. Sadly, it's not worth $20. It's pretty, but that doesn't make it valuable. So I'm afraid the stone is not a diamond, and I'm sorry to be the one who has to tell you.'

He volunteers to buy it from her for his daughter for $50, which now seems like a generous offer. Having accepted his argument, and its conclusion, she accepts the offer too, and sells him the ring.

Activity

Discuss whether the argument is valid and/or sound.

Commentary

The answer is that the argument is not sound, but it is valid. Make no mistake about this. What makes it valid is that *if* its premises had both been true, there would have been no escaping the truth of the conclusion. For no large, genuine diamond would have so low a value, and this ring, according to the friend, has practically none. If the second claim were as true as the first, then the stone could not have been a diamond.

Of course we know, from the story, that the conclusion is false. But that doesn't make the argument invalid. Its form, when we cut it down to the bare bones, is the same as that of [9]:

If (*d*) the stone was a diamond,
then (*v*) the ring would be valuable.

The ring is not valuable (Not-*v*).

The stone is not a diamond (Not-*d*).

What practical use is the assessment of an argument's validity, if we already know the premises are false? As far as judging its soundness, none at all. It would be unsound even if it were valid. No argument can be considered sound if it is based on a lie, as this one is. But if we are giving a critical evaluation of an argument, we must be able to say why it is unsound; and it would be incorrect to say that this is invalid. What the above example also shows is that valid reasoning can be abused and exploited for persuasive purposes. It is partly because [12] is valid that it looks and sounds plausible. Andrea is persuaded, dishonestly, to part with a precious possession for a fraction of its value.

The validity or otherwise of an argument is also important if we do not know the truth or falsity of the premises. To see this, look at the next example. It is about a ring, too, but this time one that evidently does have a high value. The question is: *why* does it have a high value?

Activity

Read the following carefully and decide if you think it is sound or unsound.

[13] No ring with a diamond that size would sell for less than $20,000. Miranda Marchi's ring fetched $50,000 in an auction, so the stone in it has got to be a diamond.

Commentary

This time we are not told whether the reasons are true or not, but let's suppose they are, for the sake of argument. Therefore we accept that the stone in the ring is big enough to be worth at least $20,000, *if* it's a diamond; and we accept that the ring really did fetch well over that figure in an auction. Could these two premises be true and still lead to a false conclusion?

Yes, they could. There are all kinds of circumstances under which the ring could have sold for a very high price without being a diamond. The buyer could have been a fool. Alternatively Miranda Marchi could have been a celebrated film star, who had worn the ring (with a fake diamond in it) in her best-known film. No one had ever pretended it was real; it fetched a high price as a collector's item.

There are many plausible scenarios under which the premises could be true and the conclusion false. So the argument is *not* reliable. Unlike the ring that featured in the previous example, which could not have been a real diamond and have a value of $20, this stone could have been a fake and still sell for thousands. That possibility makes the argument invalid – along with all arguments that follow the same pattern.

A fair assessment of this argument would therefore be: we don't know if the premises are true or not, but we can say that the argument is unsound anyway, because the reasons do not support the conclusion. Even if we later find out that the conclusion was in fact true, and the ring did contain a genuine diamond, the argument would still remain a fraud!

Deductive standards – and limitations

The arguments examined in this chapter – even the more natural ones – have been deductive in character. The standard of validity required for a deductive argument is very strict and unbending. Deductive arguments are intended to draw conclusions with absolute certainty. The kind of proofs that logicians and mathematicians use depend on rigid deductive arguments, and nothing less will do. But some quite ordinary reasoning can also be interpreted as deduction, as we have seen in several of the examples.

Partly because deductive arguments are so watertight, they can be rather limited, too. For a conclusion to follow validly from its premises, the premises have to be stronger than the conclusion. To use the more technical term, the premises must *entail* the conclusion. It is often said that if we know the premises of a deductive argument, the conclusion itself tells us nothing we did not know already. There is something in this. Certainly if we know that all true fish do have gills, *and* that whales have no gills, then we really do not need to add that whales are not fish. In a way, deductive arguments are more like proofs explaining *why* something is true, than means to discovering new facts or supporting new hypotheses.

By no means all argument is deductive. Moreover, not all reasoning requires the same level of certainty from the conclusion. Often it is sufficient to be able to say that the truth of a claim that is supported by an argument is beyond reasonable doubt, or even that it is more likely than not to be true – i.e. true on the balance of probabilities.

In the next chapter we turn our attention to certain kinds of non-deductive reasoning, and arguments which fall short of deductive validity, but still have powerful persuasive force.

Summary

- The soundness of an argument depends on two factors: (1) the truth of the reasons; and (2) whether or not they adequately support the conclusion.
- There are different standards for judging (2), depending on the type of inference being made.
- The highest standard for judging (2) is that of deductive validity. But there are other standards by which to judge the reliability of an argument.

End-of-chapter assignments

1 Are either, neither, or both of these two arguments sound – and why?

[A] Citrus fruits have a sharp, acidic taste. Lemons taste sharp and acidic. Therefore lemons are citrus fruits.

[B] Citrus fruits have a sharp, acidic taste. Lemons are citrus fruits. So lemons have a sharp, acidic taste.

2 Comment on the following argument.

A real diamond is so hard it will scratch glass. But when we drew a line on the glass with the stone in your ring it didn't leave any mark at all, however hard we pressed. Therefore it is not a real diamond.

3 What can you say about the soundness of this argument?

If the vice-president were guilty of corruption, as you say he is, he would be in prison, not on an official state visit to South America. He is not in prison. In fact he is in Chile right now and is flying on to Argentina tomorrow, and he will not be back until next week. Therefore he is not corrupt.

4 Suggest a conclusion – if there is one – which can validly be drawn from each of the following sets of premises. If it is valid, show or explain the form that the argument would take.

[A] If an athlete has accepted prize or sponsorship money, that makes him or her a professional. Nathan is sponsored by a major software company. So . . .

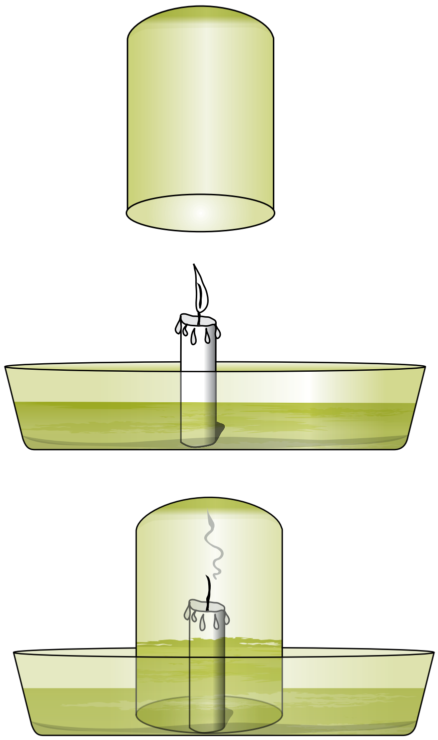
[B] If an athlete has accepted prize or sponsorship money, that makes him or her a professional. Eunice is not a professional. So . . .

[C] If an athlete has accepted prize or sponsorship money, that makes him or her a professional. Abbas has not accepted any prize or sponsorship money. So . . .

Answers and comments are on pages 339–40.

7.3 Non-deductive reasoning

Consider the following simple demonstration. (You probably saw it at elementary school.) A candle is placed in a shallow dish of water and lit. A jar is then held over the candle so that its rim is underwater to seal it from the air.



After a short while, the candle flame dies, and some of the water rises inside the jar. The procedure is exactly repeated three or four times to demonstrate that it wasn't a fluke.

The reason why the candle goes out, in non-technical language, is that the flame burns up the oxygen in the jar, and without oxygen it can no longer burn.

You don't need an argument to persuade you that if you repeat the experiment a fifth or sixth time, the candle will go out. But, if you were asked to spell out the argument, it might go something like this:

[1] Every time a lighted candle is placed in a sealed and restricted space (such as a jar) it has been observed to go out shortly afterwards. Therefore we can infer that it always will.

Activity

Is this a valid form of argument? Is [1] a *good* argument?

Commentary and continuation

The answer to the first question is no. It is not valid. [1] is an example of a fallacy that is sometimes called 'appealing to history'. It is claiming that because something has been observed to be the case in the past, it will always be so in the future. We can assume that the single premise in [1] is true. It is based on direct evidence, verified by a great many experiments and demonstrations, none of which has ever been observed to have a different outcome. But the inference that this will always be the case cannot be verified by direct evidence. Therefore the premise could be true and the conclusion false (under some freak circumstance).

You could argue that the conclusion of [1] was a practical certainty. The laws of physics would have to change to make it false. But logically it is still an uncertainty. Its truth may be beyond *reasonable* doubt, in the world as we know it, but it is not beyond *all* doubt, in all *possible* worlds. Again, you could argue that possible worlds don't count; only the real world counts. But in logic, and to many scientists as well, possible worlds *do* count. We human beings are quite intelligent, but our knowledge is still restricted to a tiny bubble of space-time. Using words like 'always' and 'everywhere' *literally* in our reasoning cannot be justified by evidence or experience. So, although [1] may seem a safe bet, it is not a safe inference in any deductive sense.

Induction

You should have noticed that the first question was about the *form* of reasoning in [1]. Even if you were satisfied that [1] itself made a good case and justified its conclusion beyond reasonable doubt, that does not mean that every argument with the same form as [1] would be as reliable or as persuasive.

We call arguments like [1] 'inductive', to distinguish them from deductive arguments. Induction does not establish certainty. Instead its conclusions come with varying degrees of *probability*. In strictly logical terms an inductive argument is invalid, because it is theoretically possible for the premises to be true and the conclusion false, however unlikely this may be. Inductive arguments are therefore judged not by their validity or invalidity, but by the strength of the evidence that they provide and the degree of probability it gives to the conclusion.

One problem with induction is that evidence for any general hypothesis is always limited to a finite number of experiments or observations or examples. No matter how many times a hypothesis is confirmed by an observation, there is always the possibility that the next one will be the exception. One of the best-known examples of this weakness in inductive reasoning is the case of the black swan. For many centuries it was believed – with good reason – that all swans were white, because every swan that had ever been observed was white. That was until a species of black swan was discovered in Australia.

The premise that only white swans had been observed *up to that date* was true, and remains so. But the inference that therefore *all* swans are white was then seen to be false. The great Scottish philosopher David Hume pointed out the general problem with induction. The problem, put very simply, is that to argue in the way that [1] does, one has to assume that the future will be like the past. But the only real evidence that we can have for this assumption is that the future has always been like the past, in the past! So the argument is circular, and we are back where we started.

But although inductive reasoning does not *guarantee* the truth of its conclusions, and sometimes yields false ones, we still draw inferences from repeated experiences and observations. Indeed, scientific reasoning is routinely based on such evidence, and proves to be highly reliable. Also, the problem of induction is not really a practical problem. Rational people, including scientists, do not make a habit of making such crude inductive arguments as the one above. There is no need to say that candles in sealed jars will always go out. It is enough to say that they always have, and that there is no reason to think that this will change. The problem can be avoided by simply not overstating conclusions, and recognising that good inductive arguments increase the probability of the conclusion up to and beyond any *reasonable* doubt.

Argument to the best explanation

Anyway, what is of interest scientifically is not whether candles will always go out in sealed containers, but *why* they go out. It has not always been known that burning, or combustion, involves the absorption of oxygen (oxidation). From classical times until relatively recently combustible materials were believed to contain a mysterious undetectable substance called 'phlogiston', which they gave off when they burned, to be absorbed by the air. The reason the candle goes out was thought to be that the air in the jar could only absorb so much phlogiston: quite the reverse of the oxidation theory that we now learn at school. One problem with the phlogiston theory was that burning should have resulted in a loss of substance to the air, and hence a loss of weight. The discovery that combustible matter when burned *gained* weight was the beginning of the end for the theory – although some scientists clung to it by claiming that phlogiston had *negative* weight, making it even more mysterious.

The idea of phlogiston was accepted for centuries because at the time it seemed to *explain* combustion. It was the best explanation around, until oxidation was understood. The argument for phlogiston was that if there were such a substance, it would explain why the candle went out in a confined space. It did not, however, explain why burnt matter (ash etc.) gained weight without extra complications such as negative mass. Nor could it explain, simply, why the water level rises in the jar. If phlogiston were added to the air during burning, then arguably it should have forced the water level down! You will recall (from Chapter 4.2) that explanations are generally assessed by their scope (how much they can explain) and their simplicity. Once understood, the theory of oxidation explained more than phlogiston did, and much more simply. It didn't need implausible extra accounts as to why it couldn't be detected, or weighed less than nothing. The argument for oxidation is therefore much more compelling.

The fact that something is the best or most believable explanation is often used as a reason for arguing that it is true. Not surprisingly, reasoning of this kind is known as argument (or inference) to the best explanation. As this is a mouthful, we'll shorten it to ABE. (It has also been given the name 'abduction'.) ABE or abduction is not found only in science. It is actually one of the commonest ways in which we reason in everyday situations; so much so that we are often barely conscious that we are reasoning at all. A classic example is my coming out of the house in the morning and finding the ground in the garden soaking wet. If it had rained heavily in the night, that would explain this observation simply and plausibly; so I take it that it has rained, and think no more about it. In fact, if it had not rained in the night, I would be very surprised.

ABE is a powerful and familiar method of reasoning. But as we have seen in several previous chapters, it carries a high risk of jumping to conclusions. It therefore has to be used and evaluated with care. ABE supports hypotheses; it does not establish facts. Recall the example of the origin of 'posh' (Chapter 4.2, page 141). The claim that it was an acronym from 'Port Out, Starboard Home' seems such a plausible explanation that it is often accepted without further thought. It turns out there is little evidence to support it other than its elegant explanatory properties. So, we must either abandon it or look for additional supporting evidence. ABE is not sufficient on its own to make an inference safe. Returning to my wet garden: if I later discovered that the ground everywhere else in the neighbourhood was dry, I would obviously have to think again about the seemingly obvious inference that it had rained in the night. To explain the dry ground elsewhere, as well as my wet garden, I would need a more local explanation such as a burst water pipe.

Activity

Which of the following are examples of argument to the best explanation, and why? How safe are their conclusions? (The second question applies only to the examples of ABE.)

A My coffee is barely warm so it must have been made some time ago.

B These bushes won't survive because they're not getting enough sun.

C Head injuries from cycling are very common. Not only do they cost lives, they cost the health service millions of dollars of taxpayers' money every year. The wearing of cycling helmets on public highways should therefore be compulsory. It's illegal in most countries to drive without a seatbelt on. Cycling should not be treated differently.

D Most land mammals have a dense coating of fur. Humans, by contrast, have little hair and a thicker than normal layer of fat, more like aquatic mammals than the ancestral apes. Humans are also unusual in habitually walking upright. The obvious conclusion is that a large part of human evolution took place in a watery environment, where fat would provide insulation and wading on two legs would be the natural way to move.

Commentary

The two examples of ABE are A and D. Having observed that my coffee is barely warm, it would then be no surprise to learn that it had been made some time before. That would be a plausible explanation, and therefore a plausible hypothesis. However, that is by no means the only possible, or even plausible, explanation for lukewarm coffee. The water may not have been allowed to boil. It may have been made in the belief that I don't drink my coffee hot. And so on.

B and C are not cases of ABE. ABE proceeds from an observed fact to a hypothesis which would explain the fact, and explain it better than other hypotheses would. In B the conclusion is a prediction based on an observation. C is a recommendation based on a claim about cost, and supported by a comparison between cycling and driving. In neither case is the conclusion justified by what it supposedly explains.

D is a clear case of arguing from observed facts to an explanatory hypothesis. There are three observations: (1) that humans have less hair than most land mammals; (2) that they have more fat; (3) that they walk upright. D is giving support to what is known as the 'Aquatic Ape Theory': a claim advanced by some anthropologists that for a considerable time during their evolution humans took to the water as their natural environment. D claims that this theory is the obvious conclusion because if it were true it would explain all three of the observed facts at a stroke. The Aquatic Ape Theory is a nice one, and it certainly does offer a plausible explanation for many differences that are found between humans and other primates or mammals generally. However, D itself is a one-sided argument. It does not acknowledge that there may be other explanations that are just as persuasive. It is overstating the case therefore to say that it is the obvious conclusion.

Argument from analogy

A third line of non-deductive reasoning that is very frequently used is argument from analogy (AfA). An analogy is a comparison, an observed similarity. In C, above, an analogy is drawn between cycling helmets and seatbelts. The comparison is an obvious one: both devices are designed to reduce injury in the event of an accident. The assumption in the argument is that they do. Its conclusion is that the same rule should apply to both cycling and driving.

Argument from analogy has the following general form: If such-and-such a thing is true of X, and Y is like X (*in the relevant respect*), then the same thing is true of Y. So, if one ought to be made to wear a seatbelt in a car, one ought to be made to wear a helmet on a bike, because – arguably – there is no *relevant* difference.

Activity

Do you accept this argument? Is the analogy in C fair, or fit for purpose as a premise in the argument?

Commentary

This will be quite brief. The answer to the first question is up to you: evaluating arguments of this sort very often comes down to whether you *think* the analogy is a good one or not. But that does not mean that arguments from analogy cannot be evaluated with some objectivity. The heart of the matter is whether or not the analogy is a *fair* one: whether the two things being compared are sufficiently alike for the conclusion to apply to both of them. The key phrase in this is the one in brackets and italics above: '*in the relevant respect*'. Why must this be added?

The reason is this: an argument from analogy does not depend on the compared objects being exactly alike, or alike in *every* respect, for they would then be identical. Indeed, some of the best AfA compare objects which are in many respects quite different. (We'll see an example of one shortly.) In C the analogy is between cycling helmets and seatbelts. It does not demolish the argument to point out that one goes on your head and the other across the lap and over the shoulder. The relevant respect is the alleged reduction of injury that both devices are meant to bring; and in that respect, they are closely analogous.

When evaluating an AfA, therefore, it is essential to bear this qualification in mind. An objection to the argument that merely points out random differences has little value. The relevance of the comparison must be considered. With that in mind, it is really quite difficult to fault C. If it is right to make people wear seatbelts, on the grounds that it saves both lives and public money, then it is a very fair point that cyclists should take the same precautions, and face the same compulsions. You might say that bicycles are slower than cars; but unless that translates into fewer accidents or injuries, that is *irrelevant*. You might be tempted to object that cyclists have a right to take risks with their own lives. But that would apply to drivers and the wearing of seatbelts too. The analogy remains fair – in the relevant respect.

Argument from analogy is especially effective in counter-arguments and debates. Here is an example of two people – we'll call them K and J – disputing the merits of reality television, especially the programme called *Big Brother*, in which a number of people are confined to a house and filmed night and day. We take up the debate at a point where J has just said that the Big Brother housemates are 'manipulated and exploited like circus animals'. (There is one analogy already.) She goes on:

J: That pathetic lot in the house think they are celebrities, when really they are just sad little victims making fools of themselves for public entertainment. And the only *reality* is they're too stupid to know it.

K: Don't you think that's a bit patronising?

J: It's the truth.

K: How do you know? You never watch it; you've admitted that. You can't criticise something you've never watched.

J: Yes I can. I've never watched a public execution, but I know it's wrong. Therefore I wouldn't watch.

K: That's different, and you know it.

J: What's different about it?

K: No one's killed on *Big Brother*.

J: Give it time.

K: Don't be ridiculous. It's just harmless fun.

J: Now there you *are* wrong. Fun it may be: that's a matter of opinion. But *Big Brother* is not harmless. People are seriously damaged by being in that house. Not physically, but mentally. You can't imprison people together like that, knowing they are on camera day and night, without it affecting their personalities. You only have to look at them when they're interviewed afterwards to know they are not the same person they were when they went in. So it's a very dangerous game they're playing. Any psychiatrist will tell you that.

K: They have psychiatrists monitoring the housemates all the time, looking out for danger signs.

J: They have doctors watching boxing matches, but boxers still get brain-damaged. Some even die from their injuries.

K: There you go again: executions, violent sports. What are you going to drag in next?

J: I'm just making the point that *Big Brother* is a sick spectacle. And it's people like you who watch it that keep it going . . .

K: Well if that's the case, you are in no position to point the finger. You watch motor-racing.

J: So?

K: People only watch that because they're waiting for an accident to happen, preferably nasty. Fatal even. And you call *that* entertainment. . .

Activity

Identify two or three examples of the use of analogy in the above dialogue, and discuss what they contribute to each speaker's argument.

Commentary

The first and perhaps most interesting analogy is the one that J makes when she is told that she can't criticise what she hasn't watched. The analogy she draws is with watching a public execution. This is the example promised earlier, in which there is a major difference between the items being compared. K is quick to point this out: *Big Brother* is different, she says, because no one is killed on the show. J jokes that it may happen one day. K dismisses this as ridiculous.

But J's argument is not ridiculous, despite the difference in physical harm to the respective 'victims'. At this point J is countering the claim that people cannot criticise something they have never watched. But, she says, you *can* criticise public executions without going to see them. If you can criticise one you can criticise the other. J is not seriously saying that the two spectacles are the same in their consequences or extremity. She is just saying that they can both legitimately be criticised. Nonetheless you might have felt that the analogy goes too far, implying that reality TV is in some way brutal, and that you can know this without even watching it. There is room for disagreement about this part of the argument, and that is what makes it an interesting exercise.

The next example is more straightforward. J says that there are psychological dangers in the reality show; K says that psychiatrists are there to spot them and prevent them. J draws the analogy with ringside doctors at boxing matches, who do not always spot the harm before it happens. It is a fair comparison to draw, since both are medical safeguards. If one can fail, it is at least reasonable to question the reliability of the other.

There is possibly a third analogy that you may have identified towards the end: the comparison between watching motor-racing and watching *Big Brother*. But if this is an argument, it is a fallacious one. It is basically arguing that if it's all right to watch cars crash for entertainment, it's all right to watch *Big Brother*, the implication being that identifying nasty things about one spectacle justifies the unpleasantness of the other. But surely K is supposed to be defending *Big Brother*. It is a weak defence to say that it is no nastier than something else that is nasty.

Tu quoque

This is another classic fallacy to add to your file. *Tu quoque* means literally 'you too'. More explicitly it means responding to a criticism or objection by saying that the other person, or other people, are guilty of the same thing. At the lowest level it is quite a childish argument, effectively breaking the rule that two wrongs do not make a right. It is also a form of *ad hominem* argument (see Chapter 4.9) when directed at someone personally, as in this case. K is saying: 'You watch a dangerous sport, so you can't criticise me for watching reality TV.'

Note that *tu quoque* arguments can take a more general form. I am committing the same reasoning error, for example, if I say that I cannot be criticised for doing something because lots of people do it too. The fact that lots of people break the speed limit or drop litter or tell lies does not make any of these acts less wrong.

Summary

- In this chapter we have looked at three frequently used forms of non-deductive argument:
  - induction
  - argument to the best explanation
  - argument from analogy.
- It is an important analytical skill to be able to recognise these forms and to evaluate them appropriately.

End-of-chapter assignments

1. Clive is an experienced hill-walker. For 25 years he spent most of his leisure time backpacking in wild country, living off the land, sleeping in the open and finding his way, sometimes in uncharted regions. He refuses to use satnav. His most valued possession is a compass, which he says has saved his life on numerous occasions, especially in bad weather and poor visibility. Only once, on a ridge in Scotland in thick cloud, did he get dangerously lost, not knowing that the rock in certain places contained minerals which can attract a compass needle and distort the reading. When the cloud lifted he realised that he had strayed a long way off course.

   What does the above anecdote imply about inductive reasoning?
2. Analyse and critically evaluate J's longest argument in the dialogue on pages 266–7 (the speech which begins 'Now there you *are* wrong . . .').
3. **(Harder task)** Find out more about the Aquatic Ape Theory, and some of the arguments that are raised for and against it. Write a short essay either supporting or challenging the theory.

Answers and comments are on page 340.

7.4 Reasoning with statistics

A leading politician once summed up his approach to law and order with the now famous slogan:

[1] 'Prison works.'

But does it? Does it, for example, reduce crime? Do the authorities make law-abiding citizens safer by locking up criminals? Does prison deter people from committing crimes in the first place, or from reoffending after serving a sentence?

In this chapter we shall be considering ways in which questions like these can be answered: what sort of evidence is required to support or to challenge the claim expressed by [1]? We shall be looking in particular at the use of *statistical* evidence, and statistical reasoning. As well as considering ways in which statistics can legitimately be used to support claims, we will also be looking at ways in which they can give false or misleading impressions. It is fairly obvious why statistical evidence is needed in a context such as this. It would be hard to see how any grounds could be given either for or against [1] without producing facts and figures: numbers of prisoners, levels of crime, lengths of sentence, rates of reoffending and so on. If someone said to you: 'Prison works because it reduces crime,' you would be entitled to ask for some *proof* of this, or at least some indication that locking people up does bring down the crime figures.

You would also be entitled to ask whether some observed reduction in crime would be a *sufficient condition* for claiming that prison works. (Necessary and sufficient conditions were discussed in Chapter 7.1.) For a start, we would need to be sure that it is prison that is responsible for the reduction. It would be wrong to assume that because crime numbers were falling, and prison policy getting tougher, that one was the cause or the consequence of the other. And even if we were satisfied that prison sentences do reduce crime, we might still want to know by how much they reduce it. If it turned out that a very large increase in the number and severity of prison sentences was needed to achieve a small reduction in crime, we might well question whether this showed that prison was really as effective as the author of [1] would have people believe.

