Assessing Process Capability in the Presence of Systematic Assignable Cause

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A process must be considered free from variation due to assignable cause before its capability can be determined. In those situations where variation due to an assignable cause occurs and is tolerated, process capability cannot currently be assessed. By allowing the process capability to be considered dynamic, a procedure for assessing capability is proposed and an estimator developed for those situations where a systematic assignable cause occurs. Some statistical properties of the estimator are examined and a technique for managing a process exhibiting a systematic assignable cause is discussed. Examples from a particular class of systematic assignable cause known as tool wear are included.

Introduction

PROCESS capability studies are conducted and their associated measures determined under the assumption that the process variation is due only to random causes, and are in fact valid only once the process under investigation is free from any special or assignable cause (i.e., in a state of statistical control). For those processes affected by special or assignable causes, process capability must be assessed during periods where the special causes are not present. The frequency and effect of special causes can vary from infrequent minor changes in the process (that are often undetectable) to large sustained systematic influences. Regardless of the frequency or magnitude of the assignable cause, traditional process capability measures should not be used in the presence of variation due to assignable cause.

In processes where variation due to a systematic assignable cause exists and is tolerated the overall variation consists of (i) variation due to assignable cause (e.g., the physical deterioration of a tool) and (ii) variation due to random causes (i.e., inherent variation). The variation resulting from a systematic assignable cause is usually identifiable and quite predictable. However, because the overall process variation arises from two sources (i.e., random causes and assignable causes), traditional measures of process capability are invalid as they confound the true process capability with some measure of assignable cause. In order to overcome this difficulty it is proposed that process capability be considered dynamic. That is, for a process exhibiting variation due to a systematic assignable cause, the capability of the process is considered to be constantly changing as the process ages. This represents a fundamental change from the traditional approaches used in assessing process capability in the presence of systematic assignable cause.

In the dynamic model, the ability of the process to meet specifications will vary. This variation is often quite predictable if no other shocks to the production process occur. The changing ability of the process can be monitored using a process capability index that considers both process variation and proximity to the target value. C_{pk} (Sullivan [1984, 1985], Kane [1986a, 1986b]) and C_{pm} (Chan, Cheng, and Spiring [1988]) are two of several competing indices which possess the ability to consider proximity to the target as well as process variability when assessing process capability. Both have reasonable point estimators that are easy to determine; however $\hat{C}_{pk}$, as defined by Kane (1986a) has a very complicated probability density function (pdf), making statistically-based inferences difficult. The estimator of C_{pm} suggested by Chan, Cheng, and Spiring (1988) has a pdf that can be used to derive statistically based inferences from the sampling results. For this reason the focus will be on C_{pm}, however the procedure is easily adaptable to C_{pk} once the pdf of $\hat{C}_{pk}$ is developed.

Assuming the process capability to be dynamic, the goal will be to maintain some minimum level of capability. This minimum level will vary from process to process and should reflect aspects such as (i) shut down costs/time, (ii) repair/replacement costs/

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time, (iii) cost of producing and identifying non-conforming product, (iv) sampling/monitoring costs, and (v) production levels and speeds. In order to maintain a minimum level of capability the process will be monitored using procedures similar to those used in control charting. When the process reaches some specified minimum level a warning should be issued indicating that the ability to produce conforming product is near an end.

Dynamic Process Capability

The most general case discussed will assume only the existence of a systematic assignable cause possessing a reasonably predictable recurring pattern with known upper specification limit (USL), lower specification limit (LSL), and target value (T) (Figure 1). The process specifications (i.e., USL, T, and LSL); the starting, stopping, and change times (i.e., t_0, t_1, t_2, t_3); and the process output have been included in Figure 1. The variation is depicted in a non-linear, increasing fashion but could be any reasonably consistent recurring shape. The change times may represent chronological time but are more likely to represent production quantities. The variation due to the assignable cause, although similar in shape in each cycle, is not identical, emphasizing the heterogeneity that may exist among cycles.

Permitting the process capability to be dynamic results in the capability index rising and falling over each cycle. Maximum capability becomes a parameter of the process and will occur at some combination of the inherent variation and proximity to the target. The capability of the process depicted in Figure 1 has been sketched in Figure 2. At t_0 the process is below target and as the process ages the systematic assignable cause produces a shifting of the process toward the target, causing the process capability to increase. As the tool continues to age, eventually movement is away from the target and consequently the process capability diminishes. This pattern is repeated over each cycle. The general model allows both proximity to the target and the inherent variability to be dynamic and as a result the maximum capability will be a parameter of the process and may occur at any point over a cycle. For the special case where the inherent variation is considered to be constant over a cycle, the maximum capability will occur when the process is producing at the target.

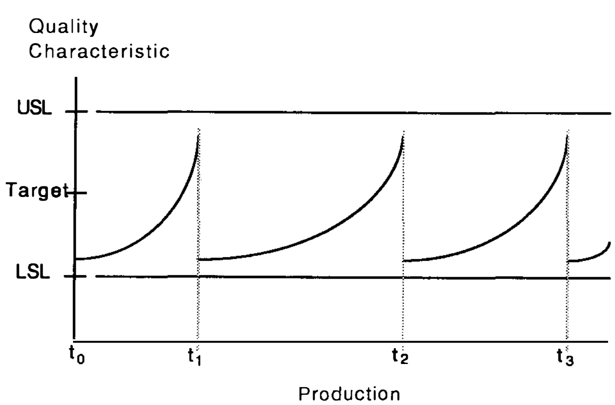

FIGURE 1. An Example of a Toolwear Problem.

