Section 3.2

Understanding and Interpreting Confidence Intervals

Outline

- Interval estimate
- Interval estimate based on margin of error
- Confidence interval
- 95% confidence interval using the SE
- Understanding confidence intervals
- Common misinterpretations

Employer-Based Health Insurance

- A report from a Gallup poll in 2011 based on a random sample of n = 147,291 adults says

"Forty-five percent of American adults reported getting their health insurance from an employer…"

- How accurate is 45%???

http://www.gallup.com/poll/148079/Employer-Based-Health-Insurance-Declines-Further.aspx

Interval Estimate

An ***interval estimate*** gives a range of plausible values for a population parameter.

Margin of Error

One common form for an interval estimate is

statistic ± margin of error

where the ***margin of error*** reflects the precision of the sample statistic as a point estimate for the parameter.

Employer-Based Health Insurance

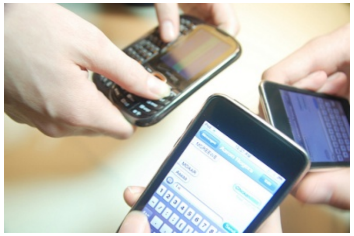
"Forty-five percent of American adults reported getting their health insurance from an employer…"

- Later in the report, it says

"the maximum margin of sampling error is ±1 percentage point"

- Interval estimate: 45% ± 1% = (44%, 46%)
- *The percentage of American adults getting their health insurance from an employer is probably between 44% and 46%*

Text Messages



- A 2011 random sample of n = 755 US cell phone users aged 18+ found that the average number of text messages sent or received per day is 41.5 messages, with a margin of error of 12.2.

- Give an *interval estimate* for the average number of text messages sent or received per day for US adult cell phone users.

41.5 ± 12.2
$= (29.3, 53.7)$

Smith, A. "Americans and Text Messaging," Pew Research Center, http://www.pewinternet.org/Reports/2011/Cell-Phone-Texting-2011/Main-Report/How-Americans-Use-Text-Messaging.aspx , accessed 9/19/11.

Election Polling

General Election: Romney vs. Obama

RCP Electoral Map | Changes in Electoral Count | Map With No Toss Ups | No Toss Up Changes

Polling Data						
Poll	Date	Sample	MoE	Obama (D)	Romney (R)	Spread
RCP Average	9/4 - 9/11	--	--	48.6	45.2	Obama +3.4
Rasmussen (Wednesday)	3-Day Tracking	1500 LV	3.0	46	45	Obama +1
Gallup (Wednesday)	7-Day Tracking	3050 RV	2.0	50	43	Obama +7
ABC News/Wash Post	9/7 - 9/9	710 LV	4.5	49	48	Obama +1
CNN/Opinion Research	9/7 - 9/9	709 LV	3.5	52	46	Obama +6
IBD/CSM/TIPP	9/4 - 9/9	808 RV	3.5	46	44	Obama +2

- Why is the margin of error smaller for the Gallup poll than the ABC news poll?

http://www.realclearpolitics.com/epolls/2012/president/us/general_election_romney_vs_obama-1171.html

Election Polling

- Using the Gallup poll, calculate an interval estimate for the proportion of registered voters who planned to vote for Obama.

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Election Polling

- The 2012 presidential election already happened, so this is one of the rare situations in which we actually know the true population parameter, *p*!
- In the actual election, 50.4% voted for Obama.
- Did your interval estimate contain the true population parameter?

Margin of Error

- How do we determine the margin of error???

- We can use the spread of the sampling distribution (the standard error) to determine the margin of error for a statistic