Interpreting statistical data

In Chapter 4.3 the distinction was made between *raw* data and *processed* data. Raw statistics are just numbers, or quantities. If we want to use them we have to interpret them, and draw inferences from them. They do not come with inferences and interpretations attached. Statistics on their own don't make points or support arguments or answer questions. They are used by people to do these things, and for that purpose they usually need to be *processed* in some way: for example, combined or contrasted with other statistics;

multiplied, divided, rounded, converted into percentages, plotted on graphs and so on.

Raw data is not necessarily altered by processing – unless, of course, it is deliberately falsified. Even so, the same data can be presented in ways that support different inferences, some perhaps more justified than others. It is how statistics are used and presented therefore that requires critical attention. As far as the raw material is concerned we either believe it or we don't. (Grounds for believing or disbelieving a claim were discussed in the chapters on credibility in Unit 4.) But even if we believe the data, and are satisfied with its accuracy, we may still question the way it has been interpreted. Like any argument, the premises can be true but the reasoning still flawed. Statistical reasoning is no different in this respect.

Here is a simple illustrative example. Take the raw statistic that in 2010 there were 2.3 million people in prison in the USA. (To be precise this has already undergone some processing because it has been rounded to the nearest 100,000, and presumably averaged over the year. But within these bounds, it is either true or false; we'll assume it is true.) It is another fact that in Germany the corresponding number was a little over 67,000. These facts may come as a surprise. They may prompt someone to argue that the number of prisoners in the USA is excessive or unnecessary, or inhumane, given that the contrast is so striking between two developed, and in many ways similar, countries. But the numbers themselves do not carry those implications. What is more, they cannot be used in their raw form either to strengthen or to weaken any such conclusion.

'Like with like'

One way in which statistics may mislead is by comparing total numbers with proportions. Suppose I did want to argue that the rate of imprisonment in the USA was excessive, by comparing it with that of another developed, prosperous, democratic country. The two figures above would be quite inadequate, simply because they are not *proportions*: they are bald totals. If I supported my claim by simply observing that there were 34 times as many prison inmates in the USA as in Germany, that would not be a false statement, but it would be a misleading one in the context of my argument. To compare the two facts in any fair and meaningful way we need the populations of the two countries as well as the number of prisoners. The population of the USA, as of 2011, was 312 million (in round figures); that of Germany 82 million. We can enter these numbers into a table, and calculate the rates of imprisonment as follows:

TABLE 1

	Total population (millions)	Prisoners	Prisoners per 100,000 population
Germany	82	67,000	82
USA	312	2,300,000	737

The first two columns of the table contain the (more or less) raw data; the third the processed data. The processed data permits us to compare like with like. We can now argue legitimately that the proportion of the US population that is in prison is around nine times that of Germany: still a significant and striking difference, but a long way short of 34 times! The difference may still fail to establish that the number and length of prison sentences are *excessive*. That remains a value judgement, depending on what one means by 'excessive', and requiring rather more information than we have in the table. But at least the intermediate conclusion – the contrast between the Germany and USA *rates* – now has a firm evidential base.

Selectivity

A second way in which data may mislead is due to selectivity: choosing facts which suit a theory or hypothesis and/or omitting those which do not. One of the obvious weaknesses of the data in Table 1 is that it compares the USA with just one other country. What is more, the country in question contrasts sharply with the USA in terms of its rate of imprisonment. If there are other countries with similarly low prison populations to that of Germany then we might have a stronger case for the claim that the USA figure is excessive, on the grounds that other countries can get by with many fewer and shorter prison sentences. If instead we found that Germany was atypical, and the USA was more in line with international levels, then we would have to concede that the argument was weakened.

Activity

Study the following bar chart.

CHART 1

1 Discuss how you would interpret the statistical data in Chart 1.
2 Does it lend support to the following claim?

[2] The USA locks up too many people.

In particular, look for ways in which the data might be used or presented in ways which could mislead or influence the reader.

Commentary

The first thing to do when faced with any statistical document is to clarify precisely what it conveys. This is particularly important with graphs or other visual documents, because they create impressions as well as presenting facts. A table of figures is closer to the raw data than a graph or chart. A graph or chart, on the other hand, is easier to read, because some of the interpreting has already been done. That is both an advantage and a danger, since visual representations can obscure important details and/or exaggerate others.

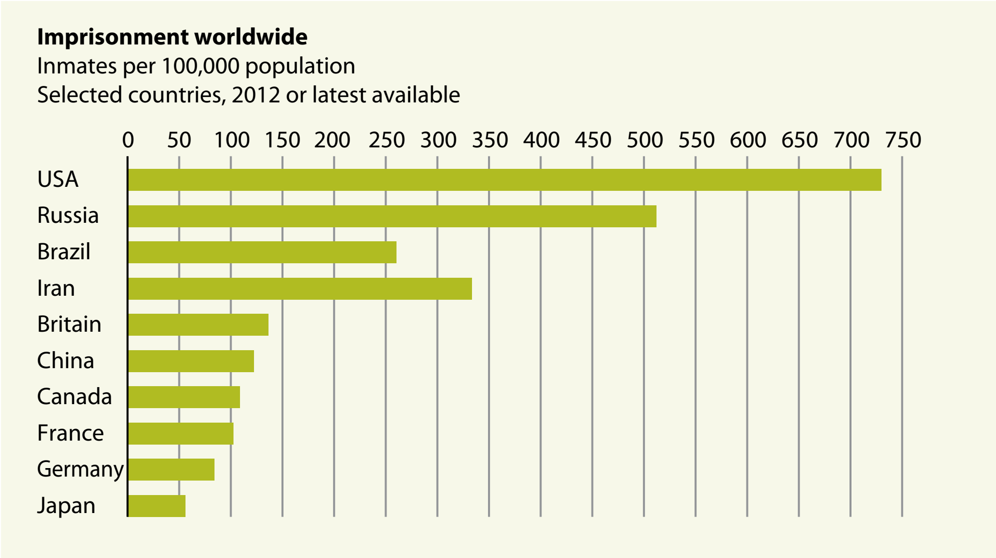
Chart 1 extends the information in Table 1 to ten countries instead of just two. Its calculations are based on raw data, rather than rounded figures, so the rates are slightly different from those in Table 1. It tells us that among these ten countries the USA has the highest proportion of prisoners. But it also shows that Germany is well down the list, with only Japan imprisoning fewer; and it shows that there are many countries with numbers closer to the USA than Germany has.

Scale

At a glance the message might appear that there is a fairly even spread, with three countries, Russia, Brazil and Iran, not very far behind the USA. It looks from the chart as if the numbers increase gradually from the lowest rate of imprisonment to the highest. On closer inspection, however, we see that two of the bars are not drawn to the same scale as the others. The jagged white line indicates that a section of the bar is missing, or that it is 'truncated' (meaning shortened). Drawn to scale, as in Chart 2 on the next page, the bar for the USA should be nearly three times as long as Brazil's, and over seven times as long as France's. Even in comparison with its nearest rival, Russia, the USA imprisons 25% more of every 100,000 population.

Chart 1 is not inaccurate, but *visually* it could be misleading. It illustrates the need to study graphs carefully, and not be influenced

CHART 2



by purely visual impressions. Interestingly, it is not clear whether the truncating device actually minimises the difference between the USA, Russia and the other countries, or exaggerates it by showing them as 'off the scale'. In its original context this graph came with the caption 'Exceptional America', and was part of a report that was critical of high rates of imprisonment. So it may be that the author wanted to make a point by emphasising the gap. You can compare the two charts, Chart 1 and Chart 2, and decide which you think presents the data more strikingly.

Selectivity again

So does the data in Chart 1 support the inference, or strengthen the argument, that the rate is too high? Not really. For one thing it still represents only a *selection* of countries, and we have no information on why the particular selection was made. It would have to be established that there were not other countries with comparable or even higher proportions of their populations in jail. Besides, for a statement like [2], we need more and different data than mere comparisons between countries. For instance, we would need to know why the rate is higher in one country than another, whether there were special circumstances which necessitate or justify a tougher prison policy. It is time to get back to our first question: whether or not prison *works*. If the rate has to be high to be effective, then perhaps it is wrong to say the rate of imprisonment in the USA is excessive.

Activity

Read the following short comment from a law-enforcement website:

> [3] Prison works. Not only are those inside prison prevented from committing crimes, those outside are deterred from committing crimes by the knowledge that they will face long sentences if caught. Besides, the facts speak for themselves: more prison, less crime.
>
> *John Keyes, Indiana, USA*

To what extent do Charts 3 and 4, based on official records, support the above argument?

In answering the question you will need to critically assess the statistics represented by the graphs *as evidence* for [3]. You are not asked to assess the accuracy of the data: assume it is correct. But do ask yourself how clear and transparent the presentation is. Does it hide or distort any of the relevant information?

CHART 3

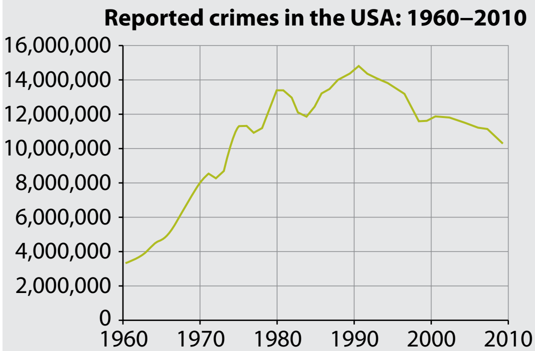
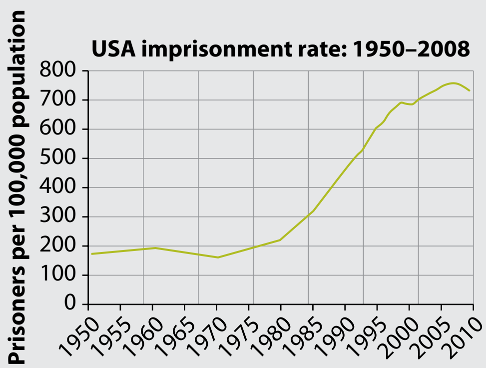


CHART 4



Commentary

Well, the facts (as presented in the graphs) may speak for themselves, but do they speak for Mr Keyes of Indiana? Chart 3 indicates that reported crime in the USA rose sharply and increasingly through the 1960s; and, with occasional temporary falls, throughout the 1970s and 1980s too. It peaked around 1991 at close to 15 million reported crimes. Over the next two decades it fell back to just over 10 million, a decrease of 4.5 million, or 30%. Meanwhile the numbers in prison (Chart 4), which had been under 200 per 100,000 of the population prior to the 1970s, rocketed over the next three decades, with one small reduction in the late 1990s and another around 2009. The increase between 1970 and 2008 was over 360%.

So, for two decades – 1970 to 1990 – crime rates and imprisonment both rose. But whilst the imprisonment rate rose continuously, the crime rate fell back three times before reaching its peak. If the increasing imprisonment rate was 'working' it looks as though it was working for a time, then failing again. Then, after 1991, with imprisonment still on the same steep rise, crime began a more or less steady descent. But we do not know what happened after that, or what will happen in the future. You must decide whether there is a sufficiently strong pattern or trend in Chart 3 to make a reliable prediction, or to support Keyes' hypothesis.

Remember that the task you were set was to criticise the data *as evidence*. This does not mean that there is anything wrong with the data itself. The graphs are based on official statistics, and therefore come from reputable and reliable sources. You are not asked to assess their credibility. It is the *interpretation* that we are concerned with. The question is whether the statistics:

- **A** positively support [3]
- **B** fail to support [3]
- **C** contradict or disprove [3].

The answer is almost certainly B; and here are some reasons why. Firstly, even if the data is interpreted as a strong correlation between the rise in imprisonment rates and the fall in crime rates (which is questionable on the basis of Chart 3), there is nothing to indicate which is the cause and which the effect. In Chapter 2.10, and several times in Unit 4, the fallacy of assuming cause on the basis of correlation was discussed, and you should have realised that [3] makes this assumption. Surely it is just as plausible that the causal connection is the reverse: that an explosive rise in crime has pushed the prison population higher and higher. If so, it is crime that is 'working', and the slogan should be: 'More crime, more prison!'

Why would reported crime then fall as it did from 1991? Well, there are plenty of possible reasons. One is that the police may have become better at solving crimes, and that conviction rates have risen accordingly. That would reduce crimes *and* result in more criminals going to jail, and thus explain both graphs. It would not mean that prison was working, but that detection and prosecution were working. It is possible, too, that there was merely a reduced rate of *reported* crime, or a change in the way crime is classified and recorded. That sometimes happens as politicians try to reassure the public that the fight against crime is being won, and they have less to fear. So long as there are other plausible ways in which the trend in Charts 3 and 4 can be explained, the claim that prison is the driving force is weakened.

Be careful, however, not to swing too far in the other direction, towards option C. Doubt about the support that the statistics give to [3] does not mean that [3] must be false. In fact the data that can be read off from the graphs gives no more support to the claim that prison does *not* work than to the claim that it does.

No control group

Another way in which statistical information may mislead is by giving only one side of the picture. What is missing from the data is what researchers refer to as a control group. If we think of the period of time during which the prison numbers rose as an 'experiment', we can see what this means. The experiment was performed on a whole population, and the observed outcome was that as prison numbers rose, crime figures rose and then fell. What is lacking from the experiment is a second group in which prison sentences are reduced, or held at the same level, to see what effect that has. If this, the 'control' group, shows no reduction in crime, then it would support the case for the effectiveness of prison. But if the outcomes in the control group were the same as in the main group – or even resulted in a bigger reduction – then the argument that prison works would be severely weakened.

Obviously one whole population cannot be subjected to both experiments, main and control, at the same time. But different regions with different crime-fighting policies can be compared. Similarly, different periods in history, when different methods were in operation, can also be compared. Chart 5 on the next page is an example of such a comparison.

Activity

Comment critically on the statistical information in Chart 5 and the claims made on the strength of them.
Can the following claim, from the headline of the document, reliably be inferred?

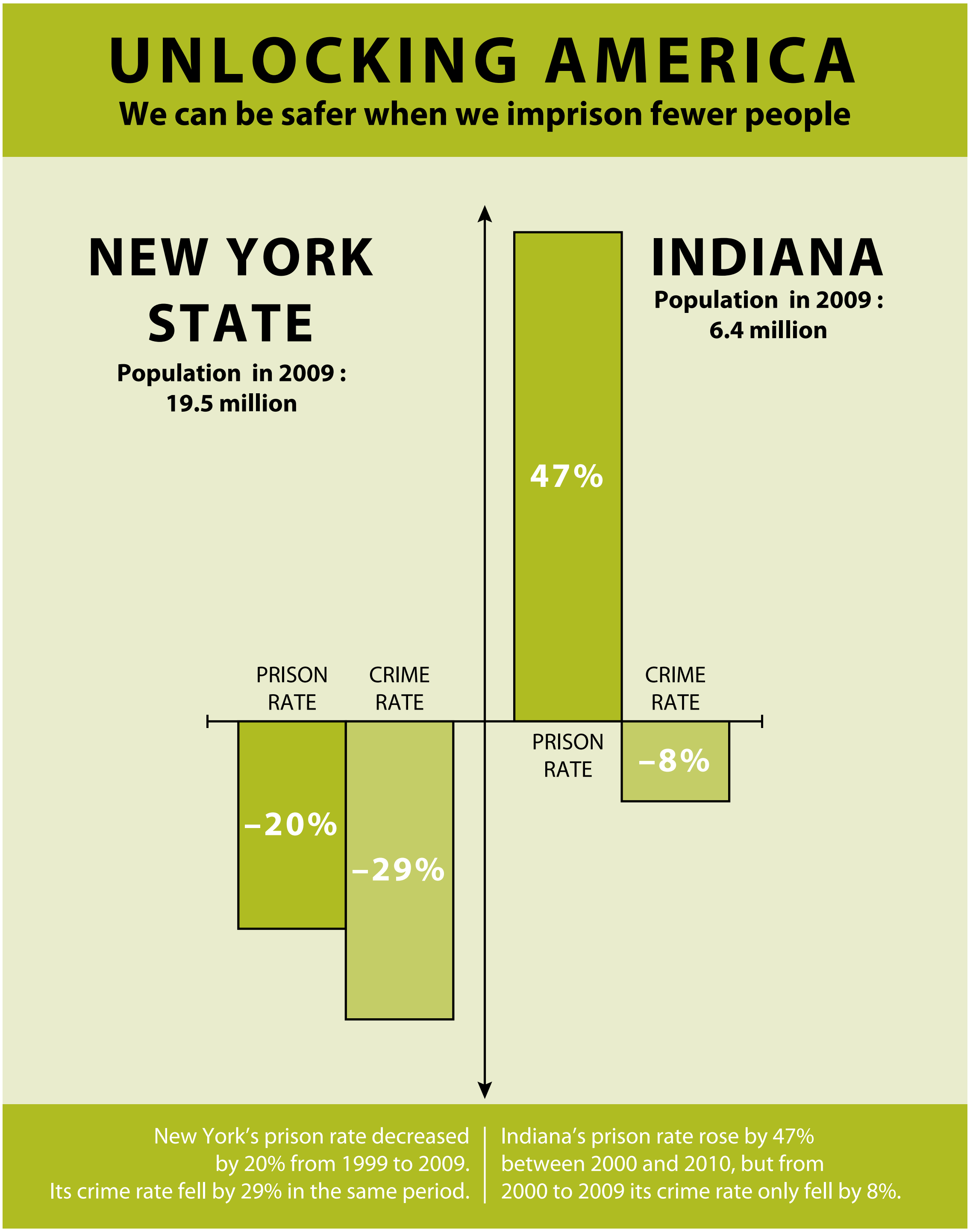
[4] We can be safer when we imprison fewer people.

(Keep in mind what you already know from Charts 3 and 4.)

Commentary

Clearly this bar chart is intended to counter the claim that prison works. As they stand, the statistics are impressive. Over the ten-year period from 1999 to 2009, when imprisonment was rising generally across the USA and crime falling, the state of New York saw a reduction in its prison population and an *accelerated* fall in crime, compared with the state of Indiana which had a huge rise in its prison population and a much smaller fall in crime.

CHART 5



New York prison rates from 'Downscaling Prisons', a report by The Sentencing Project
Indiana prison rates from the Justice Reinvestment Project
Uniform Crime Reporting Statistics from the US Department of Justice.

Let's look at the numbers. From Chart 3 we know that in the same decade crime fell nationally by around 1.8 million from 12 million, which is approximately 15%. New York's crime rate fell by almost twice that, 29%. Indiana's fell by a mere 8%. From Chart 4 we can calculate that the national increase in prisoners per 100,000 was around 7% over the relevant period. Indiana's was a massively inflated 47%, whilst New York, as we see, saw a *reduction* of 20%. This amounts to one in every five prisoners being released without being replaced.

New York certainly 'bucks the trend'. Compared with the national pattern, it is an anomaly. But does it prove anything in general terms? The answer has to be no. Generalisations drawn from particular cases are always questionable, as you will recall from discussions earlier in the book (see Chapter 2.10). Anomalies, likewise, can very often be 'explained away' (see Chapter 4.2, pages 140–1), which lessens their impact. In the last section it was suggested that falls in crime can have many other causes besides high rates of imprisonment. New York's police may have done a better job than Indiana's. New York may have fewer of the social problems that lead to crime. The fact is that if there are fewer crimes – for *whatever* reason – there will be fewer people being sent to prison and replacing those who are leaving; so of course prison numbers will fall. That does not mean that releasing prisoners lowers crime. We have the same problem as we had with claiming that *more* prison meant less crime.

The problems with Chart 5 have more to do with what we don't know than what we do. For one thing, the statistics do not tell us *why* prisoners were released in New York. If they had simply reached the end of their sentences, and crime was declining anyway for other reasons, then the prison population would fall naturally and have nothing to do with a deliberate policy to reduce offending. But what we lack most of all is other statistics for other states. Indiana and New York may be no more than dramatic exceptions to the national picture. There may even be states in which the crime rate is falling more rapidly than in New York, but in which prisons are also getting fuller.

Sample may not be representative

This criticism of Chart 5 is one that can frequently be levelled against statistics which take samples. Firstly, the sample may be too small to be representative of wider trends. There are 50 US states with a total population of over 300 million. New York and Indiana, though large, account for less than 10% of the national population. Secondly, because the statistics come from just two states, they are not a *random* selection, meaning that 90% of the population are not represented at all. Thirdly, it is very likely that the two states have been selected deliberately because they support the claim or claims being made. Selection bias is almost certainly an issue with these statistics.

Despite these critical comments, the data in Chart 5 is not without significance. There are inferences that can be drawn from it, though not broad generalisations.

What can be inferred?

The specific inference that you were asked to assess was not as strong as 'Prison works' or 'Prison does not work'. It was simply the contention that we *can* reduce our reliance on prisons and be safer. With the emphasis on 'can', [4] can be understood as a much weaker proposition than, say, [3]. It *challenges* the claim that long prison sentences are the best or only answer to crime, and *suggests* that there may be other ways to tackle the problem. On that understanding, the evidence for [4] is much more compelling, because it is merely registering that there *may* be another way of doing things. It is not saying that we should throw open the prison doors tomorrow and expect to see law and order swiftly return. It is saying that we should not assume that just because crime rates have been falling, tough sentencing is necessarily the explanation.

As far back as Unit 2 you were warned that claims need to be measured so as not to require too much from the reasons or evidence which are given for them. This has been a useful example. Statistics are powerful reasoning tools. But what we infer from them needs to be kept within bounds. When assessing statistical reasoning, the big question is whether the data is adequate for the claim or claims being made.

Summary

- Statistical data is a form of evidence that can be used to support claims and underpin arguments.
- There is a difference between raw data, which is neutral, and data which has been processed for a particular purpose.
- Selecting statistical data may reflect bias.
- Critical assessment of statistics involves looking for ways in which presentation can mislead, by exaggerating, simplifying, sampling selectively, etc. We must be careful not to be 'taken in' by seemingly impressive evidence.
- It also involves interpreting statistics fairly, and assessing inferences that are made on the strength of them. We should be especially wary of inferences which are too strong and/or too general, or which assume a causal explanation from a mere correlation or trend.

End-of-chapter assignments

1 Read the short extract. Also examine Chart 6 below, which is related to the same topic. Then answer the questions that follow.

> Peaks in crime rates tend to be associated with a significant reduction in the prison population. Although this trend can be observed in several countries, for instance Denmark and Portugal, the paradigm example is Italy. In 2007, the total number of police-recorded offences catapulted by over 160,000, following a mass pardon of prisoners the previous year. The crime rate only began to fall once the prison population crept up towards its 2006 level.
>
> *Carolina Bracken (UK Daily Telegraph)*

a Give a critical assessment of the evidence provided in Chart 6. How much corroboration does it give to the claims in the newspaper extract? (Look particularly carefully at the scales on the graph: robberies on the left axis and prison population on the right.)

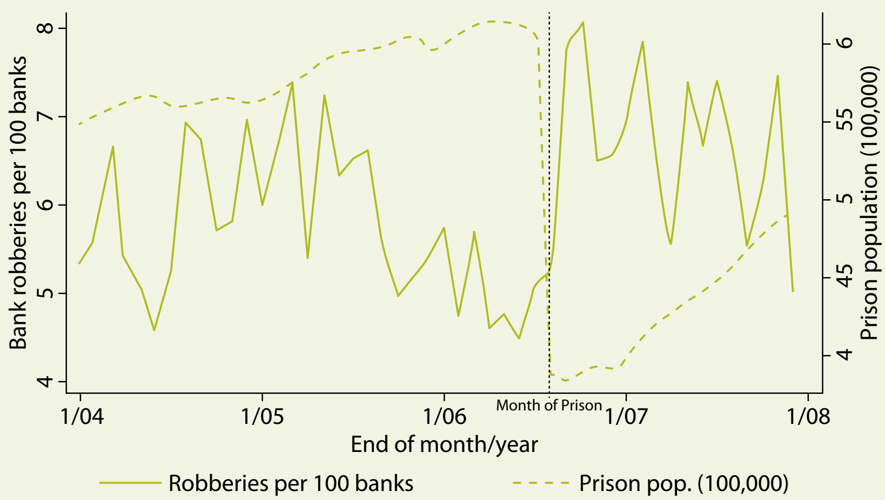
b Based on the statistical data from both sources, draw one precise, credible inference about the relationship between prison and crime.

2 Find a newspaper or magazine article which uses statistical data to support a claim or claims. Make one or more critical comments on the way the data is interpreted and presented, and give an overall assessment of the claim(s) made on the strength of the evidence.

Answers and comments are on pages 340–41.

CHART 6

Graph showing incidences of bank robbery before and after the mass pardon of prisoners in Italy in July 2007



7.5 Decision making

In some ways a decision is like a conclusion: a judgement that can be supported by giving reasons. A decision, together with the reasons, makes up a kind of argument. On other occasions, however, we find reasons being given to *explain* a decision: to say why it is, or was, the *right* decision in given circumstances.

It is interesting to note that the reasons for a decision can be given *before or after* it has been made and/or acted upon. A bank manager looks at a company's finances and, because they are in poor shape, concludes – decides – that it would be unwise to lend the company any more money. But she could also say, after having refused the loan, why she refused, giving exactly the same reason. We can call the first of these decision making; the second explanation. Both involve *justifying* the decision.

It is also interesting to note that decisions are *not* necessarily made on the strength of reasons. Sometimes we make 'snap' decisions, act on impulse or on the spur of the moment; or even against better judgement. On occasions we might look at all the reasons for and against some course of action, and convince ourselves that it is better than others, yet still decide to do the exact opposite. There is a difference, therefore, between reasoning to a decision, or making a *rational* decision, and just deciding without good reason to do one thing rather than another.