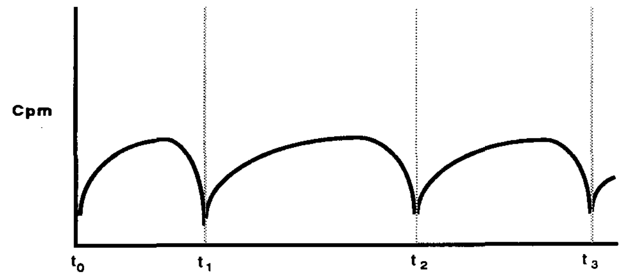

FIGURE 2. Plot of the Changing Capability of a Process Exhibiting Toolwear.

Consider a tool wear example taken from Grant and Leavenworth (1974) where the USL = 0.6480, LSL = 0.6400, and 13 subgroups of sample size five resulted in the data in Table 1. Assuming the tool does not deteriorate within a subgroup, $\hat{C}_{pm}$ has been calculated for each subgroup using T = 0.6440 and the algorithm

$$\hat{C}_{pm} = \frac{\text{USL} - \text{LSL}}{6\sqrt{\left(\frac{R}{d_2}\right)^2 + \frac{n(\bar{x} - \text{T})^2}{n-1}}}.$$

Plotting the values of $\hat{C}_{pm}$ versus its associated subgroup number (see Figure 3) illustrates the general

TABLE 1. Tool Wear Data from Grant and Leavenworth (1974)

Subgroup	$\bar{x}$	R	$\hat{C}_{pm}$
1	0.6417	0.0011	0.5110
2	0.6418	0.0016	0.5243
3	0.6424	0.0010	0.7271
4	0.6431	0.0015	1.1357
5	0.6433	0.0009	1.5456
6	0.6437	0.0010	2.5421
7	0.6433	0.0014	1.3817
8	0.6436	0.0004	2.8046
9	0.6441	0.0006	5.0028
10	0.6444	0.0011	2.1168
11	0.6456	0.0009	0.7305
12	0.6457	0.0007	0.6939
13	0.6454	0.0009	0.8298

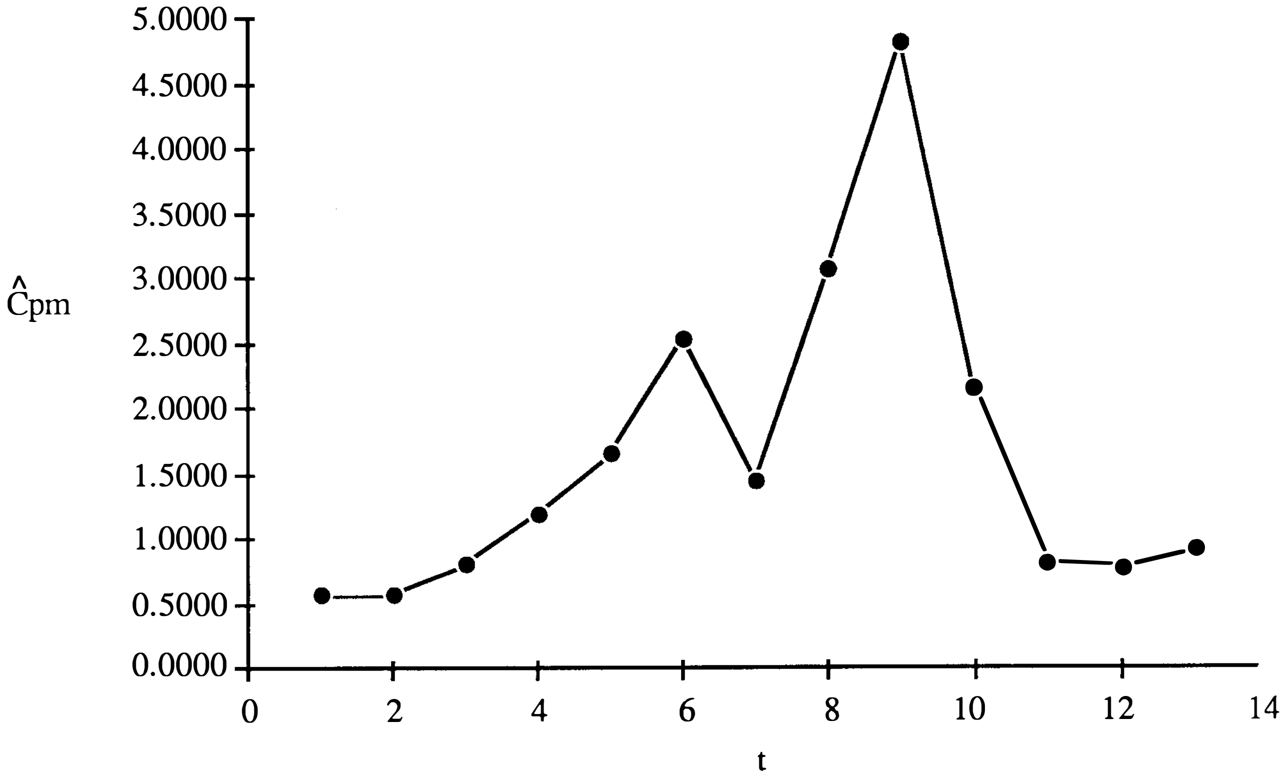

FIGURE 3. Plot of Estimated Capability over one Lifecycle.

rising and falling relationship that exists between process capability and the tool degradation within a single lifecycle of the tool. The dynamics offer insights into process performance as well as aids one in identifying those subgroups where other causes may have affected the process. For example in Figure 3, subgroup 7 appears to behave somewhat differently than expected which may indicate the presence of another assignable cause.