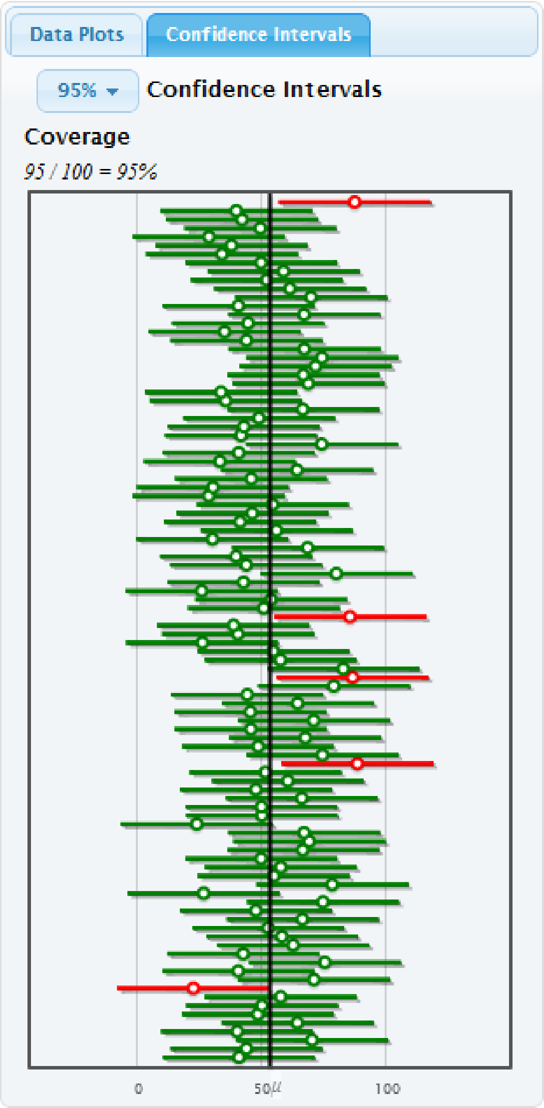
Confidence Interval

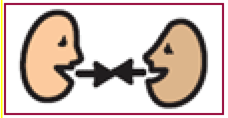
A ***confidence interval*** for a parameter is an interval computed from sample data by a method that will capture the parameter for a specified proportion of all samples

- The success rate (proportion of all samples whose intervals contain the parameter) is known as the ***confidence level***
- A 95% confidence interval will contain the true parameter for 95% of all samples

Confidence Intervals

- StatKey
- The parameter is fixed
- The statistic is random (depends on the sample)
- The interval is random (depends on the statistic)
- 95% of 95% confidence intervals will capture the truth





Sampling Distribution

If you had access to the sampling distribution, how would you find the margin of error to ensure that intervals of the form

$$\textit{statistic} \pm \textit{margin of error}$$

would capture the parameter for 95% of all samples?

(Hint: remember the 95% rule from Chapter 2)

95% Confidence Interval

If the sampling distribution is relatively symmetric and bell-shaped, a 95% confidence interval can be estimated using

statistic ± 2 × SE

Economy

A survey of 1,502 Americans in January 2012 found that 86% consider the economy a “top priority” for the president and congress.

The standard error for this statistic is 0.01.

What is the 95% confidence interval for the true proportion of all Americans that considered the economy a “top priority” at that time?

$$\text{statistic} \pm 2 \times SE$$
$$0.86 \pm 2 \times 0.01$$
$$0.86 \pm 0.02$$
$$(0.84, 0.88)$$

http://www.people-press.org/2012/01/23/public-priorities-deficit-rising-terrorism-slipping/

Interpreting a Confidence Interval

- 95% of all samples yield intervals that contain the true parameter, so we say we are "95% sure" or "95% confident" that one interval contains the truth.

- *"We are 95% confident that the true proportion of all Americans that considered the economy a 'top priority' in January 2012 is between 0.84 and 0.88"*

Carbon in Forest Biomass

- Scientists hoping to curb deforestation estimate that the carbon stored in tropical forests in Latin America, sub-Saharan Africa, and southeast Asia has a total biomass of 247 gigatons.
- To arrive at this estimate, they first estimate the mean amount of carbon per square kilometer.
- Based on a sample of size n = 4079 inventory plots, the sample mean is $\bar{x} = 11{,}600$ tons with a standard error of 1000 tons.
- Give and interpret a 95% confidence interval.

Saatchi, S.S. et. al. “Benchmark Map of Forest Carbon Stocks in Tropical Regions Across Three Continents,” *Proceedings of the National Academy of Sciences*, 5/31/11.