It is not always wrong, however, to make a snap decision or to act against better judgement. It depends on the circumstances. If nothing hangs on the decision that you make, then there is no need to spend time weighing up the pros and cons. Also there are occasions when there is insufficient time to reason things through: something has to be done, and it is better to do something than nothing, whatever the 'something' may be. Sometimes the rational decision means doing something that is less fun or less exciting. A not-very-talented golfer might go for a near-impossible shot that will probably cost him the hole, rather than a sensible one which may result in winning. On the basis that golf is just a game, and if it is not a serious competition, the decision to gamble is not stupid, even though the odds are against its succeeding.

Reasoned decisions: choices and their consequence

An important part of decision making is judging what does and does not matter: what is and is not important. That itself is a critical judgement, similar to determining the standard of proof that is needed to justify a claim (see Chapter 2.2). When outcomes do matter, and there is time to deliberate, we want a reliable methodology to maximise the chances of making the right choice. That methodology is the topic of this chapter.

The central concepts affecting decision making are *choice* and *consequence*. Obviously, if we want to reach the best decision, we need to be aware of what choices are available. You will remember the fallacy of 'restricting the options' in Chapter 4.7 (page 173). Decision making is a practical reminder of why reasoning can be undermined if all the relevant possibilities are not considered. The argument that if we cannot do X we must do Y is valid only if there is no Z that is as feasible as Y. Although this is a very obvious observation, it is often overlooked in practice.

Consequences are what follow from a decision: the outcomes of actions. In practical terms consequences are what determine whether a decision is a good one or not. This, too, is a very obvious point; but again it is easy to ignore or play down the importance and/or likelihood of some potential outcome, especially when trying to justify a decision already partly made, or favoured more than others.

A familiar example is the scenario of deciding which of various products to buy, particularly when it is a major item like a car, a new bicycle, or computer. It is very easy to let ourselves be persuaded by advertising, or by pre-existing preferences, rather than by predictable consequences. Take the choice between buying a comparatively new and therefore quite expensive car, or an older but much cheaper one. If as a result of buying the newer car you find you have taken on a debt that you can't meet, you may regret the decision. On the other hand, if the older car promptly breaks down and lands you with a massive repair bill, or worse still has to be scrapped, you may wish you had chosen the more expensive but more reliable model.

Expressed in these general terms it seems like a lottery. How can we know in advance which of these possibilities will be the actual outcome? We don't. No one can pretend that decision making, or prediction, is an exact science. But that does not mean it is not a *rational* activity, nor that it cannot be made more reliable by approaching it in a methodical rather than a random way. A sound decision – as opposed to a random choice – can only be made if it is *informed*; and to be informed it must be based on some kind of factual or statistical or quantifiable data.

Assessing consequences

This brings us to two key criteria by which consequences can be critically assessed. The criteria are:

1 probability (likelihood, chance)
2 value (importance, seriousness, cost).

(You will sometimes see the word 'utility' used instead of 'value'. Note also that the value, or utility, of something can be zero, or even *negative*: a liability rather than an asset.)

How do these two concepts figure in rational decision making? The answer, if not already obvious, is as follows.

Firstly (1): since we cannot always be sure what the outcomes of a particular decision will be, the best we can usually do is to estimate how likely they are. In the case of the two cars, we would naturally like to know what the chances are of its developing a serious fault in the foreseeable future. Taking the older car first, the kind of factors we would consider would be not just its age but the mileage it had done, the number of owners it had had, its service record, and so on. We might also want to look at information in an auto magazine, or ask someone with expertise how reliable such makes and models are above a certain age and mileage. We may want to consider the reputation of the seller. If the answers to these questions are all, or mostly, positive, this raises the likelihood of getting – say – three good years of use from the car. (Fewer than that would mean the car had been poor value; more would be a bonus.) If the answers are mostly negative, the chances of this positive outcome would be lowered.

'Raised', 'lowered' and 'likely' are still rather vague notions. Ideally we would want a more precise, *quantifiable* measure of the probabilities. Statistically such figures will exist, and can be found if you are prepared to go to the trouble. Suppose a representative sample of cars of a certain make, age and mileage have been assessed for their reliability, and it turns out that around 60% of them gave their owners three years of trouble-free use, whilst 40% developed one or more serious problems, some irreparable. Now let's suppose that the statistics for the other, newer car in our scenario, were 90% : 10% using the same criteria. Which car would you buy?

Decision time

If you were asked at this point which car is the *better buy*, you would be right to think it was a silly question. Obviously the newer car is the better buy. As that rather over-used saying goes, it is a 'no-brainer' – meaning you don't need any intelligence to work it out. But the question 'Which would you buy?' is a different one and without more information it is unanswerable. Yes, we can estimate from the statistics that the likelihood of getting three years of reliable use from the newer car is 30 percentage points higher than for the older model. But we have no way of placing a measurable value on this. Value was the second of the two criteria for assessing consequences. The most obvious missing information is the *cost* of the two cars respectively, because it is that which is at stake if the one we buy proves faulty. Nor is it just the cost itself that is relevant, but the cost *to the buyer*. If the buyer has lots of money, the relative value is less than for someone on modest income who has to watch what they spend, and will feel the effects of an unfavourable outcome more acutely. It is for this reason that 'importance' is often a more appropriate term to use than 'value' or 'cost'.

So, let's place a value on each car. Let's say that the older car is priced at \$1200, and the newer one at \$4500. We can now pose the question again, only this time with something more concrete to go on. Which of the cars, Old Car or New Car, would you opt for – and why?

Activity

Pause and discuss this question. In purely practical and economic terms, which car is the better buy for someone to whom financial considerations matter significantly?

Commentary and continuation

Not all values and probabilities are quantifiable ones. But in the example we are considering they are quantifiable, which makes the task more objective than it would be otherwise. All we are asking is: Which option, Old Car or New(er) Car, makes the better *economic* sense? We can answer it by setting the cost of each option against the likelihood of a favourable outcome (or the risk of an unfavourable one) and we can express all this in numerical terms. We are defining a favourable outcome as three years of trouble-free use, and an unfavourable outcome as anything less than this. The statistical evidence suggests that there is (up to) a 0.4 probability that the older car will fail within three years, with a worst-case scenario of losing all of the \$1200. The evidence also suggests that there is a 0.1 probability that the newer car will fail, with a worst-case loss of \$4500.

Mathematically this can be expressed as follows:

Older:	$\$1200 \times 0.4 = \$480$
Newer:	$\$4500 \times 0.1 = \$450$
Difference:	\$30

In other words, if I multiply the value (i.e. the cost) of an unfavourable outcome by the chance of its happening, this tells me there are slightly better grounds (statistically) for buying the newer, more expensive car. However, the difference is so small that it does not provide a powerful reason for deciding one way or the other. The conclusion we would draw from this exercise is that there is very little to choose between the two options when viewed in these purely economic terms. This is not so surprising, when we stop to think about it, because by and large you get what you pay for, and the marketplace reflects this: the reduced reliability of an older car is matched by its lower price; conversely the higher price of a newer model is reflected in the likelihood of greater reliability.

The above example is very simple, but it provides us with a model of the way in which consequences bear on decisions. If we wanted to give a more general, verbal account of the factors affecting our decision, we might say something along these lines:

> 'If you buy the cheaper car you run a bigger risk of wasting your money; but then you have less to lose if it lets you down. Buy the newer one, and you've much less chance of wasting your money, but a lot more to lose if the worst does happen.'

Although the numbers have been left out, it is clear that this statement still applies the same basic principle (or methodology) of balancing risk against cost, probability against value.

Quality not quantity

The principle can be seen at work even when we consider a very different kind of scenario, and one in which qualitative rather than quantitative judgements have to be made. Imagine you are on a trekking holiday:

> With 5 km to go to your next camp you come to a river-crossing with wet, slippery rocks. Partway across there is a large gap. You are not confident that you can jump the gap, but the water is slow-moving, so in the likely event of falling short, you will just have to walk the remaining 5 km in wet clothes. There is of course some risk of a minor injury: cut, bruise, strain. You would have to be extremely unlucky for there to be anything more serious, though it is not strictly speaking impossible. Freak accidents do happen: you could slip and crack your head, or break your leg. A rational decision must allow for this, but must be kept in perspective too. The alternative option is a 4 km walk upstream to the nearest bridge, which would add 8 km to an already long day's trek. Basically you have to decide whether the risk of wet clothes, or worse, is a less desirable consequence than the longer hike.

Can you quantify this? Perhaps not with the precision with which we were able to assess monetary values in the previous case, but there is nothing to stop you making qualitative judgements in the same way.

Depending on other relevant factors – the weather, the terrain, how tired you feel, the distance remaining and so on – you might decide that an 80 : 20 chance of getting wet outweighed the certainty of having to trek eight extra and unnecessary kilometres.

But next imagine a similar situation, only this time you are practically certain you can jump the gap. The difference is that this time the river is a raging torrent and there is a large waterfall just below the crossing point. Falling in would carry a serious risk of fatality. In both cases there is a long walk at stake if you decide not to jump. But in the first case the probability of failure is high and the seriousness of a bad outcome low; in the new one the risk is very low and the seriousness of a bad outcome very high. What is the right decision now?

How a real person would decide would depend to a degree on temperament. Some people are natural risk-takers and even thrive on adrenalin; others are naturally cautious. But we are not talking here about feelings or personalities, but about rational choices. Most people would say it was perfectly rational to avoid a long trek at even a high risk of falling into a slow-moving river, but irrational to take even a tiny risk when the consequence of failure could be death. We cannot put fixed numerical values on the seriousness of the chance of death compared with the annoyance of wet clothes, but we can say with justification that a small risk of death outweighs a big risk of wet clothes. Although the actual values, and the ways of expressing them, may differ, the underlying principle of measuring seriousness against likelihood is broadly unchanged. To summarise:

1. Consider the available options.
2. For each option consider the consequences – the pluses and the minuses.
3. For each consequence think 'likelihood versus seriousness'.

Decision trees

Mathematically, consequences can be measured by multiplying the value (importance) of a particular outcome by its probability. (This is basically what we did in the simple case of buying a car.) If all the possible outcomes of a given decision are added up, that gives us an idea of its overall desirability, which can be compared with that of the other available choices, calculated in the same way. A formal, graphical representation of this can be made by means of a decision tree diagram, like those used in problem solving (see Chapter 6.3).

Tree diagrams are used in a range of real-life situations where decisions are influenced by factual data or evidence. We find examples in business, politics, economics, medicine, sport, and many other widely different disciplines. (Watch a baseball coach studying pages of percentages before deciding when or whether to bring on a new pitcher.) Real-life decisions can be highly complex. They can also have very important and far-reaching consequences. If you look up 'decision trees' on the internet, you will find some bewilderingly complicated examples. But the underlying principle is simple, as we have seen.

Here is a fictional, but broadly realistic, scenario. A small energy company, Zenergies, has discovered a deep deposit of shale gas, with unknown commercial potential. The board have to decide whether to proceed with extraction of the gas, at a cost of \$3 million, or abandon the project because it may be unprofitable.

The key factors are the *known* costs and the *possible* returns. The returns, and therefore the possible profits, depend on the size of the gas deposit. Although this is unknown, geologists and market analysts have estimated that on the lowest estimate the gas would have a value of \$2 million. They call this a 'Level C' result. This, of course, would mean a net loss for the company when the exploration costs are subtracted, but the analysts also calculate that the probability of a Level C result is very low. They set it at 0.1 (or 10%). They also claim that there is a similar (10%) probability of a large gas deposit – a 'Level A' result – with a value as high as \$12 million. The most likely prediction, however, is somewhere between: a 'Level B', worth around \$7 million.

If the company abandons the project and looks for a safer venture, there is a second option of putting the extraction rights up for auction, in the hope that a richer company, able to take bigger risks, will want to buy them. Zenergies' accountants have estimated that there is a 40% chance of selling the rights for as much as \$5 million, and a 50% chance of a sale for around \$3 million. (That leaves a slim, 10%, chance that there will be no sale, or an offer so small that selling is not a viable option.)

Activity

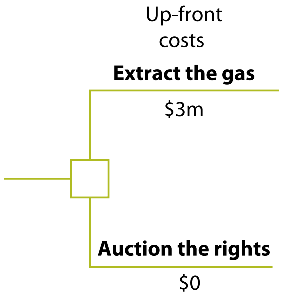
Discuss what the company should do, and why.

Commentary and continuation

Statistically there are big gains to be made, but also significant risks involved. The question is which is most likely, and by how much. It is unlikely, though not impossible, that the yield will be as low as \$2 million, with a consequent loss of \$1 million. That is the worst-case scenario. It is likely to be about \$7 million, with a profit of \$4 million; and it may be as much as \$12 million, with a profit of \$9 million. Compared with this there is the less risky option of selling the rights to extract the gas.

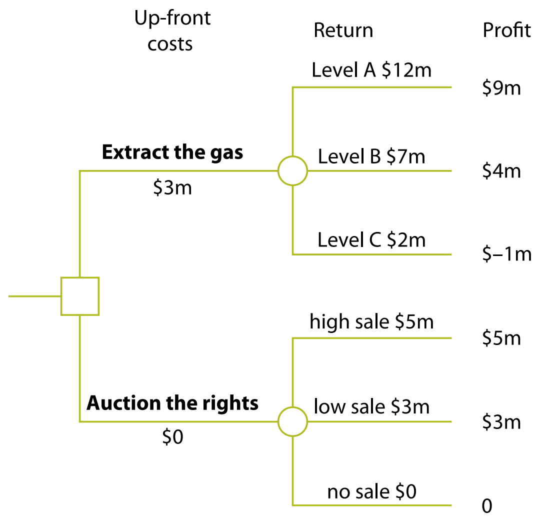
To represent this mathematically we can construct a tree diagram. We start with what is known as a decision node, which by convention is a rectangle, with the choices branching out from it – hence the name 'tree'. At the same time we can write down the known costs of each of these options next to the respective branches.

Step 1



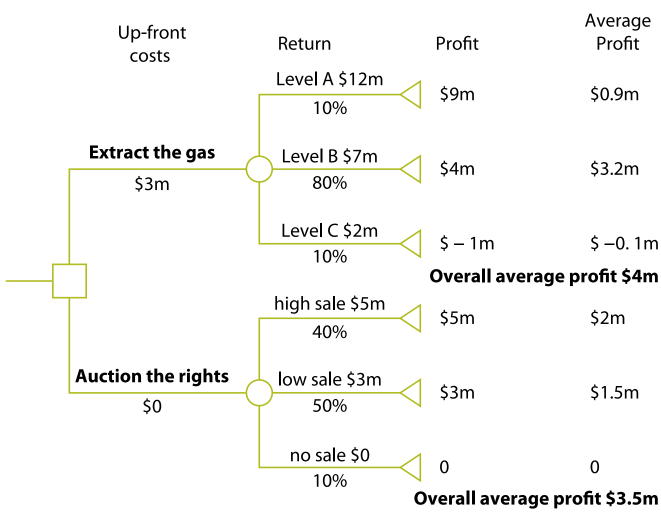
The next step is to add branches for each of the possible outcomes. These branch out from a second type of node, conventionally a circle, called a chance node. We have data for three levels of return, depending on the size of the gas deposit, giving us three levels of possible return, C, B and A. We can then do the same for the three possible levels of return that could come from an auction of the extraction rights.

Step 2



Now all that we have to do is insert the likelihood of each of these outcomes in the form of a percentage, and to multiply the estimated value by its probability. (The probability could be written as a number from 0 to 1, if preferred.) A triangle, or end node, is placed at the end of each branch with the product of the calculation beside it. These are then added together to give the most likely overall outcome of each decision.

Step 3



So what does the exercise tell us? It suggests that going ahead with the project would probably be more profitable than selling by about $500,000. This is not very much, in modern business terms; certainly not enough to make Zenergies' decision an easy one. The board might say that with such small margins it would be better to take the safer course of selling, which at least could not end in a loss. On the other hand it might argue that since the odds look about equal, the risk is worth taking. Calculations like these assist decision making, but they don't guarantee success.

Other factors

As observed earlier, real-life decisions generally involve many more factors than we have considered in this simplified example. Nor are direct financial consequences the only factors which may need to be taken into account. There may be environmental issues at stake. The current way in which shale gas is extracted – a process known as induced hydraulic fracturing, or 'fracking' for short – is believed by some scientists to increase the risk of earthquakes. Public opinion, fear of lawsuits, or even ethical principles may deter Zenergies' board members from getting involved in shale gas. Considerations such as these are the subject of the next chapter.

Summary

- Decision making, on a practical level, can be assisted by assessing consequences.
- Consequences are measured in terms of: (1) their seriousness / importance / value / utility; (2) their likelihood / probability / risk. Judging the impact of consequences on a decision means balancing these two factors against each other.
- Decision trees demonstrate a formal methodology for decision making. They work best when the values and probabilities are quantifiable.

End-of-chapter assignment

Suppose a new team of analysts have reassessed the shale gas deposit in the light of fresh evidence and improved technology. The cost of extraction remains the same, but the team now estimates that there is:

- no danger of a Level C result ($2m return)
- only a 30% probability of a Level B result ($7m return)
- a 40% probability of a Level A result ($12m return)
- a 25% chance of a Level AA result ($24m return)
- a 5% chance of a Level AAA result ($40m return).

A rival company called Ygen has bid $10 million for the extraction rights.

Calculate the new probable returns, using a decision tree if it assists you. Then decide which of the following can most reliably be inferred from the data.

A On economic grounds alone Zenergies should accept Ygen's bid.

B On economic grounds alone Zenergies should decline Ygen's bid and go ahead with extraction.

C It makes no difference economically which decision is taken by Zenergies.

Answers and comments are on page 341.

7.6 Principles

One particular type of claim on which arguments and decisions are often based is an expression of principle. A principle is a *general* claim that, if true, is true under all circumstances. A principle is not something that can be applied selectively.

Activity

Read the following dialogue, preferably aloud with a partner, taking a part each. Then complete the activity that follows.

Carla: Can I borrow your new CD?

Dieter: What do you want it for?

Carla: To make a copy. I'll give it straight back.

Dieter: But it's illegal.

Carla: So what? No one is going to find out.

Dieter: They might. And if they do it's not just you that gets in trouble, it's me as well.

Carla: Dieter, am I hearing this? Do you think the police are going to burst into your room in the middle of the night because they suspect you of lending me a CD? Get real.

Dieter: I am getting real. I work for a shop that sells CDs, or had you forgotten? If I get caught making pirate copies, I'll lose my job.

Carla: But I've told you, you won't get caught. No one will know.

Dieter: *I'll* know.

Carla: You mean you'll inform on yourself!

Dieter: Don't be silly, Carla. I mean I'll know I've done something wrong. I'll be guilty of cheating.

Carla: Cheating! Cheating who?

Dieter: The record company, the song-writers, the band, the retailers . . .

Carla: Oh, them! Do you know how much profit they all make out of people like you and me? If they didn't charge so much, we wouldn't need to copy CDs. They're greedy. And if piracy is denting their profits, good for piracy! Anyway, it's not like I'm walking into a shop and taking something off the shelf.

Dieter: It's still theft. You're helping yourself to something without paying for it. And you are cheating the owners of the copyright out of what is theirs. If it's all right for you to take from them, you can't complain if someone takes something of yours. Remember how you felt when your mobile phone was stolen. Are you now saying that was all right?

Carla: That was different. You know it was. It cost a lot of money to replace. If I copy your CD, some fat-cat bosses are going to lose a fraction of a cent that they won't even miss.

Dieter: Well, then, where do you draw the line? One cent? Twenty? A dollar? If it's OK to take a small amount, it's OK to take a little bit more. Then a

	little bit more, and a little bit more still . . . In the end you'll be saying it's OK to walk into a shop and fill your pockets with anything that takes your fancy – as long as no one finds out.
Carla:	That's shoplifting. And if you really think it's the same as copying one little CD you've got some very mixed-up ideas.
Dieter:	I've got mixed-up ideas?! You're the one –
Carla:	Oh, I'm not listening to any more of your self-righteous rubbish. Keep your CD. I'll borrow one from someone else.

Compare Carla's argument with Dieter's. How would you describe their different standpoints, and the kind of reasons they offer to support their positions? Do you think there is a winner in this argument, and if so who?

Commentary

The main difference is that Dieter's is an argument from principle. At least, it *becomes* an argument from principle as a result of Carla's persistence. At first Dieter simply resists lending the CD on the grounds that it is illegal to make pirate copies and you can get into trouble for it. When Carla points out that there is no risk of being found out, Dieter changes his direction and argues that it is wrong to do it – on principle. He claims that pirating CDs is cheating, and really just the same as any other kind of theft: it makes no difference what the amounts are or who the loser is. Stealing is stealing, whether you take a fraction of a cent from the profits of a huge corporation, or take goods from a shop, or steal someone's mobile phone when they are not looking.

Carla's argument invokes no such principle. She clearly believes that there is a significant difference between copying a CD and committing a serious theft. She even implies that because of the very small loss involved, and the very large incomes of those who incur the loss, that there is some justice served by pirating CDs. It is not that she thinks stealing is all right: she thinks copying a CD is not the same as stealing.

Who you pick as the winner depends on whether you agree with Dieter that this issue is wholly a matter of principle. If it is, then Carla's argument cannot stand up to it: clearly, the pirating of a CD is a form of theft, and Carla is wrong to do it, however negligible the sums are. That is what distinguishes a principle from other kinds of claim. You can't wriggle out of a principle by saying that it applies under one set of circumstances and not under others, especially if the 'other' circumstances are ones that happen to suit you. So, if you agree with Dieter that this is a question of principle, you would really have to say that he wins the argument. If, however, you think that the principle doesn't stretch to 'harmless' actions like copying a CD, then possibly you would say that Carla's argument shows more sense of proportion, and that Dieter's is too extreme and inflexible.

The point to remember, however, is that arguments from principle *are* inflexible. If something really is a principle, then there are no exceptions. You could not have it as a principle that stealing is 'all right sometimes', and that people have to decide when it is and when it isn't all right. You might agree with Carla that it is not the biggest crime in the world to cheat the music industry out of a few cents, but you couldn't defend it on principle. In fact, if you accept that cheating is wrong, and that what you have done is cheating, then you also have to accept that you are in the wrong – even if you think it is a very minor offence.

So how might Carla defend her position? One of her lines of argument is to claim that the companies who make and sell CDs charge an unjustly high price, which to some extent justifies cheating them. This is, in fact, quite a common argument that people bring against big and powerful organisations. It implies that overcharging is itself a form of theft; or if not theft, then at least an abuse of position. As Carla says:

> 'Do you know how much profit they all make out of people like you and me? If they didn't charge so much, we wouldn't need to copy CDs. They're greedy. And if piracy is denting their profits, good for piracy!'

'Two wrongs don't make a right'

The trouble with this argument is that it infringes another principle that many people rightly stand by: the principle that two wrongs don't make a right. Basically this means that if someone takes advantage of you, it doesn't make it right for you to behave in the same way. Of course, we all know of occasions when it seems quite appropriate to say that so-and-so 'asked for it', or 'deserved it', or 'had it coming to him'. Suppose a politician has come to power by spreading malicious lies about her opponents, only to meet her downfall because someone has finally done the same thing to her. You might say with good reason that she 'deserved' the shame and humiliation it caused her. But that would not make it right to publicly tell lies about her.

Spreading a malicious lie is wrong, whichever way you look at it. It is harmful; it is untruthful; and (since it is malicious) it is obviously done with *intent* to do harm. No matter how 'deserved' it may be, it remains a bad thing to do. In fact, by saying that it is 'deserved', you have already made the judgement that the original act was bad. So you can't have it both ways: it can't be a bad thing when one person does it and a good thing when another person does it – whatever the reason. That is what it means to say: 'Two wrongs don't make a right.'

If you accept the principle that two wrongs don't make a right, you can't really accept Carla's defence that the big music companies have 'asked for it' by charging inflated prices. You can sympathise with people who feel that they are being overcharged. But you can't rationally argue that *therefore* cheating is good behaviour.

Differences of degree and differences of kind

There is another line of reasoning used by Carla that we need to consider. Straight after her attack on the music industry she says: 'Anyway, it's not like I'm walking into a shop and taking something off the shelf.' To which Dieter replies: 'It's still theft. You're helping yourself to something without paying for it. And you are cheating the owners of the copyright out of what is theirs.'

Activity

Carefully consider or discuss the question of whether it is still theft. Is there a difference between shoplifting or stealing someone's phone, for example, and infringing the copyright law in the way Carla intends to? And if so, what is the difference?

Commentary

The difference, according to Dieter, is one of *degree*. According to Carla it is a difference in *kind*. If these expressions are not familiar to you, their meaning should soon become clear.

A difference in degree is just a difference that can be measured or counted: for example, degrees of temperature, or degrees of strength, or of intelligence, or of wealth. The list could go on and on. If we ask two people what their earnings are, and find that one receives just a little more than the other, we would call the difference one of degree, not one of kind. If we ask the same two people what they do for a living, and one says he is a doctor, the other says a farmer, that is a difference in kind. There aren't degrees of being a farmer: you either are one or you aren't.

Here is another example. The capital of Canada is situated on the Ottawa river, which not only divides the city in two, but also forms the border between the English-speaking province of Ontario and the French-speaking province of Quebec. Judged on the basis of where you live, you are either an Ontarian or a Quebecer. You are not more of an Ontarian if you live three kilometres from the river than you are if you live one kilometre from the river. In other words, the difference is of kind, not degree. The river draws a line between the two residential areas, and you live in either one or the other.