Measuring Process Capability in the Presence of a Systematic Assignable Cause

In those processes where the assumption of no deterioration over a subgroup is invalid the traditional measures of process capability fail to acknowledge that portions of the overall variation will be due to assignable causes. Hence any estimates of process capability will again confound the true capability with some measure of assignable cause. In order to obtain a true measure of process capability, any variation due strictly to assignable cause must be removed. Denoting the overall variability by σ^2, in the presence of variation due to a systematic assignable cause σ^2 consists of at least two discernible contributions. Letting σ_r^2 represent the variation due to random causes and σ_a^2 the variation due to assignable causes, $\sigma^2 = \sigma_r^2 + \sigma_a^2$. In assessing process capability only σ_r^2 and proximity to the target value should be considered.

Practitioners have generally taken one of two paths when attempting to assess capability in the presence of a systematic assignable cause. Some advocate that measurements taken over a cycle of the process represent the actual variability in the process. For example all measurements taken between t_1 and t_2 in Figure 4 would be used in assessing overall variation. As a result these practitioners effectively use σ^2 as a measure of variability when assessing capability. The resulting measure seriously confounds the inherent variability with the variability due to the systematic assignable cause.

A second, and more widely accepted, approach attempts to remove the variability associated with the systematic cause. This is usually accomplished by first modelling the process over a cycle (e.g., Quesenberry [1988], Long and DeCoste [1988]) and then removing the variability (i.e., σ_a^2) from the computations. For example in Figure 4 a curve would be fit to the observed points within a cycle (e.g., from t_1 to t_2) and the variability attributable to the assignable cause removed. These techniques work well but include

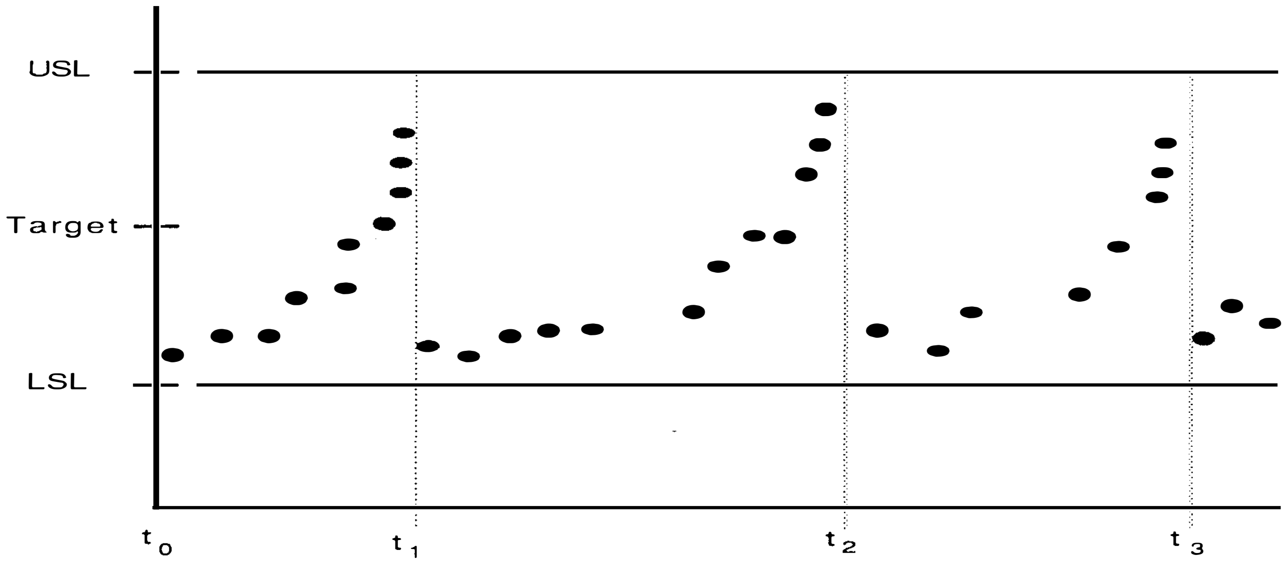

FIGURE 4. An Example of Monitored Tool Performance, with Measurements Taken Systematically over each Lifecycle.

drawbacks such as (i) the problem of highly complex models being required to fit a particular cycle and (ii) differing models for each cycle.

Both of the above approaches assume a static process capability over a cycle. Allowing the process capability to be dynamic within a cycle, as well as from cycle to cycle, circumvents some of the problems encountered in the above techniques. However assuming a constantly changing capability essentially requires that an instantaneous capability measure be made. By examining the process capability over small time periods, reasonable estimates of the dynamic capability can be obtained.