Carbon in Forest Biomass

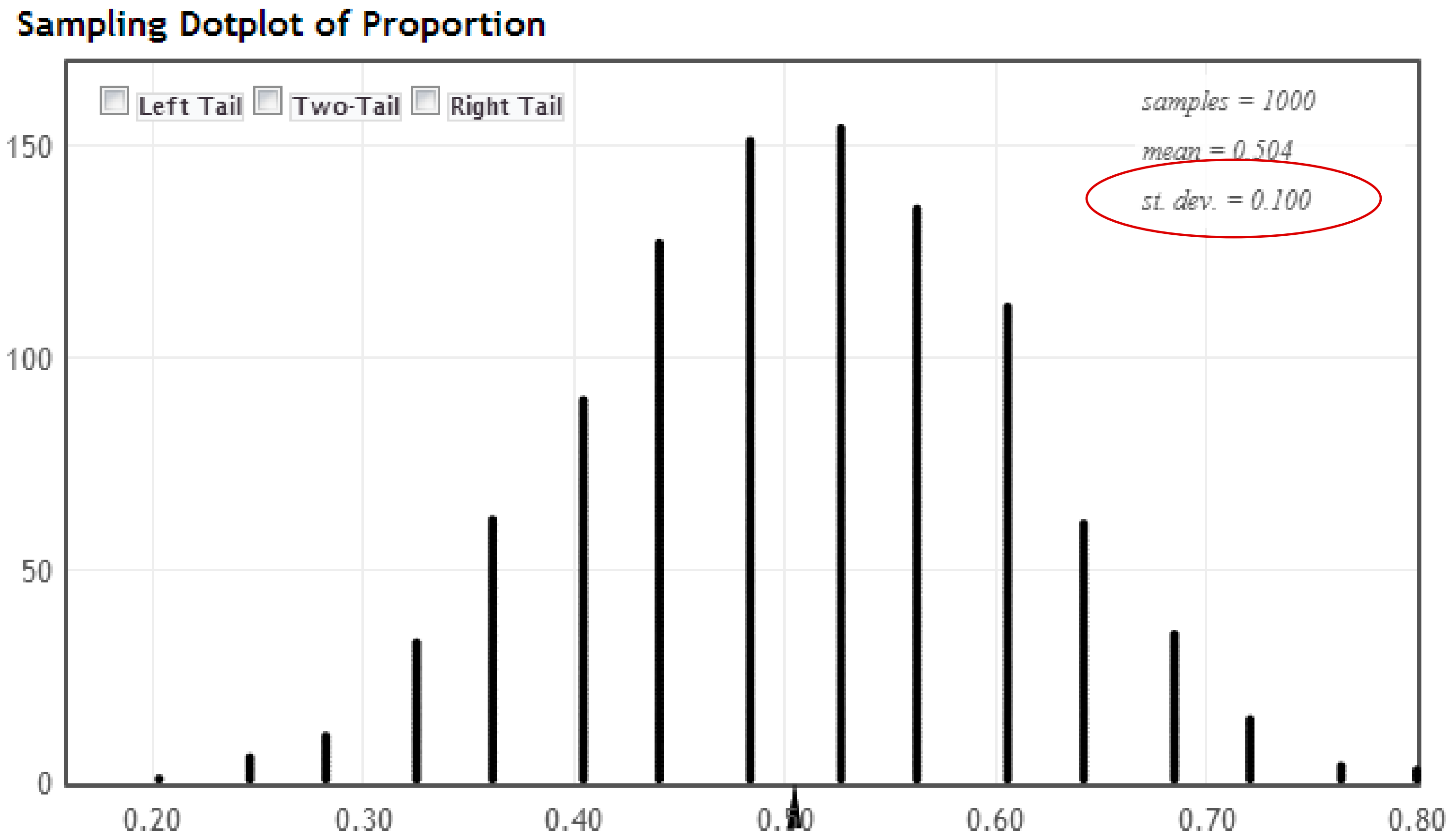
- 95% CI: $11{,}600 \pm 2 \cdot 1000 = (9{,}600, 13{,}600)$
- *We are 95% confident that the average amount of carbon stored in each square kilometer of tropical forest is between 9,600 and 13,600 tons.*

Proportion of Heads

- p = Proportion of time a coin lands heads
- We know $p = 0.5$, but let's pretend that we don't know that, and want to estimate it
- Flip a coin 25 times, and compute your sample proportion, $\hat{p}$