If we apply this distinction to Dieter's argument we see that he thinks the difference between copying a CD and stealing goods from a shop is just a matter of degree. In effect he says there is no difference, other than the amount that is taken. Petty cheating is the same as stealing – in principle. And on principle it is dishonest to do either.

Carla, by contrast, sees a difference in kind. She fails to come up with any sort of definition that shows how they are different, but she clearly assumes that they are. Comparing the copying of a CD with the theft of her mobile phone, she says: 'That was different. You know it was. It cost a lot of money to replace.' And comparing it with shoplifting: 'If you really think it's the same as copying one little CD you've got some very mixed-up ideas.'

Drawing the line

Dieter's response is a rhetorical question: 'Where do you draw the line? One cent? Twenty? A dollar? If it's OK to take a small amount, it's OK to take a little bit more. Then a little bit more, and a little bit more still . . . In the end you'll be saying it's OK to walk into a shop and fill your pockets with anything that takes your fancy – as long as no one finds out.'

In other words, Dieter sees no difference in principle between the two ends of the scale, because there is no point at which you can draw a line and say, 'This is where petty cheating ends and where real, grown-up stealing begins.'

Who is right? In the strict sense Dieter seems to have a better case. If all that Carla can say is that her mobile phone cost much more than the small amounts she is going to take from the music corporations, and that they can afford it much more than she can, then it looks like a difference of degree and not of kind. And therefore the principle applies. But it is not always as simple as that.

Consider, for example, degrees of wealth. Although there is only a difference of degree between one person's income and another, no one would say that there is therefore no difference between wealth and poverty. Just because we cannot say exactly where one ends and the other begins, it doesn't mean that the adjectives 'wealthy' and 'poor' do not signify differences in kind. Similarly, if an employee takes a paperclip home from work, surely she is a different kind of offender from someone who systematically swindles the company out of millions. Even if our principled friend Dieter would say that they are both taking something that isn't theirs, and are therefore both thieves, no rational person would say that they were in the same league.

And so Carla has a point. Sometimes differences in degree are large enough to become differences in kind. The truth is that we can distinguish between minor offences and serious crimes, just as we can distinguish between the wealthy and the poor. Dieter is right to say that they do differ in degree, but wrong to argue that we can't tell the difference.

The slippery slope

Dieter's argument in fact contains quite a well-known flaw: a version of what is called 'slippery slope' reasoning, which we met in Chapter 4.9. The underlying assumption in a slippery slope argument is that if you accept one conclusion you have to accept another conclusion that is only a little bit different, and so on. But if you do that you eventually have to accept some completely outrageous conclusion. For example, if you tell me that putting one grain of sugar in my tea won't make it taste *noticeably* sweeter than it did with no sugar at all, I would have to agree with you. I would also have to agree that putting two grains of sugar in the tea wouldn't make it taste sweeter than one grain did, and so on. I would even agree that there will not be a single point at which the tea tastes noticeably sweeter than the moment before. The 'logical' conclusion would seem to be that the tea will never taste noticeably sweeter, however much sugar I put in. This is obviously untrue, which means that a string of evidently true premises have led to a false conclusion! This shows that the argument is unsound.

Dieter does something similar by saying that if you accept the premise that a very small-scale offence is fairly harmless, then we have to accept that a slightly more serious one is also fairly harmless, and so on until we end up being stuck with the conclusion that any offence, however serious, is harmless. Stealing a paperclip is not significantly different from a massive fraud!

Note: logically, of course, there is no absolute difference. This creates a puzzle, or paradox, which has been discussed by thinkers since ancient times. It is known as the *sorites* paradox, after the Greek word for a heap or pile. If one or two grains of sand don't make a heap, adding one more won't make a heap. So when does any number of grains of sand become a heap? In reverse, if a man loses one hair from his head he is not instantly bald. How many hairs must he lose before the description is accurate?

Principles vs pragmatics

A more general way of criticising Dieter's reasoning would be to say that he pushes principle too far. He may have right on his side, strictly speaking, but his use of the principle is too heavy-handed. There are further arguments he could have used which might have been more appropriate, and which would have left him looking less 'self-righteous', as Carla calls him when she runs out of more reasoned arguments.

For example, he could have developed the argument that copyright infringement is against the law for good reasons, even if it is not taken as seriously, by most people, as directly stealing goods. If copyright isn't respected, the best singers and songwriters may not find it worthwhile producing records, causing the general quality of musical output to fall. Alternatively, the recording companies may respond by charging even more for their products to cover the costs of fighting lawsuits or researching ways to beat the pirates. Then, the argument would go, everyone suffers because of those who cheat; or, conversely, if people respect the law, everyone gains in the long run. This is similar to the argument against fare-dodgers on public transport, or people who make false insurance claims. It is the law-abiding passengers and policy holders who pay in the end, through higher fares and premiums, not the transport companies or big insurers whom the cheats think they have beaten.

Reasons like these are *pragmatic*, meaning practical or sensible, or leading to a desirable outcome.

Ethical arguments

The issues involved here belong to the subject of ethics. Dieter's argument is an ethical, or moral argument. Ethics is a big subject, and this book is not the place to discuss it in detail. However, there are a few quite basic principles which are relevant to critical thinking, and useful to be aware of, and which are included in some syllabuses.

Activity

Consider the following two lines of argument:

[1] Laws protecting copyright should be respected because it is in everyone's best interest to do so. We all benefit as a result.

[2] Laws protecting copyright should be respected because copyright protects intellectual property, and we are under a moral obligation not to steal anyone's rightful property.

How do these two arguments, for the same conclusion, differ?

Commentary

The first argument gives a pragmatic reason for respecting copyright. It is that we all benefit as a result. The benefits are not listed, but the argument could be reinforced by citing some. For example, the artists get paid in full; the companies make more profits and – if they are ethical too – they pass these on to the consumer in cheaper prices; and so on. If there were not these benefits, or others like them, argument [1] would be empty. Its success depends on there being better consequences when copyright is respected than when it is ignored.

The second argument cuts straight to the principle without considering consequences: we simply have a duty, or obligation, to respect copyright and not to help ourselves to something that is not ours, without payment in return. Even if there were no benefits, this would be the right thing to do. Stealing is wrong in whatever form it takes. This, effectively, is Dieter's argument.

There are two rather grand-sounding terms which distinguish these two forms of argument:

[1] takes a *consequentialist* approach

[2] takes a *deontological* approach.

The first of these is self-explanatory. If the consequences of some act are beneficial, then we say it is a good thing to do. Conversely things which have unwanted consequences are bad. But this raises an awkward question: whose benefit are we talking about? Generally some people benefit more than others from certain actions; some may lose altogether. Habitually selfish people will do what benefits them, or benefits those they want to please. But few people would call that a moral principle. It can be a moral principle only if it benefits more than just one person and his or her chosen group. The net gains, overall, must outweigh the losses.

This is sometimes called the utility principle, or *utilitarianism*. Its most famous exponent was the philosopher Jeremy Bentham. In its simplest form the principle is that we should always act in ways which result in the most happiness and/or least distress for the maximum number of people. It is clearly a defensible principle. For a start it is not selfish: it seeks good outcomes for as many people as possible. You have seen already how it could be applied to the copyright question. If you add up *everyone's* benefits, large and small, you get a better total score if copyright is respected than if the law is routinely broken. But utilitarianism in some contexts can be seen to have worrying side-effects. For example, it may entail that a small minority, or single individual, has to suffer disproportionately for the benefit of the greater majority.

In Chapter 7.4 the utility principle could have been applied to the debate on excessive imprisonment: harsh penalties for a minority arguably make the streets safer for the majority. So on that score an enlarged prison population would be morally justified, even if it meant some people being locked up for longer than they deserve, or on suspicion rather than proven guilt. A few pay the price of preventing the supposedly worse consequences of a crime wave. Many if not most governments operate this principle to some degree.

By contrast deontology – or the ethics of duty – involves judging acts not by their consequences, but by their own value. There are certain norms of behaviour which we have a duty to abide by: not stealing, not killing, not falsely imprisoning, not lying, and so on. If it really is right or wrong to do something, it doesn't matter what the outcomes are: if they are right we must do them and if they are wrong we must not. If Carla could show that making pirate copies of CDs and DVDs did actually benefit more people than it harmed, it could be justified in consequentialist terms. But a deontologist would argue, as Dieter does, that it is still wrong. A deontologist might also argue that imprisoning someone for a day longer than his or her crime warrants is wrong, however many crimes it prevents.

The great German philosopher Immanuel Kant is the name most strongly associated with this ethical system. He argued that an act can be justified only if it applies universally. It cannot be all right to lie occasionally, for a good cause or to help someone out. You might do it for such a reason, but it would be wrong nonetheless. If harming someone is immoral, it doesn't suddenly become acceptable if it is done to save or benefit another. One of Kant's most famous maxims was that we must never use people as a means to an end. Over-punishing offenders cannot be justified on the grounds that it makes others feel safer.

Considering principles

In your own essays and discussions there are often opportunities to introduce values and principles alongside, or as objections to, practical arguments. Think of the decision-making arguments in the previous chapter. They were almost all economic arguments, affecting only the financial interests of one company. No principles were involved other than the principle that decisions should be made that give the best chance of a healthy profit. But there are other issues surrounding the energy industry which are not purely profit-oriented. Is it right to plunder yet another of the Earth's fossil fuel reserves? Is it safe to drill down into these deep reservoirs of gas, with the risk of setting off tremors – even very minor ones? Should we take such risks? Will the natural environment suffer in any way? Is our first obligation to the health of the planet, or to the world economy, which is dependent on future energy supplies?

Questions like these, which contain words such as 'should', 'ought', 'right' or 'wrong', 'obligation', 'duty', 'must', are questions of principle. In almost any extended critical assignment that you are given, principles and values will be relevant at some level. The basic question that you will need to address is whether the principles involved are important and powerful enough to override the practical reasons for or against your conclusion. Make a point, therefore, of always asking yourself, at the planning stage of an essay, which principles are relevant to the question, and include them in your thinking. It is a dimension of critical reasoning that is often overlooked by students, and can cost valuable marks.

Statements of principle

What distinguishes a statement of principle from other kinds of claim is its generality. It must apply to more than the single particular case you are considering in order to count as a principle. It is not a statement of principle that it would be wrong for Carla to copy Dieter's CD. It is a statement of principle that it is wrong to steal, which makes it wrong to copy CDs without permission, which makes it wrong for Carla to copy Dieter's CD. The argument is downwards, from the overriding principle, to more specific principles, and eventually to the particular case. If citing a very general principle as a reason for some conclusion or decision, you may need to explain how it applies to the particular case, for example by explaining the sense in which infringement of copyright is a form of stealing.

Not all principles are ethical principles, although typically they are. There are legal principles for example. It is a legal principle, in most developed countries, that a person is innocent until proven guilty. However, it is often argued that many or most legal principles are themselves derived from ethical principles. There are natural scientific principles, too, which have nothing to do with right and wrong. Sometimes they are called 'laws', as in the laws of nature or of physics. The old saying 'What goes up must come down' is a law of nature or scientific principle, now more generally explained by the law (principle) of universal gravity. There are even logical principles. You met the paramount one in Chapter 7.2: the rule that, to be valid, an argument must never have true premises and a false conclusion.

Arguing from principle

The general nature of a principle makes it a powerful premise in an argument. If one accepts the truth of a statement of principle, then it applies to any particular case which falls under it. When evaluating an argument from principle the two questions are:

1. Does the principle really hold for every case?
2. Is the case in question really an example of the principle?

Summary

- A principle is a general claim that, if true, is true under all circumstances.
- Principles, especially those that are generally accepted, make strong premises. However, if principles are stretched too far, the effect can be to weaken the argument.
- Principles are especially relevant in ethical arguments. There are two broad types of ethical arguments: those that centre on the consequences, and those that do not.

End-of-chapter assignment

Find or construct an argument that has a principle for its main premise. Consider some of the counter-arguments that could be raised against it.

7.7 An argument under the microscope

This chapter, and the next and final chapter, take a slightly different form from previous ones. They are working chapters, and their function is to give you the opportunity to bring together the skills and understanding that you have gathered from the earlier ones.

On the next page is a piece of journalistic text. It addresses an issue that raises its ugly head every four years, and which has done so for over a century. The article asks what the fuss is all about, and offers a no-nonsense solution.

Activity

This first activity consists of five questions focusing on analysis of the argument. Suggested answers are then given in the commentary that follows. You can look at the answers after each question; or, if you prefer, treat the whole activity as a structured exercise and consult the commentary afterwards.

1 What is the overall conclusion of the argument?
2 Reread the first paragraph. How would you describe its style, or tone, and how does the author achieve it? What effect does the first paragraph have, and how might it influence the reader?
3 The author offers various reasons for choosing a permanent site in Greece. Identify:
   a a pragmatic reason
   b a principle.
4 In paragraph 2 the author makes the explicit assumption that money and national pride should have nothing to do with the debate. What *implicit* (i.e. unstated) assumption does she also make – and is it warranted?
5 What is the function of paragraph 3?

Commentary

This commentary is in the form of specimen answers to each of the five questions. Compare them with your own answers, and revise yours if you find you need to.

1 The conclusion comes at the end of the second paragraph. It is the whole of the sentence: ‘There is only one sensible and justifiable place to have the Games, and that is Athens, the capital of Greece – this time, next time and always.’

   If you choose (or you are asked) to paraphrase your answer, rather than lifting it word-for-word from the text, remember that you must still give the conclusion *in full*. This is not a simple, one-part claim: there are several elements to it. It is not enough to say that the Games should be in Athens. The actual conclusion is that there is only one ‘sensible’ *and* ‘justifiable’ location for the Games, *and* that Athens should become the permanent site.

   The need to capture the whole of the conclusion becomes clear when you move on to evaluating the supporting

DOC 1

WHOSE ARE THE OLYMPICS ANYWAY?

It's that time again when everyone starts running and jumping with excitement over the Olympic Games. I don't mean running and jumping on the athletics track, either. This is not sports fever, it's politics. Nor is the excitement about the next Olympics, but the one after the one after next. Yes, it's that time when the International Olympic Committee (IOC) decides which city will stage the world's biggest sporting extravaganza eight years from now.

So why all the fuss? One simple answer – money. National pride may have something to do with it, too; but money is the real driving force. However, the truth is that neither money nor national pride should play any part in the debate. The Olympic Games rightfully belong in one country, Greece, for the very good reason that Greece is where the Olympic Games were invented and where the name comes from. This is not a political or an economic issue. There is only one sensible and justifiable place to have the Games, and that is Athens, the capital of Greece – this time, next time and always.

Of course some of the competing nations will ask why all the benefits of holding the Olympics, especially the huge revenue that they allegedly generate, should always go to one country. Alternatively, it is often pointed out that hosting the Olympics is a risky business, requiring massive investment to make it a success. A country the size of Greece cannot be expected to bear those costs every four years. Sharing the burdens, as well as the benefits of the Games, is the fair and proper way to do it, with the richer countries being the safest choice.

But these self-seeking and contradictory arguments are precisely what you would expect to hear from big business. Of course those with most to gain from the building programmes needed to provide the facilities and infrastructures will say that the present system is the most workable. It is a view that gets much of its support from North America and Western Europe, which have had more than their fair share of playing host to the Games. The economic case for retaining the existing arrangement is therefore flawed from the start.

The Olympic Games, properly understood, are an international movement dedicated to friendship and peace worldwide. The Games are no nation's property. The countries that take part should pay for the Games according to their wealth, with the poorest nations contributing least and benefiting most. That approach alone would reflect the true Olympic ideal. But it is only possible if the Games have a permanent site.

Last but not least, there is a practical but compelling reason for returning the Olympic Games to their ancient roots, and that is the ever-present threat of terrorism. Everyone who is old enough remembers the tragic events that marred the 20th (Munich) Olympiad in 1972. Today the Games are an obvious target for an atrocity that would put 1972 in the shade, especially if the games are seen, rightly or wrongly, as a symbol of US world dominance. By holding the Games in the historical location, rather than a different national capital every four years, the issue becomes depoliticised, and the danger of a terrorist attack is greatly reduced.

Janet Sender

argument. If the reasons supported only the claim that it was justifiable, without saying why it was also sensible, the argument would be unsound, because it would be incomplete. Similarly, if the argument didn't establish that one permanent site was more justifiable and sensible than a different site each time, again the reasoning would be inadequate.

'Athens should be the site of the next Olympic Games' would not be a sufficiently accurate and inclusive answer. 'The Greek capital should be the permanent home of the Olympic Games; no other solution can be justified or makes sense' would be fine.

2 The first paragraph is introductory. It sets up the context for the argument as a whole without giving either the conclusion or any supporting reasons.

You could describe the author's style of writing in the first paragraph in a number of ways: for example, humorous, sarcastic, scornful, dismissive, pejorative. It is achieved by means of phrases like: 'running and jumping . . . (not) on the athletics track', which makes the excitement she is talking about seem childish; and the word 'extravaganza', which suggests that the current Olympic Games are over-glamorised. Janet Sender is probably trying to make the reader feel that the 'fuss' over the hosting of the Games is all a bit unnecessary, and a bit ridiculous. If it works, this can have the effect of 'softening the reader up' for the reasoned argument that is to come. In other words it is a rhetorical device, rather than straightforward reasoning.

When you are evaluating an argument it is important to look out for features of persuasive writing and distinguish between them and the reasoning itself. By the 'reasoning itself', we mean the underlying claims, which could be expressed in any number of different ways. By the writing 'style', we mean the claims as they are expressed in a particular piece of text, complete with any emotional appeals, sarcastic touches, colourful phrases and so on. In paragraph 1 there are plenty; so it is more than just an introduction.

3 **a** One pragmatic reason the author offers is that a permanent site will, arguably, reduce the threat of terrorism by depoliticising the Games. This would obviously be of practical benefit to athletes and spectators, and even to the organisers whose profits would be affected if the threat of a terrorist attack deterred people from attending the Games. The inclusion of the word 'practical' in the text marks this as a pragmatic reason.

b By contrast there is no obvious practical benefit behind the argument that Greece is where the Games were invented and where the name comes from. We are told that the Games are 'rightfully' the property of Greece for these historical reasons, and for that reason alone they should be held there. The general principle underlying this strand of reasoning is that the inventor or originator of something has a moral and/or legal ownership of it. This applies not just to this particular context, but to authors, artists, explorers and others – in fact any person or group who can claim to have discovered, created or invented something.

4 There is clearly an assumption in paragraph 2 that historical reasons should play a part in the debate. Without this assumption the conclusion just doesn't follow. Another way to say this is that there is a missing premise. If the author wanted to spell this premise out it would have to be something like: 'The issue is a historical one.' Merely saying that it is not political or economic does not establish that it is historical.

5 Paragraph 3 is a counter-argument. You may remember from Unit 4.8 that the strategy of anticipating a counter-argument – i.e. setting it up and then knocking it down – is a common argument strategy. That is clearly what the author is doing here.

Activity

The next five questions are evaluative. Again there are suggested answers in the commentary that follows.

6 Is the charge of being 'contradictory' (paragraph 4) a fair assessment of the counter-argument?

7 Paragraph 4 is a response to the counter-argument (a counter-counter-argument). What is your evaluation of it?

8 In paragraph 5, the author writes: 'The Games are no nation's property.' Is this claim contradicted elsewhere in the passage? If so, does the contradiction weaken the argument to any extent?

9 Bearing in mind exactly what the conclusion of the argument is, does the argument adequately support it?

10 'The ancient Olympic Games were for competitors from all over Greece. The modern Olympics are for competitors from all over the world.' If true, what impact does this observation have on the argument?

Commentary

6 You can see what the author means when she brands the counter-argument 'contradictory'. The way she has set up the counter-argument, it looks as if those who support it want it both ways: they want to say no one country should get the profits, and that no one country should have to bear the costs. But you could equally say that the counter-argument is simply looking at two possible outcomes, and claiming that either way it would be unfair. Thus the charge of contradiction does not really stick.

7 Paragraph 4 is a very weak response. In fact it is an example of a classic fallacy, known as an *ad hominem* argument, which was introduced in Chapter 4.9. *Argumentum ad hominem* means the argument is directed at the person who holds the belief or makes the claim, rather than at the argument itself. It may be perfectly true that the economic argument for the present system does suit big business, and that it finds favour in North America in particular. But that does not make the argument bad; and it certainly doesn't make it flawed, as the author concludes. The flaw is much more evident in the author's argument than in the counter-argument she unsuccessfully tries to demolish.

8 This is a tricky question because it appears to have a very straightforward answer. In paragraph 2 the author says, quite plainly, that the Olympic Games 'rightfully belong in one country, Greece'. This looks like a blatant contradiction of the later statement that they are the property of no one nation. And if it is a clear contradiction, it also appears to be a serious flaw in the reasoning. For surely, if the Games *do* belong to no single nation, then the present system of rotating the host country would seem the right one, and giving it permanently to Greece, as the author proposes, would seem to fly in the face of one of her premises.

But is it as blatant a contradiction as it seems? Not necessarily. You could defend the argument by clarifying what exactly is meant by the words 'belong' and 'property'. 'Property' suggests ownership or possession. If the Games were the out-and-out property of one country, that country would presumably have the right to do as it pleased with them for its own benefit – choose the time and place, make the rules, keep all the profits. But the author is not saying anything as extreme as that. Just belonging somewhere is not the same as being a possession, especially when followed by the word 'in' rather than 'to'. This may seem a small detail, but accurate analysis often depends on small detail: a word here, a phrase there. It might be perfectly reasonable to say that the Olympic Games belong *in* Greece, but that they are not the property *of* Greece. In other words the Olympics remain the property of all the nations that compete in them, but historically their rightful location is Greece. It could even be pointed out that 'Greece' means a region of the world where the ancient Olympics took place, not the modern country called Greece; and that is all that it means to say the Games belong there. Under that interpretation there is no contradiction. Of course, an opponent of the argument could just as reasonably reply that this is a quibble: 'belonging in', 'belonging to', 'property of' all mean the same when it comes to deciding whether the Games should be in one country or shared around. The author cannot have it both ways. If the Games don't belong to Greece, they don't belong in Greece either, and that is all there is to it.

Which of these is the right interpretation is ultimately for you to decide. In doing so, remember the *principle of charity* (Chapter 2.7, page 52). The way to apply it is to ask: Would the author have made these two statements if she thought they contradicted each other? The answer is, almost certainly, no. Why would she? She doesn't need to say that the Olympic Games belong to Greece in the sense of being Greece's property. It is quite sufficient for her argument to say that the Olympic Games belong in their traditional location. And she has no need to deny that they also belong to the whole world, and should be governed by the International Olympic Committee as they are now. You would only insist on the worst interpretation if you wanted to find fault with the argument, which is a form of prejudgement. Under the principle of charity you assume the best interpretation; then if you still want to make negative criticisms, or present counter-arguments, they will be fair comment. Another way to put all this would be to say that accusing the author of a contradiction in this context would be a rather cheap objection. It would be like picking someone up for a slip of the tongue, or for saying something that they never really meant. In this respect it has some resemblance to the 'straw man' argument that you saw in Chapter 4.9.

9 No, the argument does not adequately support the conclusion. The conclusion is a very strongly worded claim that the *only* sensible and justifiable place for the Games is Athens – now and *always*. Words like 'only' and 'always' require equally strong premises to underpin them. The weakness of the author's argument is that she has not eliminated all the possible alternatives, or looked at all the possible counter-arguments. She could reasonably conclude that there is some justification for a permanent site in Athens, and that it makes good sense. For that she has provided some support. She has not come near to establishing that this is the only acceptable conclusion. You could say that this imbalance between reasons and conclusion amounts to flawed reasoning. Alternatively, you could describe it as a serious weakness. Either way, the right evaluation of the argument is that it falls short of its purpose.

10 The observation may be considered fairly damaging. The historical argument is an important part of the author's case: she is using the fact that the Games were originally in Greece to support the conclusion that they should always be in Greece. If someone objects that the original Games were located in the region from which all the athletes came, and that this is no longer the case, that would be grounds for arguing that circumstances have significantly changed. However, the objection is not a fatal one. There are still defences that could be made: for example, the age of air travel has made the world a much smaller place. It probably takes less time to fly from Sydney to Athens than it took to travel from Sparta to Athens in ancient times. Therefore the place where the athletes come from is not really relevant to the case for a single permanent site.

Critical questions

Questions like the ones you have been answering provide a useful way of focusing on the key features of an argument, which is why such questions are included in thinking skills examination papers. The questions were quite tough, and required some serious critical thinking on your part. But they are also a bit of a luxury because they guide you in your analysis and evaluation. When you are confronted with real arguments – on television, in print, or just conversation – you have to know what questions to ask, as well as how to answer them.

Many of the questions above are worth remembering because they, or questions very like them, will be relevant to most arguments, not just to this one. You will almost always need to ask questions such as: What is the main conclusion? Are there any missing premises (assumptions)? Are there contradictions? Are the reasons strong enough to support the conclusion? What use does the author make of persuasive language, emotion, or popular appeal?

End-of-chapter assignment

Find an argument in a recent newspaper, or on the internet, and make a copy of it. Using some or all of the questions you were asked in this chapter, produce a list of questions based on the text you have chosen.
You can then either answer the questions yourself or exchange texts with a fellow student and answer each other's.

7.8 Critical writing

In the previous chapter you studied a single document and answered some specific questions on it. These tested your skills in analysis and evaluation.

In this unit we introduce a further skill that you need to develop for more advanced levels of critical thinking. It is the skill of bringing together information, evidence and opinion from a range of different sources to support an argument or conclusion. This is known as *synthesis*. In higher-level thinking skills examinations it is assessed by means of an extended piece of writing that you have to plan and construct yourself.