Figure 5 provides an exploded view of what might typically occur at some time t_a in a cycle of a process. The process capability at time t_a should reflect any variability due to random causes as well as proximity to the target value and it should not include any measure of variation due to assignable causes. If measurements were made on all the product produced in the "window" surrounding time t_a, clearly there would be variation attributable to assignable cause (i.e., the degradation of the process σ_a^2). In order to assess process capability at time t_a this source of variation will have to be removed.

The proposed measure of the dynamic process capability relates the allowable process spread to the actual process spread measured as squared deviations from the target, but free from variation due to assignable cause. The general form of the index will be

$$C_{pm} = \frac{\text{minimum}[\text{USL} - \text{T}, \text{T} - \text{LSL}]}{3\sqrt{\sigma_{rt}^2 + (\mu_t - \text{T})^2}}$$

where USL, LSL, and T are the usual upper specification, lower specification, and target respectively,

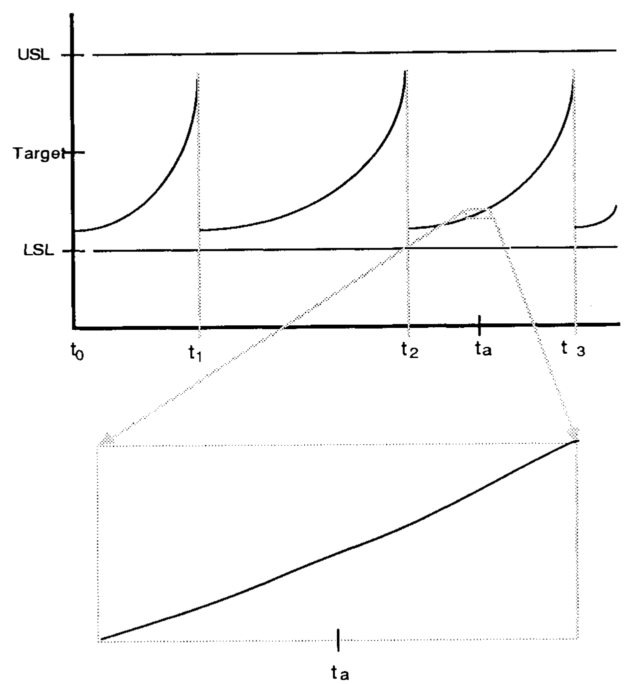

FIGURE 5. An Exploded View of Tool Performance at Time Period t_a.

used in assessing C_{pm}, while μ_t represents the mean and σ^2_{rt} the variation (due to random causes only) of the process at time t. Monitoring a process's capability will require finding the value of C_{pm} or a suitable estimate at various times over each cycle.

Assuming the effect of the assignable cause to be linear over the sampling window only (see Figure 6) allows estimates of C_{pm} that reflect only the inherent variability and proximity to the target value at any point in time. The proposed estimator is

$$\hat{C}_{pm} = \frac{\text{minimum}[\text{USL} - \text{T}, \text{T} - \text{LSL}]}{3\sqrt{\frac{(n-2)}{(n-1)}\text{MSE}_t + \frac{n(\bar{x}_t - \text{T})^2}{n-1}}}. \quad (1)$$

This measure of process capability considers only the proximity to the target value T and the variation associated with random causes as the linear effect of the toolwear is effectively removed by considering

$$\text{MSE}_t = \frac{\sum_{i=1}^{n} (x_{t_{ai}} - \hat{x}_{t_{a_i}})^2}{n-2}$$

of the sequentially selected points (i.e., $t_{a_1}, t_{a_2}, t_{a_3}, \cdots$) rather than the sample variance. In (1) MSE_t is the mean square error associated with the regression equation $\hat{x}_{a_i} = \alpha_a + \beta t_{a_i}$ and where t_{a_i} is the sequence number of the sampling unit (see Figure 7).

Consider the example given in Long and DeCoste (1988) where the process specifications were USL = 18, LSL = −18, and T = 0 and nine subgroups of size five were observed and results are given in Table 2. Using (1), the dynamic capabilities at each time period are given in Table 3. The results again illustrate the rising and declining relationship that occurs as the process first moves toward the target and then away from the target with a maximum occurring at t_5 (Figure 8).

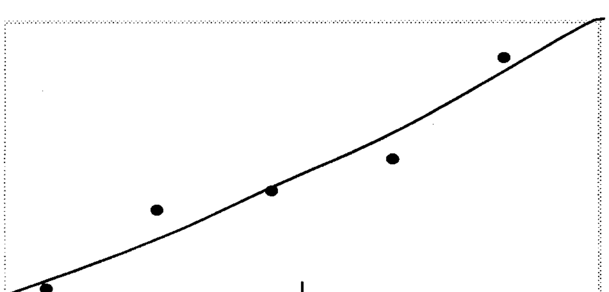

FIGURE 6. A Sampling Window with a Systematic Sample of Size 5 at Time t_o.

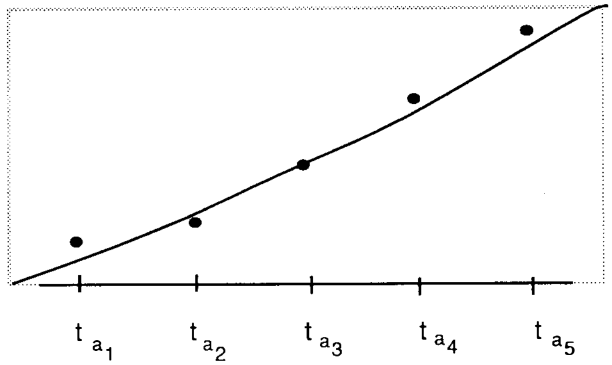

FIGURE 7. Sequential Sampling Results.