Proportion of Heads

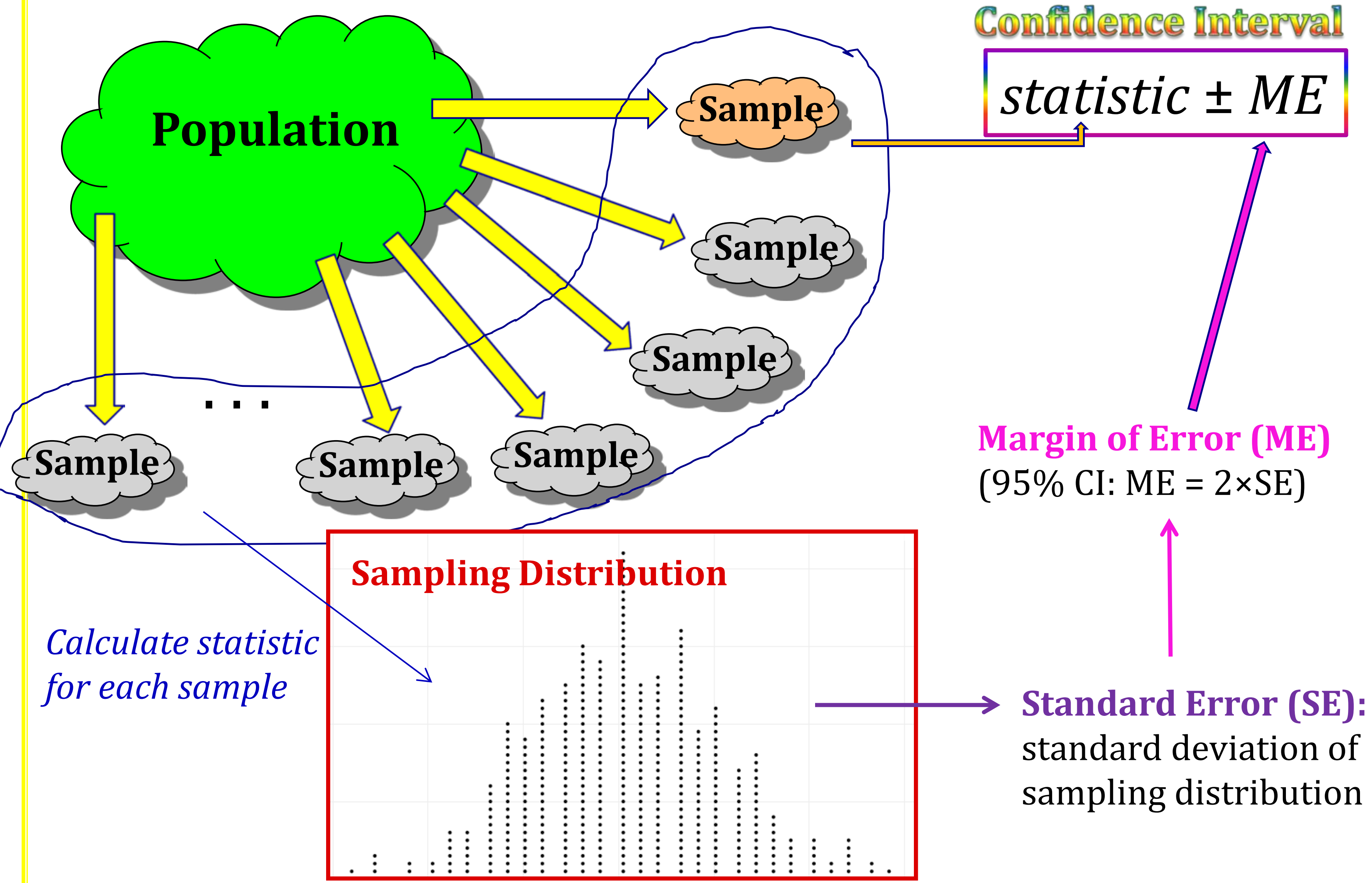
- Create a sampling distribution to calculate the standard error for $\hat{p}$



Common Misinterpretations

- Misinterpretation 1: "*A 95% confidence interval contains 95% of the data in the population*"
- Misinterpretation 2: "*I am 95% sure that the mean of a sample will fall within a 95% confidence interval for the mean*"
- Misinterpretation 3: "*The probability that the population parameter is in this particular 95% confidence interval is 0.95*"

Confidence Intervals



Summary

- To create a 95% confidence interval for a parameter:
  - Take many random samples from the population, and compute the sample statistic for each sample
  - Compute the standard error as the standard deviation of all these statistics
  - Use statistic $\pm 2 \times SE$

- One small problem…

Reality

… WE ONLY HAVE ONE SAMPLE!!!!

- How do we know how much sample statistics vary, if we only have one sample?!?

… to be continued