Synthesis requires first selecting and organising material that is relevant to a particular task. In the activity that follows, the task is to extend the debate on the Olympic Games that arises from Janet Sender's article on page 296 (Doc 1). The questions she was addressing were fairly narrow ones: 'Whose are the Olympics?' and 'Where should the Olympics take place?' Her conclusion was that they should be held permanently in Athens, and her reasoning was largely historical and political. Among its weaknesses was the fact that she gave little in the way of factual information, examples or evidence to support her claims.

The three new documents that follow are largely informative. Not every part of them is directly relevant to the debate, and there is more information in them than you would need for an argument on the specific question of where the Olympic Games should be held. Nor do the additional documents enter directly into the debate, although they contribute to it.

Read the new documents now, and if necessary reread Janet Sender's argument too. Do this quickly, to get an overview of the material, rather than trying to take in every detail. Look out for the parts of the texts that are most relevant to the debate. Then move on to the activity that follows.

DOC 2

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The history of the Olympic Games – ancient and modern

Introduction

The modern Olympic Games are always hosted by a city – not by a country. The first Olympic Games of the Modern Era were hosted by Athens (Greece). The Olympic Games were hosted by Beijing (China) in 2008 and by London (UK) in 2012.

Host cities and the calendar known as the Olympiad

The ancient Olympic Games were always in the same place – **Olympia** – a sacred city in western Greece known as **Elis**. The Games were a religious event, a festival that honored the Greek God **Zeus**. The ancient Games were hosted by the Elians who were the guardians of the sanctuary to Zeus. They tried – and succeeded for a few hundred years – to be *neutral*, that is, unallied to other Greek city-states, similar to modern-day Switzerland. But in the fifth century BCE (or BC) they allied themselves with Sparta and warred against their neighbors. The Elians lost control of the sanctuary to the Spartans, then to other Greek city-states, then finally to the conquering Romans. In 80 BCE the Roman general Sulla moved the Olympic Games to Rome and only a single race for boys was held at Olympia, the *stade race*. But then Sulla died and the next Games returned to Olympia in 76 BCE.

The ancient Olympic Games and the modern Olympic Games are **quadrennial**, meaning they are held every four years. This four year period of time is known as an **Olympiad**. To the ancient Greeks an Olympiad was their calendar, a way of designating time. However, this calendar was not used by every Greek city-state and there is great difficulty in studying ancient history because of the calendar and attempts to 'date' things. There was no accurate dating system in the ancient era and every civilization used a different calendar system. There were calendars for the Babylonians, Hebrews, Greeks and many others. The one thing the civilizations had in common was that they were conquered by the Romans. Julius Caesar created the *Julian calendar* in 46 BCE. Our modern calendar, known as the *Gregorian calendar,* is based upon revisions to the Julian calendar, made and instituted by the Catholic Church in 1582 by Pope Gregory XIII. This becomes an issue when trying to date the ancient Greek Olympiads from 776 BCE, which was 'year one' of the first Olympiad.

Just as in ancient Greece, the modern Olympic Games are held every four years at the beginning of the Olympiad. The First Modern Olympiad began in 1896 when Pierre de Coubertin revived the Olympic Games and they were held in Athens.

During the early years of the modern Olympic movement there was a disagreement over who should host the Olympic Games. The Greek government wanted the Games in Athens permanently while Pierre de Coubertin, the French 'founder' of the modern Olympic Games, wanted them to rotate around the world to major sporting cities. So the Olympic Games of the second Olympiad were held in Paris, France, and the Games of the third Olympiad were in St Louis, Missouri, USA. The Greeks went ahead and scheduled their own Olympic Games in 1906, a tenth anniversary celebration of the 1896 Games. At that time these Games were considered 'official,' in spite of the calendar – not being a **quadrennial** event. From a historical perspective, the 1906 Olympic Games must always be included in Olympic record-keeping. They happened – they cannot be ignored. However, they are not called the Games of the fourth Olympiad, because these

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were held in 1908 in London, UK. Is this confusing you? Don't worry – it was confusing to everyone back then too. The Greek government did not hold any future Olympic celebrations in the 20th century because they were too expensive. The modern Games have continued to be hosted in cities around the world. The Greeks tried to get the 1996 Games because it was the centennial (100th birthday) of the modern Olympic Games, but the host was Atlanta (USA). However, in 2004 the Games did return to Athens.

The ancient Greeks celebrated their Olympic Games without interruption for over 1000 years, from 776 BCE to 261 CE (AD). Quite remarkable! After the year 261 it is unknown what happened to the Games because records are lost. Actually, they abruptly end – probably because there was an invasion by the *Heruli*, a barbarian tribe from the coast of what is now southern Russia. Invading in a fleet of 500 ships they devastated Byzantium and Greece before the Romans forced them to retreat. The Elians erected defensive walls with towers around the Olympic sanctuary, but we have no evidence that any celebrations were held.

There must have been something still happening at Olympia. It must have remained a religious site to the Greek god Zeus. We know this because in 391 CE the Roman emperor **Theodosius I**, who accepted the new religion known as Christianity, outlawed all pagan religious festivals throughout the Roman Empire. It is believed that the last Games held at Olympia were in 393 CE. By 395 CE it is known that the great statue of Zeus, one of the Seven Wonders of the Ancient World, had been removed to a Roman palace in Constantinople, the capital of the Eastern Empire, where it was destroyed in a fire in 462 CE. But evidence has been found that there were even later Olympic Games until 425 CE. In 426 CE **Theodosius II**, grandson of Theodosius I, issued an edict to destroy all pagan temples. The temple of Zeus at Olympia was burned to the ground. Rome itself had already been sacked by Allaric and the Visigoths in 410 CE. The 'Dark Ages' had begun. Keep in mind that all these dates have been calculated by historians who have tried to use mathematics to 'date' events.

Almost 1500 years had passed when Pierre de Coubertin, of France, organized a revival of the ancient Olympic Games and the first celebration was held in Athens, Greece in 1896. In the first 50 years of the modern Games they have been cancelled three times. In 1916 the Games were cancelled due to World War I and in both 1940 and 1944 they were cancelled due to World War II. In 1980 the United States led a boycott of the Moscow Olympics and in 1984 the Soviets retaliated and led a boycott of the Los Angeles Olympics. Wars, politics, corruption – these are forces that affect the modern Games as much as they affected the ancient Games. They influence who is the host of the Games and they impact on the calendar. Although an Olympiad cannot be cancelled because it is a period of time, the Games of an Olympiad can be cancelled. Below is a list of the host cities of the modern Olympic Games with Arabic numbers being used instead of Roman numerals (21st Olympiad instead of XXI Olympiad).

Host cities of the modern Olympic Games

1896	1st Olympiad	**Athens**, Greece
1900	2nd Olympiad	**Paris**, France
1904	3rd Olympiad	**St Louis**, Missouri, USA

1906	3rd Olympiad, year 3	**Athens**, Greece (sometimes called the 'interim Games')
1908	4th Olympiad	**London**, UK
1912	5th Olympiad	**Stockholm**, Sweden
1916	6th Olympiad	cancelled because of World War I (scheduled for Berlin, Germany)
1920	7th Olympiad	**Antwerp**, Belgium
1924	8th Olympiad	**Paris**, France
1928	9th Olympiad	**Amsterdam**, The Netherlands
1932	10th Olympiad	**Los Angeles**, California, USA
1936	11th Olympiad	**Berlin**, Germany
1940	12th Olympiad	cancelled because of World War II (scheduled for Tokyo, Japan; then re-scheduled for Helsinki, Finland and cancelled a second time)
1944	13th Olympiad	cancelled because of World War II (London considered, but war continued)
1948	14th Olympiad	**London**, UK
1952	15th Olympiad	**Helsinki**, Finland
1956	16th Olympiad	**Melbourne**, Australia / **Stockholm**, Sweden (horses were not permitted to be imported into Australia so the equestrian events were in Stockholm)
1960	17th Olympiad	**Rome**, Italy
1964	18th Olympiad	**Tokyo**, Japan
1968	19th Olympiad	**Mexico City**, Mexico
1972	20th Olympiad	**Munich**, Germany
1976	21st Olympiad	**Montreal**, Canada
1980	22nd Olympiad	**Moscow**, Soviet Union (USSR)
1984	23rd Olympiad	**Los Angeles**, California, USA
1988	24th Olympiad	**Seoul**, South Korea
1992	25th Olympiad	**Barcelona**, Spain
1996	26th Olympiad	**Atlanta**, Georgia, USA
2000	27th Olympiad	**Sydney**, Australia
2004	28th Olympiad	**Athens**, Greece
2008	29th Olympiad	**Beijing**, China
2012	30th Olympiad	**London**, UK
2016	31st Olympiad	**Rio de Janeiro**, Brazil

DOC 3

File Edit View History Bookmarks Help

Hello, I found your site very informative. I was wondering if you could tell me how a city is chosen to host the Olympics? Thanx so much. Sarah, New York

Response: Cities (not countries) are chosen by the International Olympic Committee (IOC) to host the Olympic Games. There is a formal procedure that must be followed by all the cities desiring to host the Games. This process is called the 'bid'. Cities bid to host the Games. Usually a city will form a committee or a commission to prepare the bid. The bid is like a book that gives details such as sports facilities, hotels and restaurants available, transport network, and many other aspects of holding such a large function as the Olympic Games. The bid must answer questions like 'Where would the 10,000 athletes stay?' 'What sports facilities exist now, and what would have to be built?' 'What public transport exists and could it handle huge crowds for all the sports?' 'Who would finance the cost of the Games?' Hundreds of other questions need to be answered. The 'bid book' is then submitted to the IOC for review. It used to be that the entire IOC would visit all the cities that submitted bids. Six years prior to the Olympic Games in question, the IOC schedules a meeting and votes for a host city.

However, a problem has come up in this bid procedure – corruption. Salt Lake City, the host of the Winter Olympic Games in 2002, apparently earned some votes through bribing members of the IOC. The IOC has always had a very good reputation for honesty and character, but this reputation was tarnished through the bribery scandal. The IOC investigated its members and kicked some of them out. Others were warned. Then they changed their procedure. Now only a small group of the IOC (there are over 100 members) visits each candidate city, along with selected international experts and athletes, and they report back to the rest of the membership.

Are the Winter Olympics the same as the Summer Olympics? Christos, Melbourne, Australia

Response: They are in a different time and place. Obviously they have to be somewhere with snow. And there were no ancient winter Olympics either, because the Greeks hadn't invented skiing! Otherwise, yes, the same rules and procedures apply for choosing a venue for the Winter Games.

What do the Olympic rings mean and where did they come from? Ariel, Santiago, Chile

Response: The Olympic rings were designed by Pierre de Coubertin around 1913. Contrary to popular belief, the Olympic rings never existed in ancient Greece. This myth was created by an error published in a popular book about the ancient Olympic Games in the 1960s. The authors did not know what they were looking at and concluded (wrongly) that the Olympic rings were 3000 years old. In Greece, inside the ancient stadium at Delphi, there was a stone engraved (actually not engraved, but in relief) with the five Olympic rings. This stone was actually created by German stonemasons in 1936 for Leni Riefenstahl's film *Olympia*. Many authors have perpetuated this myth by including this information in their ancient Olympic chapters. But it's wrong! Just goes to show that not all historians know what they are talking about.

The Olympic rings designed by Pierre de Coubertin actually represented the first five Olympic Games (1896, 1900, 1904, 1908, 1912) when they were first used in 1913. Later they came to represent five continents. The three rings on the top row are blue, black and red with the two rings in the lower row yellow and green. When all are connected, the order of colours is: blue, yellow, black, green, red.

DOC 4

THE OLYMPIC CHARTER

Rights over the Olympic Games and Olympic properties

The Olympic Games are the exclusive property of the IOC (International Olympic Committee) which owns all rights and data relating thereto, in particular, and without limitation, all rights relating to their organisation, exploitation, broadcasting, recording, representation, reproduction, access and dissemination in any form and by any means or mechanism whatsoever, whether now existing or developed in the future.

The IOC

The IOC is an international non-governmental not-for-profit organisation. Members of the IOC represent and promote the interests of the IOC and of the Olympic Movement in their countries and in the organisations of the Olympic Movement in which they serve.

Activity

You have been asked to speak in a debate on the future of the Olympic Games to an audience of athletes, business people, sports fans and others who are concerned that the Games are falling into disrepute and straying from their original ideals. The previous speaker in the debate was Janet Sender: your job is either to support or to oppose her proposal.

Go through all the items again, including Janet Sender's article, and note down, or highlight, any points that you feel to be relevant to the argument you will be constructing. There is no need to sort or organise it at this stage: just compile a rough list of points that you could make, and others that you may need to respond to.

Commentary

Selection

Before you can begin to select and organise relevant material from sources like these, you need to be very clear what you are doing it for – the task or assignment that has led you to the documents in the first place.

There are some parts of the texts that are of obvious significance, and some that are just as obviously irrelevant. For instance, if you are going to take up Janet Sender's argument that the interests of western Europe and the USA have been served much better than those of other nations, especially in the developing world, the table of host cities would clearly be useful evidence. Even if you decide to oppose the previous speaker, you would need to anticipate the accusation that the West has had the lion's share of the Olympic cake. Hence the data in the table is relevant whether it will strengthen your conclusion or challenge it.

The list of points you select will usually be a mixture of fact and opinion, and it is important not to confuse them. Generally speaking, facts are neutral, unlike opinions or judgements. A footprint in the snow is just that, an outline of a foot, unless or until some significance is attached to it. If it turns out to have the same pattern as the boots owned by a defendant in a murder trial, the footprint becomes a piece of evidence. Similarly, the fact that the Olympic Games were held in Atlanta in 1996 is a neutral fact unless, for instance, it is coupled with the fact that they had been held in Los Angeles only 12 years earlier and that both these cities are in the USA. Something else to remember is that the same piece of evidence can be a 'two-edged sword'. It may, depending on how it is presented and interpreted, give support to either side in an argument.

Take, for example, the information about the earliest records of the Games, in the second paragraph of Doc 2:

> The ancient Olympic Games were always in the same place – **Olympia** – a sacred city in western Greece known as **Elis**. The Games were a religious event, a festival that honored the Greek God **Zeus**. The ancient Games were hosted by the Elians who were the guardians of the sanctuary to Zeus . . .

This could be presented straightforwardly as support for the claim that the Olympics belong in Greece on historical and geographical grounds. This is very much Janet Sender's take on the facts. But the few lines of information could just as well be used to argue that the ancient Games were nothing like the modern ones, and the only connection between them is that they share the same name. Therefore the event we call the Olympics now is no more a Greek invention than it is French or American or Chinese.

At this stage in the exercise you should try, as far as possible, to keep an open mind, even if you do sympathise with one side more than the other. Critical thinking should never be reduced to a game in which the sole purpose is to 'win' an argument. The primary object of learning to think critically is to make good judgements, not to score points. The right approach is to look at the facts and ask yourself: 'What conclusion does this information most strongly support?' Not: 'How can this information be manipulated to back up my already-formed opinion?'

The points you select from the documents may be similar to the bullet points below – though the exact way in which you make notes is up to you and your tutor to develop. And if you are writing them in an exam, they can be even more abbreviated, as only you need to understand them. All the same, don't rush the reading and note-making stages of the exercise: the time you spend reading, thinking and planning will save you time when you come to writing your finished essay.

Doc 1 – argument

- conclusion: should be permanent venue in Greece
- reasons: historical right / present system driven by money / would depoliticise games / lessen terrorist threat
- evaluation: contradictory in parts

Doc 2 – historical

early OGs held at Olympia – religious festival – hosted by Elians (neutral but later allied and hostile); moved to Rome in 80 BC, then back

- took place every four years and were like a calendar
- lasted 1000 yrs! then 1500 years passed before games restored
- records, especially dates unreliable / different calendars
- modern games – Coubertin – Frenchman; disagreement and confusion at first
- Games affected by wars, politics and corruption

Data table

shows most Games held in Europe or N. America

Doc 3 – internet discussion site

- complex bidding system
- IOC then decide
- open to corruption e.g. Salt Lake City winter games
- Olympic rings mean the five continents / designed in modern times

Doc 4 – official statement

Games belong entirely to IOC, a not-for-profit NGO

You may have left out some of these points and you may have included others. But it is hoped that your list will have been similar. Clearly, many of the notes above are relevant and could be used by one side or the other in the debate.

Notice how this exercise has condensed several passages, some of which were quite long, into a handful of bullet points. You may need to go back to the documents later to find specific details, but mostly you can now work from the notes in planning and writing your speech.

Inference

Once you have selected the relevant points from the documents, the next task is to decide what can be inferred from them, both individually and collectively. You must also decide what *cannot* be inferred, so that you do not jump to conclusions. Take the table of host cities in Doc 2. As raw data it just tells you each of the venues of the Summer Olympics in modern times. But the data supports a number of quite striking facts. For instance, the modern Games have never been held in Africa; and Asia and South America are also clearly under-represented in the table. You can count up yourself how many times the Games have been in Europe.

Facts like these are simply a matter of data extraction, which is treated in more detail and complexity in the problem-solving sections of this book. If someone had wished to make the point about the unequal distribution of the host cities, they could have presented the data in other ways, e.g. percentages or pie charts. But here there are no points being made overtly. The inferences are left to the reader to draw, and that is what you must do.

Drawing direct, factual conclusions from the data is one thing. Making further inferences and value judgements on the basis of the data is another, and you must do it with care. It would be a safe enough observation to say that the international spirit of the Olympic Games has not been reflected in the choice of host cities. It would not be a safe conclusion to say that there has been favouritism and corruption in the IOC. You would need evidence of a different kind altogether to go that far. The most you could infer in that direction is that the obvious imbalance towards certain regions of the world raises questions about favouritism, and that this is not good for the reputation of the Olympic movement, whether it is founded or not.

Synthesis

We now come to the final and most demanding part of the exercise: drawing together the various pieces of information, inference and opinion to make a cogent argument for one side or the other. This is the part we call 'synthesis'.

Activity

2 Review the points you have listed showing what you consider the most relevant items in the documents. (You may want to add or delete some after comparing your list with the one suggested above, but if you are happy with yours, then use it.) Look in particular for links between the points, or natural ways to group them. There are a number of different ways to do this: highlighting, numbering, drawing connecting lines, and so on.

Commentary

Not wanting to do the work for you, just one example of the kind of links that can be made is shown below. It follows up on the inferences we drew earlier from the data in Doc 2. Three points are drawn together as being relevant to the question of favouritism – or worse – in the selection process. The student has highlighted them and made a brief note as to the possible connection between them: (1) the data adds to any suspicion there may be about corruption; and (2) even if there is no corruption there is something wrong with a movement that embraces five continents but usually excludes three of them from the honour of holding the Games. Developing these themes would provide a substantial paragraph or section in the student's eventual essay.

Decision time: 'resolving the dilemma'

All worthwhile arguments have two sides to them. An argument with only one side – or an argument to which there is no reply – may exist but is hardly worth making. An argument for something that is already a known fact would fall into this category. It is uninteresting.

Interesting arguments, on the other hand, present us with dilemmas. A dilemma is a difficult choice. It is difficult either because there are good reasons for either side of the argument; or because, whichever choice you make, there are some unwanted consequences. Therefore you will often hear people talk about the 'horns' of a dilemma: if you avoid one horn, there is another waiting for you!

The choices for the Olympic movement – and for you now that you are involved in the debate – are whether it would be better to keep to the present system of rotating the Games at different venues, with all the problems and criticisms that gives rise to; or to opt for one permanent site and risk angering some member countries who want their turn to host the event. The dilemma is that whichever the IOC decides, it will not please everyone. The dilemma is compounded by the fact that there is no third way. This is a case where the options are restricted to two, so there is no fallacy in arguing that if one is not chosen,

shows most Games held in Europe or N. America

Doc 3 – internet discussion site

- complex bidding system
- IOC then decide
- open to corruption e.g. Salt Lake City winter games
- Olympic rings mean the five continents / designed in modern times

adds to suspicion

so what does symbol mean?

then it has to be the other. You have to hold the Games somewhere, or not hold them at all.

When faced with a dilemma, you just have to make a decision, or duck it and get caught on both horns. Reaching such a decision, and satisfying yourself or others that it is the right decision, is what is meant by 'resolving the dilemma'.

Shortly you will have to make a decision about which side you support, based on what you know from reading the texts. You will have to justify your decision by giving reasons why it is the better of the two choices.

Summary

Synthesis, of the kind just described, not only involves drawing together information, it also means drawing together your *skills*, the skills you have been acquiring and practising throughout this course.

In the assignment you are about to complete, you will find yourself calling on them all: analysing, evaluating, inferring, justifying, explaining, and developing further argument.

End-of-chapter assignment

Write the speech that you will give to the audience of athletes, business people, sports fans and others who are concerned that the Olympic Games are falling into disrepute and straying from their original ideals. The previous speaker in the debate was Janet Sender: your job is either to support or to oppose her proposal.

Base your argument on the four documents you have worked on, Docs 1–4. It is important that you make reference to them. This is not a test of your own wider knowledge of the subject, but you may research further if you wish.

Answers to assignments

This section contains answers and commentary for the end-of-chapter assignments.

Cambridge International Examinations bears no responsibility for the example answers to questions taken from its past question papers which are contained in this publication.

2.1 Claims, assertions, statements

1 A fact is a true claim. There is no such thing as a false fact, but many false claims are made.

2 'Assertion' and 'claim' are very close in meaning. The difference is in when you use them. 'Assertion' is a bit stronger and more emphatic; it is more *active*. A claim may be asserted, but we would not naturally say that an assertion was claimed.

3 *Variable responses*

4 A theory is perhaps more firmly established than a hypothesis; a conjecture is more tentative than either. Guessing and speculating are very similar, and can be quite random, or made without much thought. But all five words have some meaning in common.

5 An allegation is a claim that is open to challenge. An accusation is an allegation usually made against someone; it is normally negative or disapproving. Insinuations are allegations, but the word also suggests something a bit sly or suggestive, rather than direct and open. Confirmation is agreement or approval of some claim already made. A denial is an assertion that something is *not so*. A verdict is a decision or judgement: for example, a 'guilty' verdict, or acquittal, in a legal trial.

6 Claims are presented as expressions of truth, yet they are *not* always true. Critical thinking is a way of being as sure as possible about which claims to believe, and which to question or mistrust. Also, arguments consist of claims: reasons, conclusions, etc.

2.2 Judging claims

1, 2 *Variable responses*

3 The first is stronger, because it sets a precise date for the predicted extinction. It could easily turn out to be unfounded. The second claim would still be justified even if polar bears live on for centuries, provided there is *some* threat now to their existence.

2.3 Argument

1 There are several conclusions which could be drawn from this passage. But there is one obvious point to which it seems to be leading: that minor crimes are as serious as or more serious than traffic offences (despite the consequences). A plausible answer to the question could be that the police should not neglect minor crime; or perhaps even make it the priority. Note that you do not have to agree with the conclusion or the resulting argument. You are looking for a claim which the passage appears to support.

2–4 *Variable responses*

2.4 Identifying arguments

1 B is the only argument out of the three passages. Its conclusion is that the public should not expect the safety of drugs to be guaranteed by animal testing. We can see that the next two sentences express reasons for making this claim. The clue is the phrase 'These examples show that . . .', which could be understood as 'It follows that . . .' or just 'So . . .' In neither of the other passages is

there a point at which inserting such a connective would make sense.

2 The second sentence is the best expression of the conclusion: 'The machines *are* to blame.' (It would not be altogether wrong, however, to select the first sentence.)

3 *Variable responses*

2.5 Analysing arguments

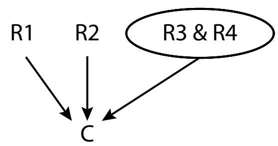
a

R1 Bottled water is meant to be safe but there have been several health alerts.

R2 Bottled water costs a lot.

R3 Tap water is just as good; and tap water is free. (*This could be two separate reasons*.)

C People shouldn't be fooled into buying bottled mineral water.

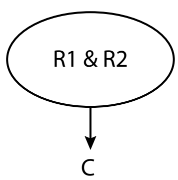


b

R1 Drugs can make the difference between winning gold and winning nothing.

R2 The rewards are so huge . . . that the risk will seem worth taking.

C There will always be some athletes who will give way to the temptation.



It is because R1 and R2 are *both* true that the conclusion follows. If drugs did *not* make a difference, or if the rewards did *not* make the risk worth taking, there would not be the same temptation. So R1 and R2 are *interdependent*.

c There are four reasons, all closely interdependent:

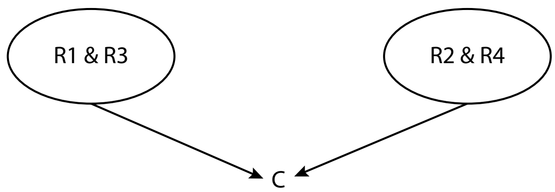
R1 No sport should be allowed in which the prime object is to injure an opponent.

R2 No sport should be allowed in which the spectators enjoy seeing competitors inflict physical harm on each other.

R3 What boxers have to do, in order to win matches, is to batter their opponents.

R4 Boxers do this in front of large, bloodthirsty crowds.

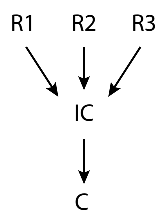
C Boxing should be one of the first sports to be outlawed.



Alternatively R3 and R4 could be reduced to one premise; but then it is quite hard to show the structure. The deeper analysis is more precise. (Note that there is some room for variation in the details of analysis.)