To illustrate the techniques used to arrive at the above estimates of dynamic capability, let us consider time period t_4. An ordinary least squares regression analysis using the measurements as values of the dependent variable resulted in the estimation equation $\hat{x}_{4_i} = -5.70 + 1.00 t_{4_i}$ for $t_{4_i} = 1, 2, 3, 4, 5$ over the window (see Figure 9) with an analysis of variance table

ANOVA

Source	df	Sum of Squares	Mean Square	F	Prob $> F$
Model	1	10.00	10.00	37.5	0.009
Error	3	0.80	0.27		

The sample mean over the window is −2.70 resulting in an estimate of

$$\hat{C}_{pm} = \frac{\text{minimum}[18 - 0, 0 - (-18)]}{3\sqrt{\frac{3}{4}(0.27) + \frac{5(-2.7)^2}{4}}} = 1.966.$$

An alternative technique for determining $\hat{C}_{pm}$ exists for the case where the measurements are taken systematically (i.e., 1 in M, $M \in Z^+$) over the window. The algorithm is

$$\hat{C}_{pm} = \frac{\text{minimum}[\text{USL} - \text{T}, \text{T} - \text{LSL}]}{3\left[\frac{\sum_{i=1}^{n} x_i^2}{n-1} - \frac{2n(n+1)}{(n-1)^2}\bar{x}^2 - \frac{12(\sum_{i=1}^{n} ix_i)^2}{n(n+1)(n-1)^2} + 12\bar{x}\frac{\sum_{i=1}^{n} ix_i}{(n-1)^2} + \frac{n(\bar{x} - \text{T})^2}{(n-1)}\right]^{1/2}}$$

TABLE 2. Results for the Example from Long and DeCoste (1988)

i	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9
1	−14	−11	−7	−5	−1.5	0	3.5	5	8
2	−11.5	−10	−6	−3	0	2.5	4.5	6	8
3	−11.5	−9	−5.5	−3	0.5	2.5	5	6.5	9
4	−11.5	−8.5	−5.5	−2	1	2.5	5	7	9
5	−10	−7	−4	−0.5	2.5	4	7.5	8	11

and does not involve formally determining the regression estimates.

Managing a Process Exhibiting Variation Due to a Systematic Assignable Cause

Utilizing the dynamic nature of the proposed capability measure and the properties of its estimator (see the Appendix), practitioners can monitor and manage processes under the influence of systematic assignable cause. Any sampling scheme used to monitor the capability of a process will need to reflect an array of factors associated with the process. Several factors, such as the frequency and magnitude of other assignable causes, can be dealt with in a manner similar to control chart sampling schemes (see Montgomery [1985]). However other factors such as the length of the sampling "window" will be influenced by factors not normally encountered in control chart sampling schemes.

The proposed sampling scheme is similar to those used in monitoring a process for control charting procedures. The general format will be to gather k subgroups of size n from each cycle (e.g., the period from t_2 to t_3 in Figure 5) over the lifetime of the tool. The value of k will be unique to each process and in fact may change from cycle to cycle. If the process under investigation is allowed to have variable length cycles, the number of subgroups used may differ from cycle to cycle. Sample sizes of less than five (i.e., $n < 5$) are cautioned against, while large samples (e.g., $n > 25$) can also pose a problem as the assumption of linearity over a window may be violated for large sample sizes. The optimal sample size for assessing process capability in the presence of systematic assignable cause will vary for each process considered. The manner in which the samples are taken will also be important. In most situations consecutive sampling is suggested (i.e., samples are collected as they come off the line). However, if desired, a systematic 1 in M sampling scheme can be used (e.g., within a window, every second unit off the line is collected).

Reaction limits based on the statistical properties of the proposed estimator can be created that provide the practitioner with criteria for adjusting the process. The reaction limits will be a function of several factors including (i) the capability measure used, (ii) the estimator used, (iii) the sample size, (iv) the parameters of the process (e.g., the portion of variation associated with the starting value ($100\%(|\text{Starting value} - \text{T}|/\sigma)$)), (v) the desired confidence levels, (vi) the specification limits, and (vii) the characteristics of the process. Letting c denote the level of C_{pm} to be maintained, α the desired level of significance, and n the sample size, the minimum acceptable value of $\hat{C}_{pm}$ associated with a C_{pm} of c for $(1 - \alpha)$ 100% level of confidence can be determined using Theorem 1 (see Appendix). The minimum acceptable values of $\hat{C}_{pm}$ have been determined for $n = 5, 10, 15$; $C_{pm} = 0.5, 1.0, 1.5$; $100\%\left(\frac{|\text{Starting value} - \text{T}|}{\sigma}\right) = 50\%, 100\%, 150\%$; and for $\alpha = 0.05, 0.01$ and are included in Table 4. The minimum values represent the reaction limits, or those values that indicate the ability of the process to produce conforming product is reaching its limit.