2.6 Complex arguments

1 E.g.



2 The map and accompanying images add weight to the argument by emphasising and/or specifying/quantifying the claim made by R3. It could alternatively be understood as *evidence* for R3, showing what a small proportion of countries

drive on the left. This would make changing from left to right a simpler procedure than right to left, thus adding support to the conclusion.

3 **a** *Context: Recently the operators of a cruise liner were fined $18m for dumping oil and other hazardous waste at sea. This may seem substantial, but . . .*

R1 In the same year the ship earned profits of $340m.
R2 The company could well afford the fine.
R3 Dumping saved them the considerable expense of storing and legally disposing of the waste.

C1 (IC from R1–R3) Emptying their tanks into the ocean was probably a risk worth taking.
R4 In the last decade only a handful of companies have been fined.
R5 Every year there are unsuccessful attempts to prosecute.

C2 (IC) Dumping is not much of a risk.
R6 The oceans of the world are in danger of becoming open sewers.

C (main) We must give the authorities greater powers and demand that they use them.

The two intermediate conclusions, together with R6, are given as reasons why the authorities ought to have and use greater powers.

b *Context: Scientists have discovered some three-million-year-old leaves preserved in the ice (at the South Pole).*

R1 The leaves are so undamaged, and preserved in such fine detail, that they could not have been carried there by wind or sea.

C1/IC They can only be from trees that once grew there.
R2 The leaves belong to a species of beech tree that grows only in warm or temperate regions.
R3 Beeches do not evolve quickly enough to adapt to changes in climate.

C The South Pole must once have been much warmer than it is today.

4 Various possible analyses, e.g.

R1 Grunting is a natural, unstoppable accompaniment to sudden effort.
R2 Some women can control grunting, others can't.
R3 Some men grunt almost as much as the women.

IC Making women play tennis in near-silence would place an unfair handicap on some but not on others.

C Grunting should not be banned (in tennis).

Note: R3 may be said to be a side issue that does not really contribute to the main argument, which is about women. On that interpretation it could be omitted.

2.7 Conclusions

1 The correct selection is C. Note that C is actually a conjunction of two sentences, one recommending the abolition of charging, the other recommending an alternative solution. Neither of these is a reason for the other: they are like parallel claims, or two sides of the same coin; and they both follow from the other claims that are made.
Distracters: A is introductory; B is one of the reasons (premises); D is not stated at all. On a casual reading the last sentence might be mistaken for the conclusion, but it is actually a premise.

2 The correct selection is A. The argument begins halfway through, after 'But . . .' It states the conclusion first, then gives reasons to support it, including the intermediate conclusion that differing fares are the only way the system can work. *Distracters:* B is the intermediate conclusion, and therefore a premise; C and D are part of the introductory information which provides the target for the main argument; E is one of the reasons which supports the intermediate conclusion.

3 The correct selection is A. The actual sentence that states the conclusion is 'This is nonsense,' but when you are asked to express the conclusion, you obviously need to say what 'This' is. 'This' refers to the target claim, 'We must be carnivores,' as A correctly includes. *Distracters:* B would be a premise, if it were correctly interpreted. The actual claim in the passage is that these foods are the natural diet of our closest relatives in the animal kingdom; C and D are premises; E is implied in the introductory sentence.

2.8 Reasons

1 This is open to debate. Some linguists and logicians flatly deny that an argument can have a question as its conclusion, unless it is a question which is obviously rhetorical, and has the meaning of a statement. But this question really looks like a genuine one: it is not saying either that the Red Sox can win or that they can't. So this is a chance for students to develop their own philosophical arguments. One line of reasoning you might consider is that the text gives a *reason* for asking the question. However, does that make it an argument or an explanation? Good luck!

2 *Variable responses*

3 Grammatically the premises are not declarative sentences. One is an imperative, the other a rhetorical question. In standard form the argument could be (e.g.):

R Statistically it can be seen that crime has been rising.

IC The current soft approach to crime is not working.

C If the courts don't get back to zero-tolerance, we face defeat in the war on crime.

4 *Variable responses*

5 This is a difficult passage, although it is short. The author is plainly approving of random stop-and-search powers for the police, but does not say so in as many words. It could therefore be said that this is an argument with an implicit rather than an explicit conclusion. Or you could say it was not an argument for that reason. There is another possible interpretation, which is that the last sentence is the conclusion. But what is it actually saying? Why would opponents of the bill be helping the guilty? This is an exercise in interpretation, which is why it is a more challenging task.

2.9 Assumptions

1 a A is clearly assumed. C is possibly implied, but it is not key to the argument; not necessary. The argument could still be sound if Raisa did not like novels much either, but just didn't hate them. B is interesting. It need not be assumed: Raisa may love mountain-climbing, but hate reading about it for one reason or another.

b A and C are both assumed, and for similar reasons. To meet the conditions Nashida would have had to suffer as a result of the changes, and have left for that reason. D is also assumed because it would have to be the case that Nashida was forced to accept the changes, i.e. had no choice. B does not have to be assumed because

Nashida is not claiming she has been unfairly dismissed.

c C is obviously assumed. A is not. If it is read carefully it should be clear that the argument would stand, i.e. the conclusion would follow, even if there had been no intention to entice children to drink alcohol. It would still be right to ban alcopops if this had been an unintended consequence of adding sweetener. D, likewise, is not necessary for the argument.
B is the interesting one. You could say it was implied in a way. You could say that there would be no need to make drinks sweet if children liked alcohol anyway. But it isn't really key to the argument: children might like the taste of alcohol, but like it more if it is sweet. If you selected B as well as C, it is not obviously wrong; but it is debatable.

2 Crucially this argument assumes that if information is unregulated and/or there is freedom of information, that is a bad thing. There are other assumptions beside this, but without this one, or something equivalent, the argument definitely fails.

3 *Variable responses* (You could try writing an argument that made no implicit assumptions at all.)

4 *Variable responses*

2.10 Flaws and fallacies

1 a The answer is B. The flaw is false cause, or cause–correlation fallacy.

b A and less obviously B both weaken the argument by suggesting that the causal connection could be the reverse: that success makes the workers less happy (because they are less well cared for in the case of B). That undermines the conclusion that making workers unhappy will lead to success. C does not weaken the argument. If anything it strengthens it.

2 a The fallacy could be described as over-generalising from the particular, or from inadequate, anecdotal evidence. But equally it could be described as a false cause, in the sense that lack of exercise did not necessarily cause Farrah Lavallier to have a long life. (She might have had a long life despite not because of it, or for some other reason.)

b B is the answer. It exposes the second of the fallacies described above, by suggesting a genetic explanation for Farrah's longevity: nothing to do with saving her energy.

3 The graphs would give little or no support to the conclusion. The conclusion is very general, whereas the data in the graphs concerns one city and one online supplier. To argue on this basis would commit the fallacy of generalising from a single case; or of assuming that the city and the supplier were representative or typical. Even if the assumed correlation were supported by the graph, it would still not follow that the games were a causal factor in the increased crime.

4 *Ongoing project*

3.1 What do we mean by a 'problem'?

1, 2 *Variable responses*

3 The key here is to be systematic: did you look at *all* the possibilities? Could you find ways to save time, for example by eliminating some orders which leap large distances on each leg of the journey?

4 a The answer is three. If the first two you pick out are of different colours (the 'worst-case scenario'), the third must match one of them.

b The answer is two. As for the situation above, if the first two are different, the third must match one of them.

c The answer is nine. The first eight you pull out could all be black; the ninth must then be blue so you will have one of each.

d The answer is eight. As above, the first eight you take out could all be the same.

e The answer is ten (note the difference from 4a).The first eight you take out could all be black. You would then need to take out two more to get a blue pair.

3.2 How do we solve problems?

1 The efficiency (in km/litre) is distance driven divided by petrol used. The calculations may be approximated as shown in brackets.

In order the cars are:

Riviera: 8 km/litre
Roamer: 8.8 km/litre (just under 9)
Stella: 9.375 km/litre (just under 10)
Montevideo: 12 km/litre
Carousel: 14.375 km/litre (over 14)

2 The object is to find how many candidates still have a chance of winning. We can do this by transferring the votes from each candidate who drops out to the next lowest. (This is the maximum number of votes that the second lowest-placed candidate could receive after the withdrawal of the bottom-placed candidate.) As the bottom candidate is withdrawn each time, we would then get the following results:

	Original	**After first withdrawal**	**After second withdrawal**
Patel	323	323	323
Brown	211	211	211
Walshe	157	157	157
Ndelo	83	83	158
Macpherson	54	75	
Gonzalez	21		

At this stage, either Walshe or Ndelo could be withdrawn, depending on the actual distribution of the votes of the lower candidates. The one who survives could go on to receive Brown's vote and win, so four candidates can still win.

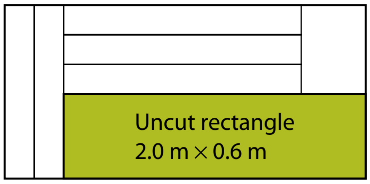
3 This is a problem where we need to work backwards. If we look at each dish in turn, we can find out its timing so that it is ready at 7 p.m.

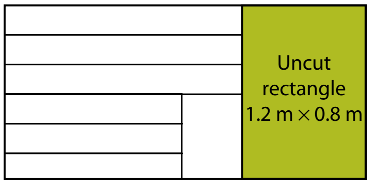
Chicken: 15 minutes rest after 2 hours of cooking. Turn on oven 15 minutes before starting to cook.
Rice: 15 minutes cooking after 30 minutes soaking.
Broccoli: 5 minutes cooking after 5 minutes preparation.
Sauce: 15 minutes cooking after 10 minutes preparation.

Working out each event time and putting them in order, we have:

Turn on oven	4.30 p.m.
Put in chicken	4.45 p.m.
Soak rice	6.15 p.m.
Prepare sauce	6.35 p.m.
Cook rice	6.45 p.m.
Cook sauce	6.45 p.m.
Remove chicken from oven	6.45 p.m.
Prepare broccoli	6.50 p.m.
Cook broccoli	6.55 p.m.
Eat	7.00 p.m.

4 As the length of the shelves is 1.6 m, they must be cut lengthwise from the sheet of wood. The 1.2 m side-pieces can be cut either way. This leaves only two reasonable options. The left-hand one clearly leads to the larger uncut rectangle (in area).





3.3 Selecting and using information

1. This graph can be drawn as either a bar chart or a pie chart.
2. In 1984, vinyl single sales were 44% of 170 million, or 74.8 million; in 1994, they were 26% of 234 million, or 60.8 million. So A is correct – they fell by 14 million.
3. The five teams played each other once, so there were ten games. The maximum total number of points scored if each resulted in a win for one of the teams would have been 30. The actual total number of points scored was 26. In each drawn game a total of two points is scored (i.e. one less than in a game with a winner), so there must have been four drawn games.
4. Each shelf requires 30 mm gap, 210 mm for books and 20 mm for the shelf thickness, or 260 mm in total. The available gap is 2.5 m less 300 mm (as the bottom shelf must not be too close to the ground), or 2200 mm.
   A maximum of 8 shelves at 260 mm total can be fitted into 2200 mm.

3.4 Processing data

1. I buy three items at a total of $110. If I deduct the least expensive ($30) before the discount, I pay $80, with no discount. If I get the discount first, the reduction is $33, making the bill $77. However, I then expect to get the least expensive item free. After discount this will have cost $21 (70% of $30), so my bill will be reduced to $56. Did you remember to reduce the price of the least expensive item rather than subtracting a further $30?
2. Sylvia's total time for the first 5 laps is 5 × 73 = 365 seconds. The time she is trying to beat is 14 minutes 35 seconds or 875 seconds, so she must run within 875 – 365 = 510 seconds for the last 7.5 laps, or 68 seconds (1 minute 8 seconds) per lap.
3. The savoury pancakes come in these types: egg; ham; tomato; egg and ham; egg and tomato; ham and tomato; and egg, ham and tomato – seven in total.
   The sweet pancakes come in three types (orange, lemon or strawberry) times two toppings (cream or ice cream), making six in total.
   The number of combinations sold by the stall is 13.
4. The monthly contract will cost me $30. Texts are free but I will have to pay for 25 minutes of calls at 10¢ per minute, an additional $2.50. The total is $32.50.
   'Pay as you go' costs me 30¢ per minute for 100 minutes of calls ($30) plus 60 text messages at 10¢ each ($6), a total of $36 per month.
   The monthly contract is better by $3.50.

3.5 Finding methods of solution

1. The difference in length between the two pieces of chain is 8 cm and this corresponds to the effective length of 10 links. The effective length of each link is 0.8 cm but, because they overlap, the actual length of each link is 1.2 cm (we need to add twice the metal thickness as can be seen in the diagram). The 34.2 cm length has a 1 cm fitting at the end, so without this it would be 33.2 cm long. This length is made up of one full link length (1.2 cm) plus a number of effective link lengths (0.8 cm); you can also see this from

the diagram. So the number of links is

$$\frac{1+(33.2-\ 1.2)}{0.8}=41$$

The 26.2 cm length would be 25.2 cm without the end fitting, so the number of links is

$$\frac{1+(25.2-1.2)}{0.8}=31$$

The total number of links is 72.

2 The plane's velocity from Los Angeles to Mumbai is $\frac{14{,}000}{22}$ km/hr (636 km/hr). From Mumbai to Los Angeles it is $\frac{14{,}000}{17}$ km/hr (824 km/hr). The wind velocity is half the difference (it adds to the velocity one way and subtracts in the other), $\frac{188}{2}$ or 94 km/hr.

3 Lighthouse 1 flashes at 0, 11, 22 etc. seconds from the beginning. Lighthouse 2 flashes at 0, 3, 7, 17, 20, 24, 34 etc. It is possible to list all the flashes of both until we find a coincidence. Otherwise, we can look at the various flashes of lighthouse 2. The first flash repeats every 17, so would coincide with the 11-second cycle at 187 seconds. The others are offset by 3 and 7 seconds from this (i.e. they repeat every multiple of 17 plus 3 and 7; the next ones are at 37 and 41 seconds), so we are looking for a multiple of 17 which is smaller than a multiple of 11 by 3 or 7. Looking at 17, 34, 51, 68, none work. However, 85 is 3 less than 88, so the two lighthouses coincide after 88 seconds. What is the next coincidence?

4 This appears to be a Venn diagram problem, and one could be used to solve it. However, there is an easier analysis. If we add the number with neither (5) to the number with a dog (13) and the number with a panda (12), we get 30. There are only 23 children in the class, so the difference (7) must be the overlap, or those with both a dog and a panda. You might like to draw a Venn or Carroll diagram to show all the subdivisions.

3.6 Solving problems by searching

1 There are a large number of ways of paying, but clearly 1 adult + 1 senior citizen + 2 children (total $32) comes to more than the family ticket ($30), so some combination using a family ticket must be used.

  Family ticket ($30)
+ 2 extra children ($10)
= $40

  Single-adult family ticket ($20)
+ extra senior citizen ($5)
+ 2 extra children ($10)
= $35.

The latter is the best option.

2 The options to search involve dividing the books as: 7, 5 + 2 or 4 + 3. (6 + 1 would be silly.) The prices, respectively, are $3.20, $2.15 and $2.10. The last of these is the best.

3 One can start listing the piles systematically:

20 × 5¢
16 × 5¢ + 1 × 20¢
12 × 5¢ + 2 × 20¢

continuing to 5 × 20¢; thus there are piles containing 0 to 5 × 20¢ coins, or 6 piles in total.

4 **a** There are two ways to approach this: to list all possible scores, and to look at the make-up of the scores listed. The first is probably safer, but more time-consuming.

28 is 6 correct and 1 wrong:
(6 × 5) – (1 × 2)
18 is 4 correct, 1 wrong and 2 unanswered: (4 × 5) – (1 × 2) + (2 × 0)
16 is 4 correct, 2 wrong and 1 unanswered: (4 × 5) – (2 × 2) + (1 × 0)
12 cannot be done
–1 is 1 correct, 3 wrong and 3 unanswered: (1 × 5) – (3 × 2) + (3 × 0).

So the Kool Kats score is incorrect.

b This requires all the scores to be listed. Other impossible scores are: 34, 33, 32, 31, 29, 27, 26, 24, 22, 19, 17, –9, –11, –13.

3.7 Recognising patterns

1 *Variable responses*

2 The numbers of my birth date could be from 01 to 31 (these are then reversed). The numbers of my birth month could be from 01 to 12 (these are also reversed). We can then look at the options in turn:

A with the respective parts reversed becomes 23 12 – this is possible (23 December).

B becomes 05 06 (5 June).

C becomes 11 14 (impossible – the month cannot be more than 12).

D becomes 12 12 (12 December).

E becomes 21 09 (21 September).

So C is the only impossible number.

3 The first table shows the situation after four of the six matches have been played. The Britons have drawn two games, so the matches B vs D and B vs S must have been draws. The Normans have lost one of their games, and this must have been to the Danes. The remaining game already played must have involved the Normans beating the Saxons. This means the two games to be played are B vs N and D vs S. There are nine possible combinations of results. The final points totals in each case are shown in the table below.

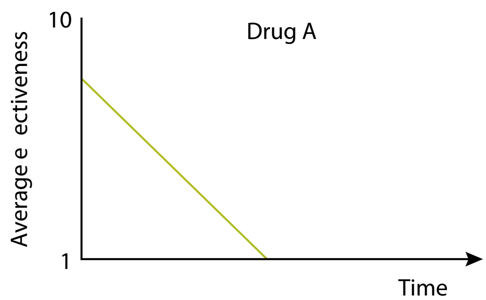
All of the final points situations given in the table on page 105 are possible except D.

This question could also have been done backwards: looking at each option given and seeing whether that combination was possible.

4 This is a question where elimination of clearly incorrect answers will help. Both B and D give a price of over $50 for one poster, so cannot be possible. Choosing between A and C, we note that C is much higher for small numbers, costing over $100 for three posters, whilst A is only $90 for three. So C is correct.

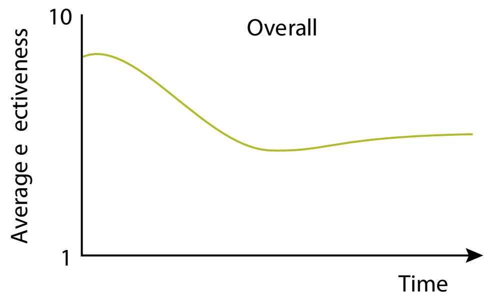
3.8 Hypotheses, reasons, explanations and inference

1 The effectiveness of drug A, allowing for the gradual withdrawal, is shown in the first graph.



	B wins / D wins	B draws / D wins	B loses / D wins	B wins / D draws	B draws / D draws	B loses / D draws	B wins / D loses	B draws / D loses	B loses / D loses
B	5	3	2	5	3	2	5	3	2
D	7	7	7	5	5	5	4	4	4
N	3	4	6	3	4	6	3	4	6
S	1	1	1	2	2	2	4	4	4

The overall effectiveness will be the sum of the first graph and that for drug B. It will appear as in the second graph.



The sizes and positions of the peak and dip will depend on the exact values for the original two curves.

2 Whilst this could be solved with a Venn or Carroll diagram, the problem is simple enough not to require either. Since the percentages studying French and German add to 115%, there must be at least a 15% overlap so D is true. Looking at the alternatives:

A It is possible that the 30% not studying French are also part of the 55% not studying German.

B There is no reason to assume that $\frac{2}{3}$ of those who study German (30% of all students) coincide with the 30% who do not study French.

C This comes from subtracting the 45 from the 70. This number has no meaning other than being the maximum percentage who could study French but not German.

3 C is the correct answer. The only firm deductions that can be made are:

- A dollar has a value between that of an orange and a lemon (but it is impossible to tell which is higher and which is lower, so neither A nor D can be correct).
- The value of a grapefruit is one orange plus one dollar (so C is correct and both an orange and a dollar must be worth less than a grapefruit).
- We can say nothing about the value of a lemon relative to a grapefruit.

4 The information we have directly is:

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
L	x					x	
M	Y	x	Y	x	x	Y	Y
N			x		x	Y	
O				Y		Y	

(We are given Moses' three days off, so he must work the other four. Liam is off on Saturday, so the other three work).

From the table, we can see that Liam and Orla must work on Friday.

Orla can then not work Wednesday or Sunday or she would work four consecutive days.

Therefore, Liam and Nadila must work on Sunday and Liam on Wednesday. We can fill in the table as far as shown below.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
L	x		Y		Y	x	Y
M	Y	x	Y	x	x	Y	Y
N			x		x	Y	Y
O			x	Y	Y	Y	x

We can now see that Nadila cannot work both Monday and Tuesday or it would be four in a row, so she must work Thursday, and Liam must work Tuesday. This leaves the only remaining flexibility that Nadila and Orla must each work one of

Monday and Tuesday, but neither can work both.

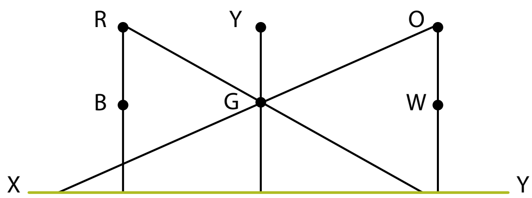
The only one of the options possible is B: Nadila could work on Sunday and Monday.

3.9 Spatial reasoning

1 The letters should look as follows:

2 *Variable responses*

3 It is best to draw lines on the diagram to show the points on the route from X to Y where the view of the flagpoles changes:

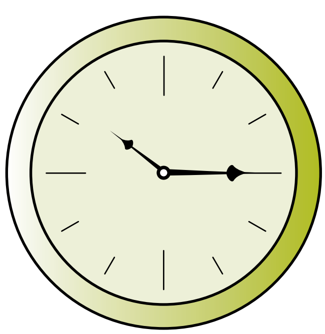


As you walk from left to right, the orders will be:

RBYOGW
RBYGOW
BRYGOW
BRGYOW
BGRYOW
BGRYWO

There are six different orders in total.

4 The clock represented conventionally will look as follows:



The time is 10.15; D is correct.

5 Only D works. If you have doubts, try them.

6 B is correct. Again, you can try it to make sure.

3.10 Necessity and sufficiency

1 I have a free choice where I put the numbers 1 to 3 (but none may go opposite a number I have already marked). Number 4 must go on one of the two faces remaining that do not contain 1 to 3 or are not opposite any of them. However, it can go on either of these two remaining faces, so I can number four faces before I am left with no choice.

2 We are looking for the highest total that can be made in only one way. The total number of pieces of fruit is 5 times the number of pear bags ($5p$) plus 3 times the number of banana bags ($3b$). We can list the total ($5p + 3b$) for various numbers of each kind of bag or we can see which numbers, successively, are possible. If we look at successive combinations (assuming there is at least one of each type of bag), we find that the first sum that can be made in two ways is 23, i.e. $(5 \times 4) + (3 \times 1)$ and $(5 \times 1) + (6 \times 3)$. 24 is unique, as is 25. 26 can be made in two ways. We need to carry on looking until we find three successive numbers that can be made in two ways – these turn out to be 31, 32 and 33. Subsequently, any higher number can be made in two ways by adding extra 3s to these. Thus the highest number that cannot be made in two ways is 30: 3 bags of pears and 5 bags of bananas. This is the number that George must have had in stock.

3 We know Kuldip had 12 coins with a different number of each; with the number of 5¢ more than the number of 2¢, which was more than the number of 1¢. All the options can now be listed:

1¢	2¢	5¢	Total
1	2	9	50¢
1	3	8	47¢
1	4	7	44¢
1	5	6	41¢
2	3	7	43¢
2	4	6	40¢
3	4	5	36¢

We need to look at each of A–D in turn.

A is not sufficient information as two options have three 2¢ coins.

B is not sufficient as two totals are multiples of 10¢.

C identifies a unique combination: in the second to last row, 5¢ coins add to 30¢ which is ¾ of the total, so this is the correct answer.

D is not sufficient because two options (rows 3 and 5) have two more 5¢ coins than 1¢ and 2¢ together.

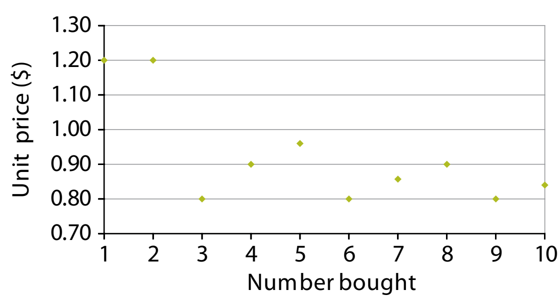
3.11 Choosing and using models

1 Both tubes should be approximately ⅓ full as 4 hours is ⅓ of 12 hours and 20 minutes is ⅓ of 60 minutes. Strictly speaking, the hours tube should be a very small amount fuller as it should show 4⅓ hours.

2 In 5 minutes Finn has travelled 0.5 km. Finn travels at 6 km/h, Alice at 18 km/hr. Alice is catching up with him at 12 km/hr so it takes her $\frac{0.5}{12}$ hours to catch up. In this time she has travelled $18 \times \frac{0.5}{12}$ km, or 0.75 km, so she overtakes him at the halfway point. The journey takes 15 minutes for Finn and $60 \times \frac{1.5}{18} = 5$ minutes for Alice, so for them to arrive together, she would have to leave 10 minutes after him.

3

Number bought	Price	Unit price
1	$1.20	$1.20
2	$2.40	$1.20
3	$2.40	$0.80
4	$3.60	$0.90
5	$4.80	$0.96
6	$4.80	$0.80
7	$6.00	$0.86
8	$7.20	$0.90
9	$7.20	$0.80
10	$8.40	$0.84



One or two alternative offers should be selected (for example the ‘buy one, get one half price’ suggested) and the tabulation and graphing procedure shown above should be repeated for these.

It is not trivial to generate a mathematical formula or rule from a graph, but a useful first step would be to reverse the above process and generate the table from the graph. We can then identify what the price of one item is, and then the unit price of each subsequent item bought. Offers such as 'buy two, get one free' will generally produce cyclic graphs like that shown in the first example. Offers such as '20% off if you spend over $20' will give a discontinuity in either the value or the gradient of the graph. Such observations may help to identify the nature of the offer.

3.12 Making choices and decisions

1 If you have a score of 17, the possible outcomes (only allowing one more throw) are:

No extra throw		Win 3
One extra throw	1	Win 3
	2	Win 6
	3	Win 8
	4	Win 10
	5	Lose 4
	6	Lose 4

Averaging the outcomes with the extra throw (all scores are equally possible): $(3 + 6 + 8 + 10 - 4 - 4)/6 = {}^{19}/_{6}$, so the average is a win of just over 3. The score with no extra throw is a win of 3, so it is marginally better to throw again.

2 If Clyde spends $29.99, he will get a 2¢ voucher that will save him 60¢ on petrol, so his effective spend is $29.39. Any spend over $30 in the shop will get him a 3¢ voucher, saving him 90¢ on petrol, so he can spend up to $30.29 (an extra 30¢) without increasing his overall bill. Similarly, if he spends $49.99, he could spend up to $50.29 without increasing the bill.

3 D is not possible as there is no subject from the third column.

4 34¢ can be made up as 22¢ + 9¢ + 2¢ + 1¢; any other combination needs at least one more denomination. 67¢ can be made up as 3 × 22¢ + 1¢, so no extras are needed. $1.43 can be made up as 6 × 22¢ + 9¢ + 2¢, so four denominations are needed in total: 22¢, 9¢, 2¢ and 1¢.

4.1 Inference

2 There are a number of clues that you could have noted. One is that the two people are walking in opposite directions. Look at their knees if you missed this point. This fact makes D quite implausible. C is not impossible, i.e. that the contact between L's hand and the bag is accidental; but if you look closely at L's fingers, it would be unlikely that his or her hand would be in that position unintentionally. The position of L's fingers also make it less likely that L was reaching into the bag; more likely that he or she was grasping it. A is therefore a more plausible explanation than B.

A is probably the most plausible of the four suggestions, but is it the best possible explanation? It is hard to see what else might be going on, and A is not far-fetched. People do, unfortunately, snatch bags. R is carrying the bag carelessly, and it would be very easy for L to pull it off R's shoulder and run. R has no grip on it that we can see.

Our answer is, therefore, that A is a reasonable conclusion to draw. However, you may have a better explanation.

There is no right or wrong answer: what matters is that you made full use of the information, and did not jump to conclusions without being able to give good reasons using the evidence available.

4.2 Explanation

1. A explanation; B explanation; C argument; D argument; E explanation. In all five cases reasons are given, but only in C and D do the reasons function as support for a conclusion.
2. B and C, if true, could explain the data. A is simply a summary or interpretation of the data: an observation. D is an inferred generalisation, and not necessarily a safe one.
3. *Variable responses*. (For example, that car claims have risen so significantly it may give the impression of a general rise. Or that it seems *likely* that claims would rise, and it is therefore assumed that they have.)
4. *Variable responses*. (In brief: primarily the passage is explaining why mountain-climbing ethics have changed. However, it could be added that in explaining why they have changed, the author is also making the case that they have changed, and changed for the worse. It is a good example of the boundary between argument and explanation becoming blurred at times.)

4.3 Evidence

1. Check your answer against the three relevant sections of the chapter, beginning with 'Types of evidence' on page 145. Examples: *variable*.
2. a. Mrs Short's evidence provides corroboration only in the sense that it concurs with Green's claim. But her evidence is somewhat vague and uncertain. We have no information about Mrs Short herself: her age, alertness, etc., or her relation to Green other than their being neighbours. It is weak evidence, possibly biased by acquaintance / friendship.
   b. The restaurant owner is not independent, so his reliability as a witness is questionable.
   c. Long, we are told, is independent inasmuch as he does not know either party involved in the incident. However, his evidence is compromised by the fact that he did not come forward until after he had seen White and his car in a news photograph, and knew that White had been arrested. This knowledge makes his claims unreliable.
   d. If Mrs Short is right about seeing a parking ticket on the suspected car, an obvious step would be to find out if a ticket had been issued to White on the day in question. If so it would mean White and the restaurant owner were lying, and one would have to ask why. A parking ticket in such a situation would come close to being the 'smoking gun'.
3. *Variable responses*

4.4 Credibility

Responses will vary, but the following very brief notes may be useful.

1. You should have considered the following criteria:
   - plausibility of the statements made
   - reputation (position, status, etc.)
   - expertise; experience
   - possibility of vested interest
   - corroboration (if any) and whether or not it is independent.
2. The most important items are the two songs, including the chords, and what can be inferred reliably from the similarities and/or differences between them.
3. Obviously what Ewbank writes is hearsay, not direct testimony. She is reporting what those involved in the case have said when interviewed, and in response to questions. However, she is allegedly quoting them directly in many cases. She also produces some factual evidence, such as the content of Berry's scrapbook, and the song itself. These factors need to be taken into account when deciding how reliable her report is.

4, 5 *Variable responses*

4.5 Two case studies

Variable responses

4.6 Critical thinking and science

Variable responses

4.7 Introducing longer arguments

1 If the bored and disadvantaged young men knew that the police were banned from chasing stolen cars, they might not find the theft of a car so exciting, and a ban may not, after all, lead to an increase in car thefts.

2 *Variable responses*

4.8 Applying analysis skills

Variable responses

4.9 Critical evaluation

1 The obvious flaw here is the straw man. It distorts the author's argument by making the conclusion much too strong and creating a soft target for its own attempted refutation. The author does not advocate denying a job to anyone who has committed a crime, but makes the more moderate claim that serious criminals should not be lauded as celebrities. If this had been correctly represented the counter-argument would be a slippery slope.

2 The argument is blatantly circular. It uses the claim that the dinosaurs were rendered extinct by a single catastrophe to draw the intermediate conclusion that they were wiped out almost overnight. But from this it then argues back to the starting premise that the cause must have been a single catastrophic event rather than a gradual process.

3 *Variable responses*

4.10 Responding with further argument

Variable responses

4.11 A self-assessment

1 If the source of a document is a person or organisation which has a special interest in an outcome, or a theory being right, then it may be slanted or biased. This article is from the magazine of a conservation society for dolphins. Its author is likely to have high regard for these creatures and may – but not necessarily does – exaggerate their intelligence to argue for their rights. The article may still make fair claims: it is from a respectable publication. But the potential for partiality must be recognised and taken into account in the evaluation.

2 It is scientific in that it is based on observation rather than mere opinion – at least up to the point where the author introduces ethical claims and ideas about 'non-human persons' in the last paragraph. The argument is sufficiently cautious to be taken seriously: it talks of 'contentions' and offers plausible explanations rather than drawing strong and unwarranted conclusions from the limited evidence. It could be described as a mixture of scientific reasoning and speculative thinking. However, there is a part of the argument that is speculative rather than scientific. What animals deserve, what rights or status they should be given, how they should be treated and so on are ethical questions, and cannot be answered within a rigorously scientific context.

3 A is assumed. If it were not true it could not really be argued that the lack of an obvious benefit means that they seem to walk on water for fun.

B is not assumed because the author implies that it is *unusual* for a cultural activity not to be linked to food (in animals).

C is not assumed. It is a different issue altogether.

D is not assumed either. Although the author suggests that dolphins walk on water for fun in the wild, it does not mean they have to enjoy performing tricks in general, or enjoy anything at all that they do in captivity. They may enjoy it, but it is not necessary for the argument that they do.

4, 5 *Variable responses*

5.1 Combining skills – using imagination

1 a (i) The last two digits of the year are split – into first and last (sixth) position, with the month in second and third and the day in fourth and fifth. 17 March 1981 (170381) would appear as 803171.

(ii) Each had a repeated digit (two zeros) so from only one of the numbers they could not be sure which one was moved to which position. Because the repeats are in different places for the two people, a pattern can be found by comparing the two numbers.

(iii) Any date where all the six digits are distinct is necessary and sufficient: e.g. 23 Jan 1945 or 17 March 1982. A number such as 11 November 1911 is a special case. It would allow the particular birth date to be predicted, but would not give the general pattern.

b This introduces a new factor: the distinction between male and female numbers, which adds a level of complexity.

(i) 662126 would give the month as 62, and 752232 the month as 52, so a five must be added to the tens of the month (the second digit of the six) for female drivers.

(ii) Jocelyn was born on 23 February 1972.

c This question requires you to generate your own method for distinguishing between different groups. To keep all the existing information, the new system would need to use values which cannot exist using the current system. The simplest ways to do this are by adding 4, 5 or 6 to the fourth digit (because currently the fourth digit will always be 0, 1, 2 or 3, so digits 4–9 would clearly denote foreign-born licence holders) or adding 2 or 3 to the second digit (in the current scheme this number could only be 0, 1, 5 or 6).
Another, more complicated, method would be to add a number between 31 and 69 (inclusive) to the date (fourth and fifth digits), as this could otherwise only lie between 01 and 31. Any unambiguous and consistent method would be acceptable as an answer.

d This question requires you to show how the number system can be used in a practical way. As almost all parents are at least ten years older, the deception is certain to be noticed. Your answer should express the probability as certain. If you state that it is a high probability, you should also explain your answer – a parent will almost definitely be more than ten years older than their child – to get the mark.

e Excluding the digits denoting the year, there are 10,000 possible numbers (0000-9999), of which only 365 (or 366 in a leap year) are valid. So (10,000 – 365) ÷ 10,000 = 96.35%. A good approximation would be acceptable. One simple error, for example using 9999 instead of 10,000 or trying to incorporate a factor for a wrong year would normally lose only some of the available marks.

2 The Fastrack bus leaves Aaland at 8 a.m., so passes through the three villages at 8.10, 8.20 and 8.30, arriving at Matsberg at 8.40. The Stagebus leaves Matsberg at 7.45 and takes 1 hour 15 minutes, including three 5-minute stops, so takes

15 minutes between villages. Thus it is at village 1 (nearest Matsberg) from 8.00 to 8.05, at village 2 from 8.20 to 8.25 and at village 3 from 8.40 to 8.45, arriving at Aaland at 9.00. They both arrive at village 2 at 8.20, so C is correct.

3 Dunrovia has 6 points – this can only be obtained by two wins and one loss. Similarly, Arbadia's and Brindling's score of 4 points can only be achieved by one win, one draw and one loss. Crittle's score of 2 points can only be obtained by two draws and a loss.

- **a** Crittle drew two matches. Two other teams drew a match each; thus Crittle must have drawn with both Arbadia and Brindling. Two matches were drawn.
- **b** Crittle drew two matches and lost one; Dunrovia won two and lost one. Thus Dunrovia must have beaten Crittle.
- **c** This leaves three games unaccounted for: D vs A, D vs B and A vs B, none of them draws. If Brindling beat Dunrovia, Dunrovia must have beaten Arbadia, and Arbadia must have beaten Brindling.

4 If Chico's bill was \$3 more than the average, Andy and Benita would have paid \$13.50 each (\$12 + \$ $\frac{3}{2}$) if all had paid the average. Thus Chico's individual bill was \$16.50.

5 If the border is the same all the way around, and there is one more square vertically than horizontally, the difference between the two dimensions must be the same as one square. Thus the squares are 0.3 m × 0.3 m. The border is 0.1 m on each side.

5 The total expenses were 2 × \$400 (two weeks' fixed costs) + \$1400 for materials, or \$2200 in total. Thus the profit was \$2700, or \$900 each. Bill had paid out \$800, so Fred owes him \$1700. Harry had paid out \$1400, so Bill owes him \$2300, leaving \$900 for himself.

5.2 Developing models

1 Let us assume that Duane walks x km. It doesn't matter whether this is done as a single stage or they swap bike and walk several times – it is only important how far in total each walks and rides.

Duane's total journey time is

$$\frac{x}{6} + \frac{(12 - x)}{15}.$$

Mervin's total journey time is

$$\frac{x}{20} + \frac{12 - x}{4}$$

If they arrive at town at the same time:

$$\frac{x}{6} + \frac{(12 - x)}{15} = \frac{x}{20} + \frac{(12 - x)}{4}$$

Multiplying both sides by 60:

$$10x + 4(12 - x) = 3x + 15(12 - x)$$

$$\text{or } 10x + 48 - 4x = 3x + 180 - 15x$$

Simplifying:

$$18x = 132 \quad \text{or} \quad x = 7.33 \text{ km}$$

The total time is:

$$\frac{x}{6} + \frac{(12 - x)}{15} = 1.22 + 0.31$$

= 1.53 hours (1 hour 32 minutes)

We still have to convince ourselves that arriving at the same time is the best strategy. Suppose Duane (the faster walker) walks the whole way. It takes him two hours. Clearly any strategy in which Duane walks more than 7.33 km will result in a slower time (nearer to two hours). It is even worse if Mervin walks further as he is a slower walker.

2 **a** Two orientations are possible.

- Along a 2 × 2 face:
  time = 1 + 1 + 1 + 1 = 4 mins
- Along a 6 × 2 face:
  time = 3 + $\frac{1}{3}$ + 3 + $\frac{1}{3}$ = 6 mins 40 secs

So four minutes is the minimum possible time.

b Three orientations are possible.

- Along the 1 × 4 face:
  distance = 1 + 4 + 1 + 4 = 10 m

- Along the 1 × 6 face:
  distance = 1 + 6 + 1 + 6 = 14 m
- Along the 6 × 4 face:
  distance = 6 + 4 + 6 + 4 = 20 m

If you give more distances than this, do not award yourself any marks.

c The principles for using the model having been developed in the first two questions, this question now asks for a real application of its use.
To minimise the number of turns required, we need to find the face of a 24 m^3 block that would have the largest possible perimeter.
Assuming the lengths are integers, this would be a 24 m × 1 m face, giving us a perimeter of 50 m.
610 ÷ 50 = 12 revolutions (48 turns of 90°) with 10 m remaining. We are able to choose whether the block is standing upright or lying down at the beginning, which allows us to choose which way it is standing at the end. If it starts each full revolution upright then it can pass the 610 m mark with one more 90° turn, so it would take 49 turns in total.
If you did not give 49 as your answer, you might still claim two marks if you gave the number of complete revolutions or half revolutions for a 24 × 1 face, or if you gave the number of 90° turns for a set of measurements which could reasonably be encountered for a 24 m^3 block (12 × 2, 8 × 3, 6 × 4, 6 × 2, 4 × 3, 3 × 2, 2 × 2, 1 × 1). Award yourself one mark if you calculated using one of the sets of measurements in brackets, but gave the number of 180° or 360° turns.

d Here, the model is used to find the optimum dimensions of a 24 m^3 block in order to move it quickly.
The optimum dimensions are 4 m × 6 m.
To calculate the time taken to complete 610 m:

$$\text{Number of revolutions} = \frac{610}{(6+4+6+4)} =$$
30, with 10 m remaining

$$\text{One revolution} = 2\left(\frac{6}{4}+\frac{4}{6}\right) = 4\frac{1}{3}\ \text{mins}$$

$$\text{Total time} = \left(30 \times 4\frac{3}{2}\right) + \frac{3}{2} + \frac{2}{3} =$$
132 mins

If you failed to find the shortest possible time, you may award yourself:

- three marks for a correctly calculated time under 300 minutes (using possible dimensions)
- two marks for a correctly calculated time over 300 minutes (using possible dimensions)
- one mark if you showed you knew to multiply the speed of a full revolution by the number of revolutions needed, but made an error in your calculation.

e There are two ways to achieve a time under 500 minutes:
Blocks sized 20 + 20 + 21 will take

(139 mins 24 secs) + 5 mins
+ (139 mins 24 secs) + 5 mins
+ (168 mins 29 secs)
= 457 mins 17 secs.

Blocks sized 21 + 24 + 16 will take

(168 mins 29 secs) + 5 mins
+ (132 mins 10 secs) + 5 mins
+ (153 mins)
= 463 mins 39 secs.

Award yourself full marks for the above solutions (rounding is acceptable).
Award yourself three marks for correctly calculated answers that add up to 61 m^3, or one mark for choosing block sizes adding up to 61 m^3 and another for calculating the time taken to move any block 610 m.

f Answering this question involves developing a new use of the model to investigate moving a large amount of stone.

The quickest way of transporting between 61 and 70 m^3 of stone is to move two 24 m^3 blocks and one 20 m^3 block: Blocks sized 24 + 24 + 20 will take

(132 mins 10 secs) + 5 mins
+ (132 mins 10 secs) + 5 mins
+ (139 mins 24 secs)
= 413 mins 44 secs.

Award yourself full marks if you correctly identified the correct combination of blocks: 24 + 24 + 20.
Award yourself one mark for 24 + 20 + 20, or 24 + 24 + 16.

3 We need to calculate the total race time for the various numbers of pit stops. For 1 pit stop, 150 litres of fuel are required for each half of the race. The average lap time (0.12 seconds slower than 75 seconds for each 5 litres of fuel) is, therefore:

$$75 + 0.12\left(\frac{75}{5}\right) = 76.8 \text{ seconds}$$

So 60 laps takes 60 × 76.8 = 4608 seconds. The time for the pit stop is

$$10 + \frac{150}{15} = 20 \text{ seconds}$$

so the total race time is 4628 seconds (77 minutes 8 seconds).
For two stops, the calculation is based on an average fuel load of 50 litres, so the average lap time is 76.2 seconds and the pit stop time is 16.7 seconds.
The total time is

$$60 \times 76.2 + 2 \times 16.7 = 4605.4 \text{ seconds}$$

or 76 minutes 45.4 seconds.
For three stops, the average fuel load is 37.5 litres, the average lap time is 75.9 seconds and the pit stop time is 15 seconds.
The total time is

$$60 \times 75.9 + 3 \times 15 = 4599 \text{ seconds}$$

or 76 minutes 39 seconds. Therefore three pit stops is optimum. Should you consider four?

As a further exercise you might consider how the problem could be tackled if the distance between pit stops was not constant (for example, it might be worth filling the car right up at the start to save on refuelling time, although this would make it slower).

4 It is more straightforward to work in proportions than percentages. Suppose the proportions are as follows: x Brazil nuts, y walnuts and $(1 - x - y)$ hazelnuts. The cost to the shopkeeper for this mix is $40x + 35y + 20(1 - x - y)$. She wishes to make 50% profit selling it at 60¢, so 60¢ represents twice this value.
We now have a model:

$$40x + 35y + 20(1 - x - y) = 30$$

Simplifying:

$$20x + 15y = 10$$

This cannot be solved explicitly for x and y, so we must investigate different values. We can note that $y = \frac{(2 - 4x)}{3}$, so this gives a relationship between the two (and implies the proportion of the third ingredient).
Putting some values into this:

x (Brazil nuts)	*y* (walnuts)	*z* (hazelnuts)
0.0	0.67	0.33
0.1	0.53	0.37
0.2	0.40	0.40
0.3	0.27	0.43
0.4	0.13	0.47
0.5	0.00	0.50
0.6	–0.13	0.53

Thus there is a range of mixes that fulfil the conditions, from 0 to 50% Brazil nuts. We can test one of these answers: 1kg can be made up of 20% Brazil nuts, costing 8¢, 40% walnuts costing 14¢ and

40% hazels costing 8¢, a total of 30¢. The most even mix is around 30% Brazil nuts – can you define it more closely?

5.3 Carrying out investigations

1 **a** This must be carried out by looking at amounts successively. There is more than one way of doing some of the amounts; only one is shown:

1¢ = 1¢; 2¢ = 2¢; 3¢ = 1¢ + 2¢;
4¢ = 2¢ + 2¢; 5¢ = 5¢; 6¢ = 5¢ + 1¢;
7¢ = 5¢ + 2¢; 8¢ = 10¢ – 2¢;
9¢ = 10¢ – 1¢; 10¢ = 10¢;
11¢ = 10¢ + 1¢; 12¢ = 10¢ + 2¢;
13¢ cannot be done in two coins.

b This part of the investigation is open-ended. A systematic approach should be taken, possibly starting with the 1, 3, 5 . . . example (in order to make 7, a 7¢, 8¢, 10¢ or 12¢ coin would be needed and others follow from this). If a set does not include a 1¢ coin, then two denominations must differ by 1¢.

2 A 2 × 2 box contains 4 + 1 = 5 oranges; a 3 × 3 box has 9 + 4 + 1 = 14. Each successive size can be worked out as the square number plus the sum of the square numbers below it, so a 5 × 5 box has 25 + 16 + 9 + 4 + 1 = 55 oranges. As advanced level mathematics is not expected for this paper, this answer would be sufficient. The general formula is

$$\frac{n(2n+1)(n+1)}{6}$$

where n represents the number of oranges on one side of the square. For a rectangular box, students should tabulate a series of values for $n \times m$ boxes. They would be expected to recognise that the pattern depends on the smallest square that would be fitted into this box (i.e. an $n \times n$ square if $n<m$). For rectangles based on a given value of n, each extra row adds a further number which is the nth triangular number. So, for a 4 × 4 square, there are 30 oranges, and we then add 10 (the 4th triangular number) for each extra row, so a 4 × 5 box contains 40, a 4 × 6 box contains 50, and so on.

3 There are 36 combinations of two dice. Winning combinations are:

1, 4   1, 5   1, 6   2, 5   2, 6   3, 6

and the reverse of these (4, 1; 5, 1 etc.), so 12 of the 36 combinations win, or $\frac{1}{3}$. If 200 people play, Milly takes \$200 and will expect to pay out $2 \times \frac{200}{3}$ or \$133, so she should raise \$67.
Investigating alternatives is again quite open-ended. We can look at the two options suggested. Multiples give:

		First die					
		1	2	3	4	5	6
Second die	1	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	18	24	30	36

She needs players to win less than half the time to make a profit. There are 17 values in the table of 12 or over, so 12 is the minimum winning score which would guarantee her a profit. If players have to score over 12 to win, this would give odds of $\frac{13}{36}$ – similar to those in the original game.

One can similarly investigate the two values written as a two-digit number (it may be necessary to colour the dice to define which is the first digit).

5.4 Data analysis and inference

1 a Statement A cannot be confirmed. Over the last 22 years, the discovery of new resources has matched the rate of depletion for both oil and gas. Whether this will continue to happen in the future, and for how long, is not certain. Statement B is true if stated in terms of years of potential supply. The 2011 proved gas reserves are equivalent to 60 years' consumption and the oil just over 40.

Statement C is true, as stated for A: the graphs of potential years of supply are approximately horizontal. D is also true. Energy consumption is rising (graph 2) so, if the reserves are constant in terms of years of supply, the rate of discovery must be increasing in a similar manner to the rate of usage. E is not true. It would lead to the potential years' reserves in graph 1 falling.

b In the 1980s there must have been a surge of exploration and discovery of new reserves. As the usage was fairly constant, this led to an increase in the known years of supply. Since then, discoveries have just matched consumption. Other factors may be involved.

c If the discovery of new reserves fails to match consumption, prices will rise. This will lead to a variety of things, one being a search for alternative energy sources (which will become more attractive as the price for energy is higher); another is a recession in world trade (this would reduce consumption and ease prices); and a third is a search for increased energy efficiency. You should comment on these, their implications and any other factors you can think of which are relevant. This is a good topic for class discussion.

2 a Greece and Spain have 4 points: this could only be achieved by one win and one draw. Portugal have 3 points: only a win and a loss would give this. Russia have lost both their games. These results are shown in the table.

Team	Played	Points	W	D	L	Goals for	Goals against
Greece	2	4	1	1	0	3	2
Spain	2	4	1	1	0	2	1
Portugal	2	3	1	0	1	3	2
Russia	2	0	0	0	2	0	3

Russia have lost to both Spain and Portugal (they have yet to play Greece). Spain must have drawn with Greece (only one match was drawn). Greece must have beaten Portugal (this is the only game not accounted for). Russia lost one game 1–0 and the other 2–0 (the only way of making 3 goals against).They could not have lost 2–0 to Spain as Spain would then have lost their other game (their total is 2 for and 1 against). Thus Russia lost 2–0 to Portugal and 1–0 to Spain. We can now work out all the results and scores:

Greece 2 Portugal 1
Greece 1 Spain 1
Spain 1 Russia 0
Portugal 2 Russia 0

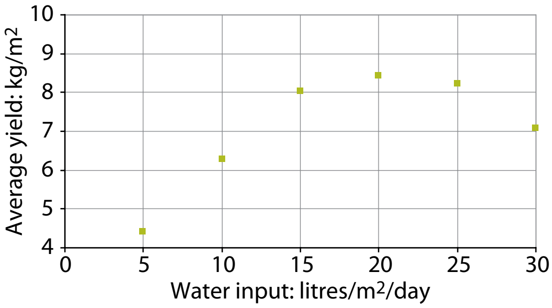
b There are nine possibilities for the remaining two games (either team can win, or the game can be drawn, giving three possible results for each game; $3 \times 3 = 9$). Russia cannot finish in the first two, but the three other teams can. The situation can be analysed backwards (e.g. if Greece win or draw they are through, as Spain are playing Portugal and both cannot get 3 points). The full analysis is given in the following table . (GbR means Greece beat Russia; GdR means Greece drew with Russia.)