To illustrate the general procedure consider a process exhibiting variation due to a systematic assignable cause similar to the process depicted in Figure 1. Suppose the practitioner has decided to (i) maintain a C_{pm} of 0.5 for $(1 - \alpha)100\% = 95\%$ where USL = 15, LSL = −15, T = 0 and (ii) take consecutive samples of size five every two hours, with the first sample consisting of the five "first-off" pieces. The minimum acceptable value of $\hat{C}_{pm}$ associated with a

TABLE 3. Dynamic Capabilities at each Time Period from Table 2

	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9
$\hat{C}_{pm}$	0.458	0.590	0.957	1.966	9.370	2.254	1.046	0.825	0.595

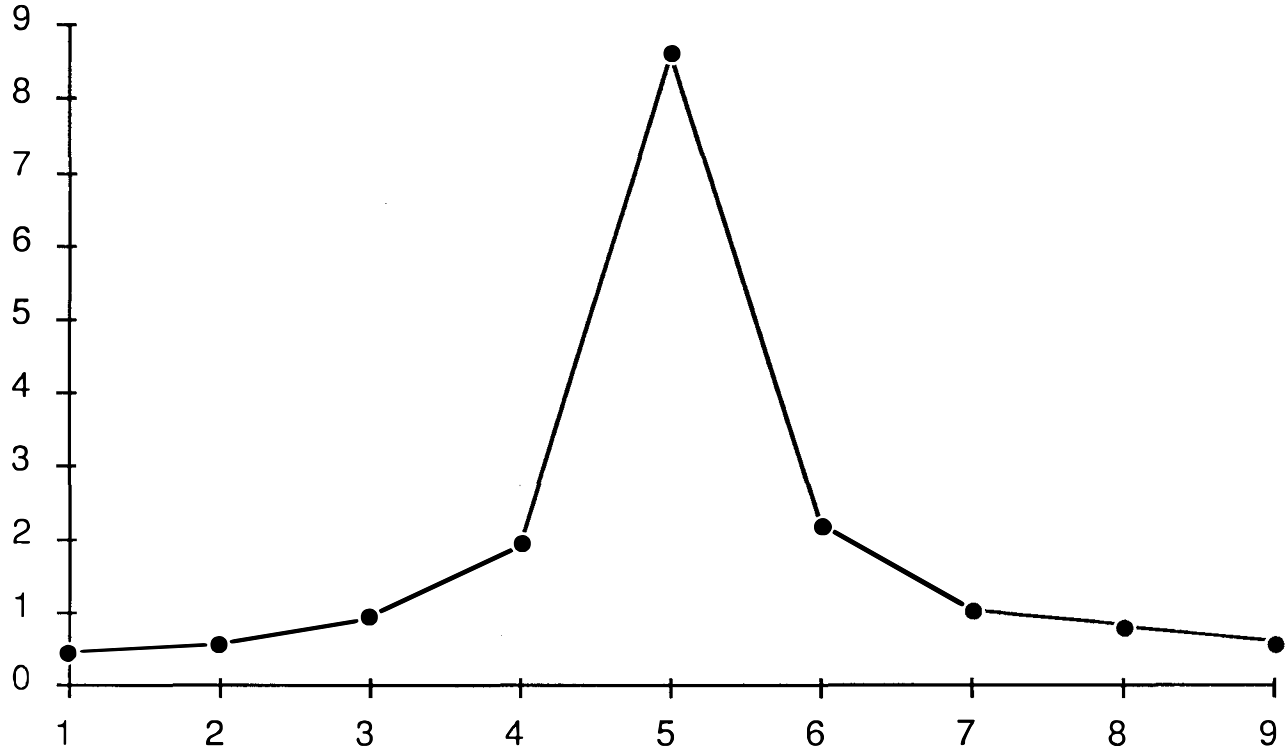

FIGURE 8. Example with Assignable Cause Removed.

$100\%\left(\frac{|\text{Starting value} - T|}{\sigma}\right) = 100\%$ is 0.77 (from Table 4). With this then the practitioner can monitor the capability of the process based on the results of the sequential samples of size 5 taken every second hour.

When the estimated process capability drops below the reaction limit of 0.77 the practitioner should stop the process and reset it as there is evidence to suggest that the process is nearing the end of its ability to produce conforming product. For values of $\hat{C}_{pm}$ greater than 0.77 the process is deemed capable and is allowed to continue. Figure 10 illustrates such a relationship and includes the actual measurements along with their specifications and process capability estimates over a single cycle of the process.

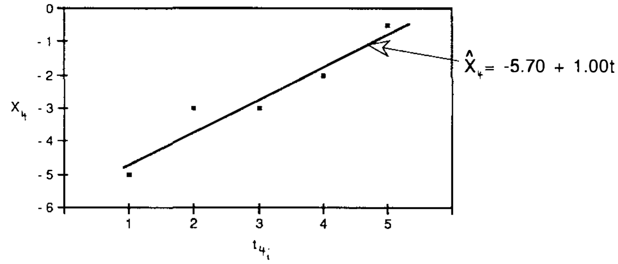

FIGURE 9. Regression Equation for Period t_4.