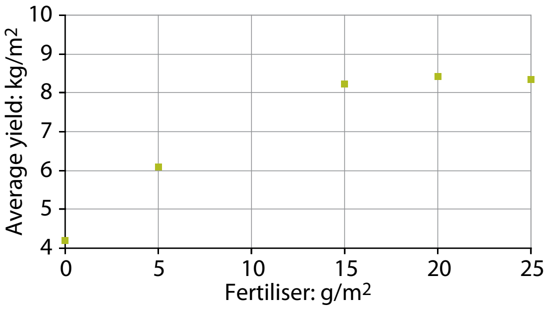
	SbP	SbP	SbP	SdP	SdP	SdP	PbS	PbS	PbS
	GbR	GdR	RbG	GbR	GdR	RbG	GbR	GdR	RbG
Greece	7	5	4	7	5	4	7	5	4
Spain	7	7	7	5	5	5	4	4	4
Portugal	3	3	3	4	4	4	6	6	6
Russia	0	1	3	0	1	3	0	1	3
Qualify	**G S**	**G S**	**G S**	**G S**	**G S**	**G S**	**G P**	**G P**	**P***

The result is clear in all but two columns. In column 6, Greece and Portugal finish level on points but Greece qualify as they beat Portugal. In column 9, Greece and Spain finish equal and they drew their game. The scores of the two games will determine who goes through, whether on goal difference, goals scored or drawing of lots. Can you determine which scores will lead to which outcome? The actual result was column 9 (Spain 0 Portugal 1, Russia 2 Greece 1), so Greece and Portugal went through. Portugal had the only positive goal difference and Greece had scored more goals than Spain.

3 This is another open-ended problem. As much as possible should be extracted from the data given: this involves averaging the rows and columns, graphing both these averages and the individual values, and drawing appropriate conclusions.
The table, with averages included, can be used to create graphs:

Crop yield: kg/m²		**Water input: litres/m²/day**						
		5	**10**	**15**	**20**	**25**	**30**	**Average**
Fertiliser: g/m²	**0**	3.55	4.58	5.76	5.36	4.04	2.04	4.22
	5	4.54	5.83	7.16	7.54	6.82	4.73	6.11
	15	5.21	7.73	9.22	9.32	9.06	8.89	8.24
	20	4.85	6.89	8.95	10.27	10.40	9.38	8.46
	25	3.97	6.42	9.04	9.62	10.83	10.32	8.37
	Average	4.42	6.29	8.03	8.42	8.23	7.07	7.08

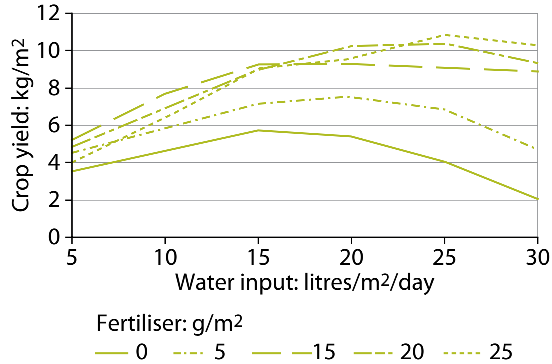




The following observations can be made:

- There is a missing row in the fertiliser levels – this is typical of experimental data, in that unexpected problems can happen.
- The effect of fertiliser is less than that of water over the range investigated.
- Both factors lead to a peak in the graph, demonstrating that too much of either water or fertiliser causes a reduction in crop yield.
- Although the peak with water input is at 20 litres/m^2/day and the peak with fertiliser is at 20 g/m^2, the highest value in the table does not correspond to these two values; in fact it is at (25, 25). This shows an interaction between the two factors, i.e. more fertiliser allows the plant to use more water or vice versa. This may be seen in the graph below, which shows all the data and also indicates some variations due, presumably, to experimental error.

There is, in fact, an area where yield is over 10 kg/m^2 and shows little variation outside the experimental error.



6.1 Using other mathematical methods

1 If Rita normally sells a packet of cornflakes for \$$x$, then her profit is $0.4x$ and she buys them for $0.6x$. Next week she will be selling 3 packets for $2x$, and she has bought them for $1.8x$. Thus her actual profit is $0.2x$, and her percentage profit is $\frac{0.2x}{2x}$ = 0.1 or 10%.

2 I originally planned to buy n rolls, so I took $25n$ cents. The reduced price is 20 cents and I can buy 3 more, for $20(n + 3)$ cents. These two amounts are the same, so:

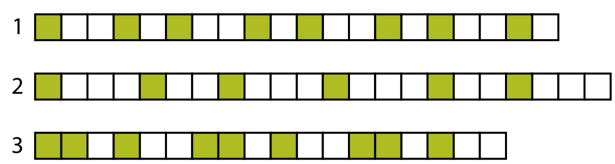
$$25n = 20(n + 3) = 20n + 60$$

so:

$$5n = 60 \text{ and } n = 12$$

I was originally going to buy 12 rolls.

3 This is most easily solved with the aid of a diagram (each box represents one second and the shaded boxes are 'on'):



Thus they all flash together at 15 seconds after starting their sequences.

4 There are four ways of picking up the first hat; then one has been removed, so there are three ways of picking up the second hat; or 4 × 3 = 12 ways of picking up the first two hats. The total number of ways they can pick up the four hats is 4 × 3 × 2 × 1 = 24. We must subtract from this the number where one person or more has the right hat.

Look first at only one person having the right hat. If A has the right hat, there are 6 combinations of hat for B, C and D (BCD, BDC, CBD, CDB, DBC, DCB). Of these, only 2 have all BCD with the wrong hats (CDB and DBC). The same applies if B, C or D is the only person with the right hat, making 8 in total.

Look now at two people having the right hat: this could be AB, AC, AD, BC, BD and CD. In each case, there is only one way the other two could be wrong, making 6 in total.

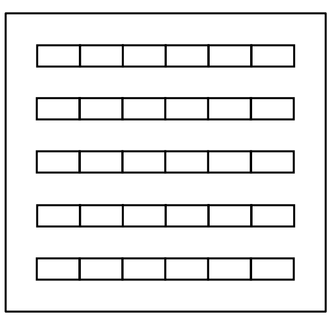
It is impossible for exactly three people to have the right hats.

There is only one way all four people can have the right hats.

This makes 8 + 6 + 1 = 15 ways of at least one person having the right hat, leaving 9 ways that everyone has the wrong hat.You could try to list these.

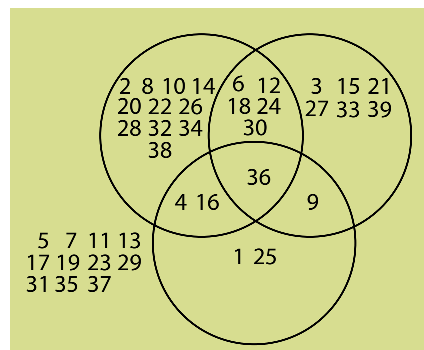
6.2 Graphical methods of solution

1 Each row of tables contains 6 tables (6 × 2 m = 12 m) with 1.5 m gaps at each end.

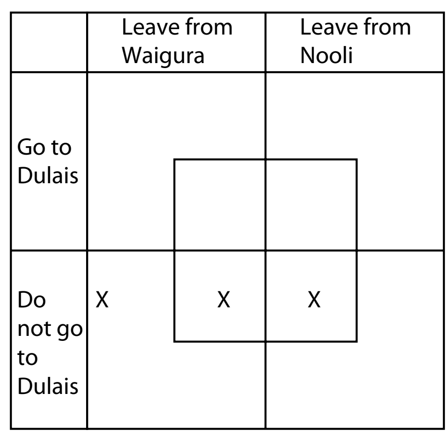


There must be a 1.5 m gap between the wall and the first row of tables. Each other row has an effective width of 0.8 + 1.5 = 2.3 m. So the number of rows that can fit in the room is the integer below $\frac{13.5}{2.3}$ = 5. Each row seats 6 × 6 + 2 = 38 people (the 2 are at the ends). 5 × 38 = 190, so A is correct.

2 The Venn diagram is as shown here. The top-left circle represents even numbers, the top-right circle multiples of 3 and the bottom circle square numbers. Those outside the three circles do not fit into any of the categories.



3 These statements may be represented as a Carroll diagram.



The inner quartered square shows the fast hydrofoil services, the outer square the slow steamboats. The Xs mark the cells that are empty (represent no service). These are ferries going from Waigura to anywhere other than Dulais and fast hydrofoil services to anywhere other than Dulais. All other cells may contain services.
We can now answer the statements:

A Hydrofoils from Nooli to Dulais are represented by the inner, top-right box and are possible. So this statement cannot be concluded.

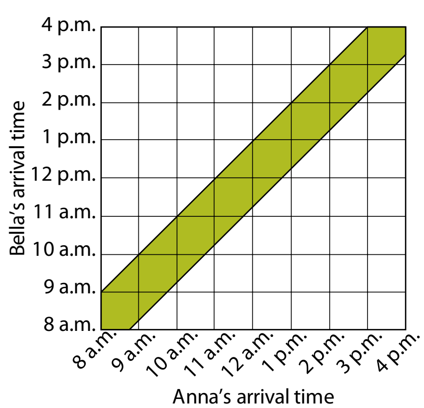
B As the inner, top-right box is possible, this statement cannot be true; hydrofoils could leave from Nooli.

C This is not true – hydrofoils from Nooli to places other than Dulais are represented by the inner, bottom-right box, which is empty.

D Steamboats from Waigura to Dulais are represented by the outer, top-left box, so this statement is possible; but it cannot be concluded from the data, as it could be that all the ferries from Waigura to Dulais are hydrofoils.

E This is true, since no hydrofoils from Waigura go elsewhere.

4 The diagram shows the arrival times of Anna and Bella.



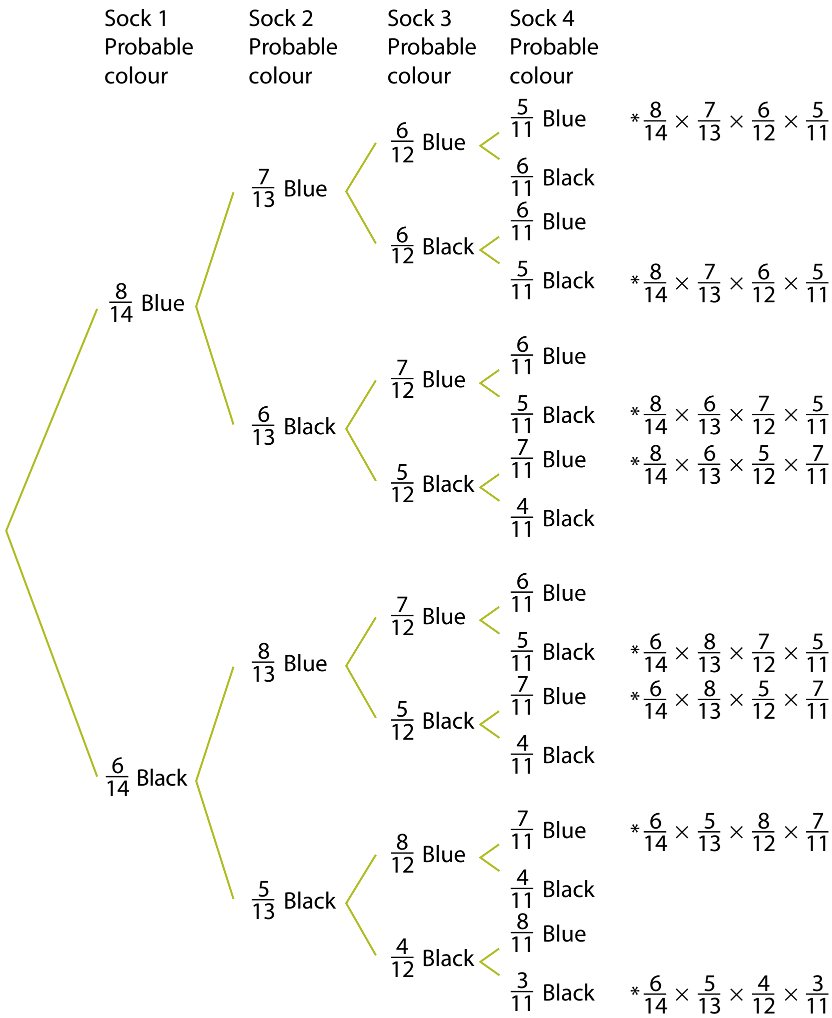
The shaded area represents the times when the two girls coincide. For example, if Anna arrives at 12, she will meet Bella if Bella arrives any time between 11.15 and 1.00; the area between these, and such equivalent times, is shaded. The probability required is the area of the shaded portion divided by the whole area of the graph. The large white triangles have areas of: $\frac{7 \times 7}{2} = 24.5$ units (upper) and $\frac{7.25 \times 7.25}{2} = 26.3$ units (lower). The whole graph has an area of $8 \times 8 = 64$ units. Thus the shaded portion has an area of $64 - 24.5 - 26.3 = 13.2$ units, so the chances of them meeting are $\frac{13.2}{64} = 0.206$ or 20.6%. This problem would be very difficult to solve without a graphical method.

6.3 Probability, tree diagrams and decision trees

1 This can be solved using a tree diagram (see page 336).
The asterisked combinations give two matching pairs.
There are 8 possibilities and the probabilities of all but the last are the same. The probabilities need to be worked out with a calculator and are as follows:
$7 \times 0.0699 + 0.0150 = 0.5043$
(The 0.0699 is the result of the first 7 asterisked calculations: $\frac{8}{14} \times \frac{7}{13} \times \frac{6}{12} \times \frac{5}{11}$, and the .0150 is the 8th.)
Thus the chance of drawing two pairs is approximately 50%.

2 The first two digits are 11 or 12. The second two digits can be 11–19 or 21–29 (regardless of the first two digits) or 31 (but only if the first two digits are 12, there being 31 days in December but not in November). There are 37 possibilities, so the chances of getting it right the first time are $\frac{1}{37}$. The chances of getting it right the second time are $\frac{1}{36}$ and the third time $\frac{1}{35}$. In order to calculate the overall probability we need to add the

Chapter 6.3 Question 1



probability of getting it right the first time ($1/37$) to the probability of getting it *wrong* the first time multiplied by the probability of getting it right the second time ($36/37 \times 1/36$), and to the probability of getting it wrong the *first two times* multiplied by the probability of getting it right the third time ($36/37 \times 35/36 \times 1/35$). The total chance in three attempts is $1/37 + (36/37 \times 1/36) + (36/37 \times 35/36 \times 1/35) = 3/37$ or 8.1%.

3 Let us suppose that the probability of hitting the nearer pole is $1/2$ and the probability of hitting the farther pole is $1/3$. (If the question can be answered, it clearly does not matter what the exact probabilities are or we would have been given them.)

If we throw near, far, near, the probabilities of throwing two in a row are as follows:

Hit, hit, miss: $1/2 \times 1/3 \times 1/2 = 1/12$
Miss, hit, hit: $1/2 \times 1/3 \times 1/2 = 1/12$
Hit, hit, hit: $1/2 \times 1/3 \times 1/2 = 1/12$

The total probability of winning is $3/12$ or 25%.

If we throw far, near, far, the probabilities of throwing two in a row are as follows:

Hit, hit, miss: $\frac{1}{3} \times \frac{1}{2} \times \frac{2}{3} = \frac{2}{18}$
Miss, hit, hit: $\frac{2}{3} \times \frac{1}{2} \times \frac{1}{3} = \frac{2}{18}$
Hit, hit, hit: $\frac{1}{3} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{18}$

The total probability of winning is $\frac{5}{18}$ or about 28%. The second strategy is better. Some may regard this as counter-intuitive as it involves two throws at the harder target. Did you expect this answer? Can you rationalise why the second strategy should be the best? Can you prove that it works for all probabilities (as long as the farther target is harder to hit)?

4 We first need to do some calculations on the various options. These are summarised in the table on page 338, with the second column showing the figures for a machine achieving a 99% detection rate and the third column showing those for a machine achieving a 95% detection rate. Fixed costs are ignored; these figures just represent the total income minus the quality control costs for the different assumptions. We can now construct the decision tree, as shown below.

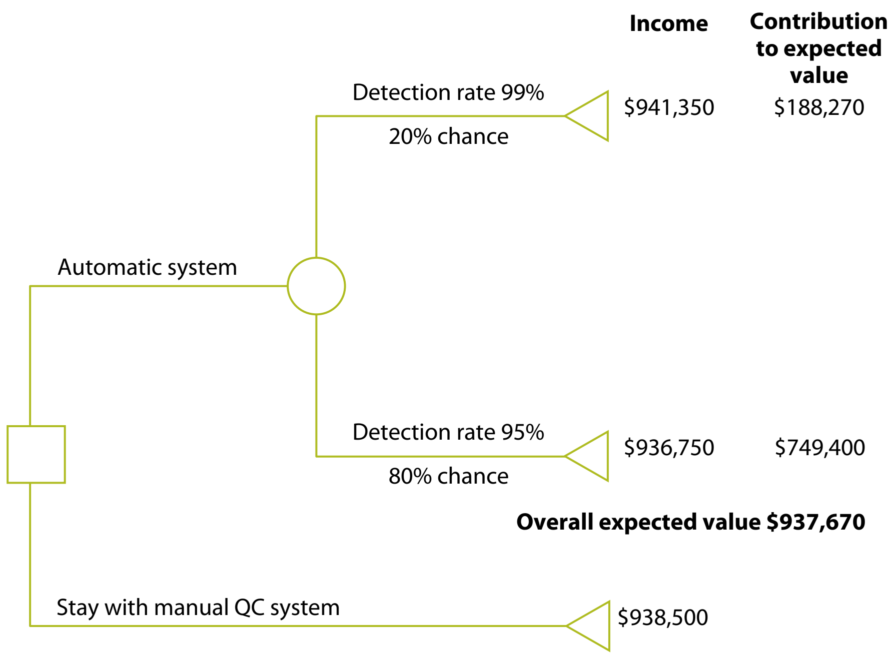
The differences are quite small – the present system shows a saving of \$830 in almost \$1 million. However, the automatic system carries an 80% chance of the loss being \$1750.

6.4 Have you solved it?

Variable responses

7.1 Conditions and conditionals

1 a Reading the book is a necessary but not a sufficient condition for passing the exam.



Costs per year over 4 years			
	Manual	**Auto (99%)**	**Auto (95%)**
Production	500,000	500,000	500,000
Unit sale value $	2	2	2
Unit compensation cost $	25	25	25
Labour cost $	40,000	0	0
Machine cost $	0	45,000	45,000
Redundancy cost $	0	2500	2500
Failure rate	0.01	0.01	0.01
Detection rate	0.9	0.99	0.95
Number faulty	5000	5000	5000
Faulty and detected	4500	4950	4750
Faulty and sent out	500	50	250
Total sent out	495,500	495,050	495,250
Income from sales $	991,000	990,100	990,500
Compensation costs $	12,500	1250	6250
Total costs $	52,500	48,750	53,750
Net income $	938,500	941,350	936,750

b B is the correct answer, because reading the book was a necessary condition only. Statement A treats reading the book as a sufficient condition, whereas it is only necessary; C would have to be true only if the prediction was that all those who read the book would pass. But that was not the prediction. In fact everyone could fail, readers of the book included, and the tutor's prediction would not have been wrong. D turns the prediction round and makes passing the exam a condition for having read the book; this does not follow from the prediction. E does not have to be true because reading the book was not a sufficient condition for passing the exam.

2
- A Yes. Being 21 or over is a necessary condition for approval.
- B No. The person might be under 21.
- C Correct. The person might not have a clean licence.
- D Yes. Passing an ADQ is necessary for anyone under 25, as Jason is; but not sufficient because a clean licence is also necessary.
- E Yes. Being under 21 is a sufficient condition for *refusal*.

3 *Variable responses,* but it should be recognised that the structure of water is a necessary (but not sufficient) condition for life as we know it.

7.2 Soundness and validity: a taste of logic

1 [A] is invalid, and therefore unsound. Lemons, as it happens, *are* citrus fruits, but many things with a sharp, acidic taste – such as pickled onions – are not. Therefore having sharp and acidic taste is not a good enough reason to say that something is a citrus fruit.

[B] on the other hand is valid and sound. Its premises are both true and the conclusion follows from them. If citrus fruits have a particular taste, then lemons, which are citrus fruits, must have that taste.

2 This is a valid argument. You can show this by simplifying it as follows: 'If this is a diamond it would scratch glass. It doesn't scratch glass. So it isn't a diamond.' As for the premises, the first is true: diamonds do scratch glass. The second *we are told* is true. Therefore the argument is sound as far as we can tell.

3 This argument is also valid. If it is true that the president really would be in prison if he were guilty, and he is not in prison, then he is not guilty. What makes this argument seem unconvincing is not that the conclusion doesn't follow from the premises but that the first premise is open to question. An awful lot of presidents have been guilty of corruption and escaped prison. That doesn't alter the *logical* fact that if the premises were true the conclusion would have to follow; but it does cast doubt on the overall soundness of the argument.

4 [A] The most obvious answer is that Nathan is a professional. (The argument would have the valid form: 'If *m* then *p*; *m*; therefore *p*' – with *m* for money and *p* for professional.)

[B] The most obvious answer is that Eunice has not accepted prize or sponsorship money. ('If *m* then *p*; Not-*p*; therefore Not-*m*.')

[C] There is no obvious conclusion. Not accepting money doesn't establish that Abbas is not a professional; he might earn money from coaching and be a professional for that reason. Logically, 'If *m* then *p*; Not-*m*; therefore Not-*p*' is an invalid form of argument, like [11] in the chapter. The best answer to [C] might be: 'So *what*?' The two sentences of [C] tell us practically nothing in relation to each other.

7.3 Non-deductive reasoning

1 Clive relied on a compass to direct him in poor visibility because, in his long experience, it had not let him down. However, he was ignorant of the fact that in some places a compass does not act in its customary way. Past experience was not, therefore, sufficient grounds for inferring that the compass would always behave predictably – as Clive discovered.

2 There are various ways to interpret the reasoning, but clearly the conclusion is that *Big Brother* is not harmless. There is a chain of reasoning leading to this. Here is one plausible way it may be understood:

R1 You can't imprison people . . . without it affecting their personalities.

R2 You can see people are not the same when they come out as they were before.

IC (So) it's a very dangerous game they're playing (as any psychiatrist will tell you).

R3 People are seriously damaged – mentally – by being in that house.

C You are wrong: *Big Brother* is not harmless.

Evaluation: If it were true that people are seriously damaged (R3), then it would follow that *Big Brother* is not harmless. Indeed it would follow deductively, or by definition, because clearly anything damaging is harm*ful*. R3, however, is not supported by R1, R2 or the IC. If it were it would be an intermediate conclusion itself. R1 and R2 do lead to IC, but just because something is dangerous doesn't mean that it actually causes damage. Our evaluation of the argument is that the chain breaks down at these points. Even if R1 and R2 are true, the conclusion does not follow from them.

3 *Variable responses*

7.4 Reasoning with statistics

1 a Various responses are acceptable. For example: the extract is making the claim that peaks in crime rates tend to be associated with a significant reduction in the prison population, and cites an incident in Italy as a *paradigm* example. ('Paradigm example' here means prime, or perfect, example.) The graph takes bank robberies as an indicator of the effect of lowering the prison population suddenly. The figures apparently shoot up by almost as much as the prison population falls. Previously, when prison numbers were rising before the pardon, and again afterwards, the bank robbery rates reduce. Look carefully however at the scales on the graph. 200,000 prisoners are released, and there is a peak of 8% bank robberies in the month after the pardon, compared with several between 6% and 7% before the pardon. Does the scale of the graph create an accurate or an exaggerated impression of the difference the released prisoners made? You may also have questioned why bank robberies in particular were selected. Did other serious crimes offer corroborating data? As for the extract, 160,000 police-reported offences again sounds impressive. But there are questions to ask, for instance about the nature and severity of the offences.

b *Variable responses*

2 *Variable responses*

7.5 Decision making

The answer is B. On economic grounds alone Zenergies should decline the offer and proceed to extract the gas. The revised projections suggest that the company would probably be better off by $1.9 million by taking this decision.

7.6 Principles

Variable responses

7.7 An argument under the microscope

Variable responses

7.8 Critical writing

Variable responses

Appendix

Applicability to various awards

*** Directly relevant
** Broadly relevant
* Some relevance

	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7
Cambridge Thinking Skills: AS Level, Paper 1	***		***		**	**	*
Cambridge Thinking Skills: AS Level, Paper 2	***	***		***			***
Cambridge Thinking Skills: A Level, Paper 3	**		***	*	***	***	
Cambridge Thinking Skills: A Level, Paper 4	**	*		***	**		***
Knowledge and Inquiry: Paper 1	**	*		*			**
Knowledge and Inquiry: Paper 2	***	***		***			***
OCR Critical Thinking: AS Level, Paper 1	**	**	*	**	*		
OCR Critical Thinking: AS Level, Paper 2	**	**		**			
OCR Critical Thinking: A Level, Paper 3	*	*	*	*	*		**
OCR Critical Thinking: A Level, Paper 4	*	*		***			**
AQA Critical Thinking: AS Level, Paper 1	**	***		***			

	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7
AQA Critical Thinking: AS Level, Paper 2	**	**	*		**		
AQA Critical Thinking: A Level, Paper 3	*	*	*	***			***
AQA Critical Thinking: A Level, Paper 4	*	*		***		**	***
BMAT Paper 1	**	***	***	**	***	***	**
TSA	**	***	***	*	**	**	*
LNAT	*	*		**			**
UK CAT	**	**	**	*	*	*	*
IB: Theory of Knowledge	*	**	*	**	*	*	**

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