TABLE 4. Reaction Limits for Dynamic $\hat{C}_{pm}$

n	$\left(\frac{\lvert Start - T\rvert}{\sigma}\right)100\%$	C_{pm}	Minimum $\hat{C}_{pm}$ $\alpha = 0.05$	Minimum $\hat{C}_{pm}$ $\alpha = 0.01$
5	50%	0.5	0.78	0.92
5	100%	0.5	0.77	0.88
5	150%	0.5	0.74	0.83
5	50%	1.0	1.56	1.84
5	100%	1.0	1.54	1.77
5	150%	1.0	1.48	1.67
5	50%	1.5	2.35	2.76
5	100%	1.5	2.31	2.65
5	150%	1.5	2.23	2.50
10	50%	0.5	0.69	0.78
10	100%	0.5	0.68	0.75
10	150%	0.5	0.65	0.71
10	50%	1.0	1.38	1.55
10	100%	1.0	1.35	1.50
10	150%	1.0	1.31	1.43
10	50%	1.5	2.07	2.33
10	100%	1.5	2.03	2.25
10	150%	1.5	1.96	2.14
15	50%	0.5	0.65	0.72
15	100%	0.5	0.64	0.70
15	150%	0.5	0.62	0.67
15	50%	1.0	1.30	1.44
15	100%	1.0	1.28	1.40
15	150%	1.0	1.24	1.34
15	50%	1.5	1.95	2.16
15	100%	1.5	1.92	2.10
15	150%	1.5	1.86	2.01

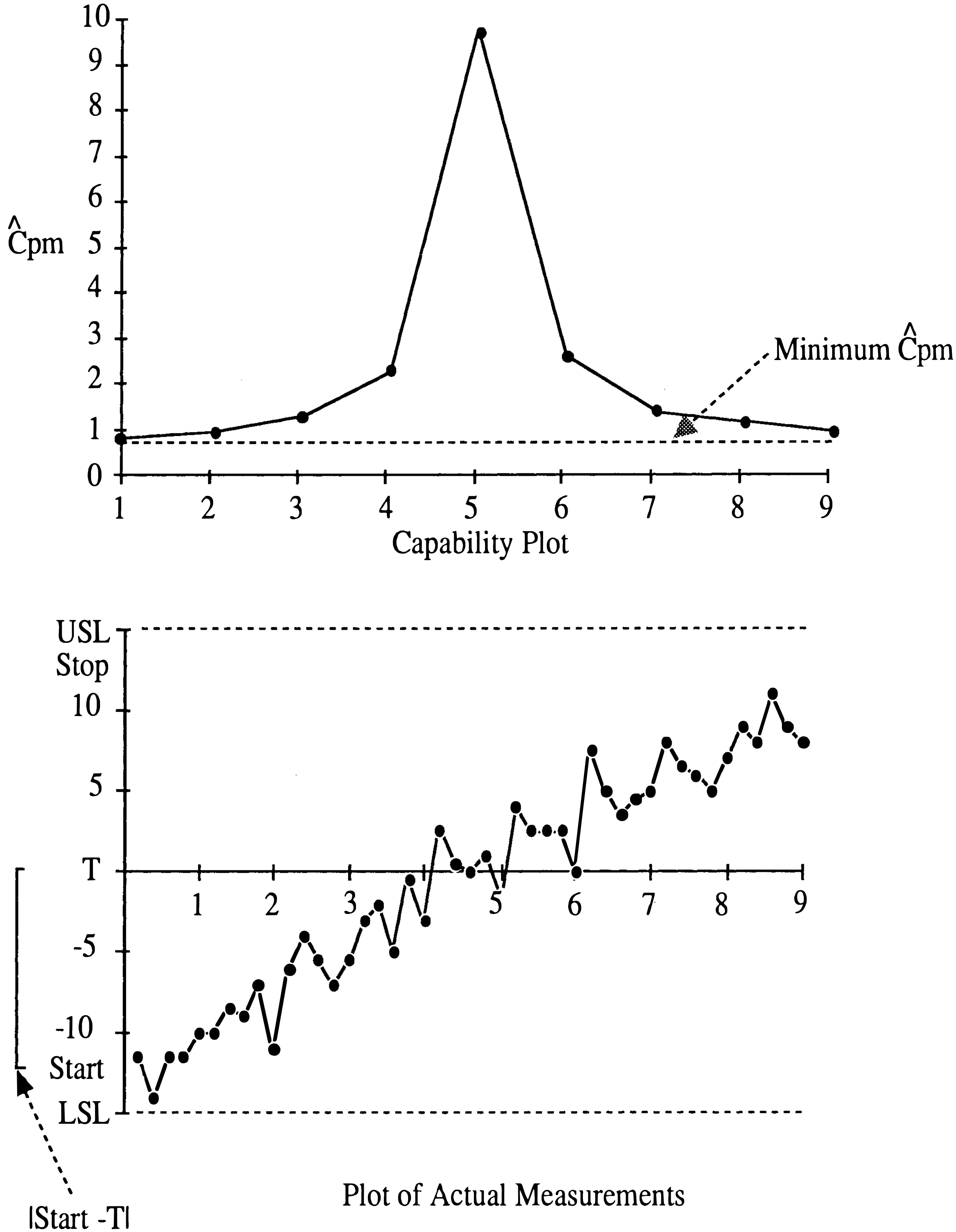

FIGURE 10. Capability Plot & Run Chart.

Comments

Allowing the process capability to be dynamic allows practitioners to monitor a process from a capability perspective. The minimum acceptable level of capability can be set and limits created to insure this level is maintained with some level of confidence. The process capability becomes a parameter of the process and can be managed using techniques similar to control charting procedures. Analogous to control charting, questions such as "How many? How often?" will have to be addressed for each process on an individual basis and will be a reflection of the process characteristics.

The single value assessment of process capability opens up the problem to other potential analysis techniques. For example the exponentially weighted moving average may allow practitioners to become pro-active in determining stopping/changing times.

Appendix

Theorem 1. Assuming the quality characteristic to be normally distributed, the pdf associated with $\hat{C}_{pm}$ as defined in (1) is a noncentral chi-square with $n-1$ degrees of freedom and non-centrality parameter $\lambda = n\dfrac{(\mu - \mathrm{T})^2}{\sigma^2}$ of the form

$$f(x) = \exp\left\{-\tfrac{1}{2}\left[\frac{(n-1)C_{pm}^2(1+\lambda/n)}{x^2} + \lambda\right]\right\}$$

$$\times \sum_{J=0}^{\infty}\left\{\frac{\left[\dfrac{(n-1)C_{pm}^2(1+\lambda/n)}{x^2}\right]^{[(n-1/2)]+J-1}\lambda^J}{\Gamma\left(\dfrac{n-1}{2}+J\right)2^{2J+[(n-1)/2]}J!}\right\},$$

$$0 < x < \infty.$$

Proof: Let $x \sim N(\mu, \sigma^2)$ so that $\sum_{i=1}^{n}(x_i - \mathrm{T})^2 \sim \sigma^2\chi_n^2(\lambda)$ where χ_n^2 denotes a noncentral chi-squared distribution with n degrees of freedom and non-centrality parameter $\lambda = n\dfrac{(\mu-\mathrm{T})^2}{\sigma^2}$. Rewriting $\sum_{i=1}^{n}(x_i - \mathrm{T})^2 = \sum_{i=1}^{n}(x_i - \bar{x})^2 + n(\bar{x}-\mathrm{T})^2 = \sum_{i=1}^{n}(x_i - \hat{x}_i)^2 + \sum_{i=1}^{n}(\hat{x}_i - \bar{x})^2 + n(\bar{x}-\mathrm{T})^2$ where $\hat{x}_i$ denotes the ordinary least squares (OLS) estimator such that $\hat{x}_i = \hat{\alpha} + \hat{\beta}t_i$. It follows that

$$\underbrace{\sum_{i=1}^{n}\frac{(x_i-\hat{x}_i)^2}{\sigma^2}}_{\mathrm{A}} + \underbrace{\sum_{i=1}^{n}\frac{(\hat{x}_i-\bar{x})^2}{\sigma^2}}_{\mathrm{B}} + \underbrace{\frac{n(\bar{x}-\mathrm{T})^2}{\sigma^2}}_{\mathrm{C}} \sim \chi_n^2(\lambda).$$

From Cochran's Theorem (Montgomery [1976]) A, B, and C are independent χ^2 distributions with $(n-2)$, 1, and 1 degrees of freedom, respectively. Then

$$\mathrm{A} + \mathrm{C} = \sum_{i=1}^{n}\frac{(x_i-\hat{x}_i)^2}{\sigma^2} + \frac{n(\bar{x}-\mathrm{T})^2}{\sigma^2}$$

is the sum of two independent chi-squared distributions resulting in

$$\sum_{i=1}^{n}\frac{(x_i-\hat{x}_i)^2}{\sigma^2} + \frac{n(\bar{x}-\mathrm{T})^2}{\sigma^2} \sim \chi_{n-2}^2(0) + \chi_1^2(\lambda)$$

$$= \chi_{n-2+1}^2(0+\lambda) = \chi_{n-1}^2(\lambda)$$

$$\Rightarrow \sum_{i=1}^{n}(x_i-\hat{x}_i)^2 + n(\bar{x}-\mathrm{T})^2 \sim \sigma^2\chi_{n-1}^2(\lambda).$$

Letting

$$x^2 = \frac{(n-2)\mathrm{MSE} + n(\bar{x}-\mathrm{T})^2}{(n-1)},$$

$$\text{then}\quad \frac{(n-1)x^2}{\sigma^2} \sim \chi_{n-1}^2(\lambda).$$

But

$$\frac{(n-1)x^2}{\sigma^2} = (n-1)\frac{a^2}{a^2}\frac{(1+\lambda/n)}{(1+\lambda/n)}\frac{x^2}{\sigma^2},$$

$$\text{where}\quad a = \frac{\mathrm{minimum}[\mathrm{USL}-\mathrm{T}, \mathrm{T}-\mathrm{LSL}]}{3}$$

$$\Rightarrow \frac{(n-1)x^2}{\sigma^2} = \frac{(1+\lambda/n)(n-1)}{\hat{C}_{pm}^2}\,\frac{a^2}{(1+\lambda/n)\sigma^2}$$

$$\Rightarrow \frac{(n-1)x^2}{\sigma^2} = \frac{(1+\lambda/n)(n-1)}{\hat{C}_{pm}^2}C_{pm}^2$$

therefore $(1+\lambda/n)(n-1)\dfrac{C_{pm}^2}{\hat{C}_{pm}^2} \sim \chi_{n-1}^2(\lambda)$ and the result follows directly.

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