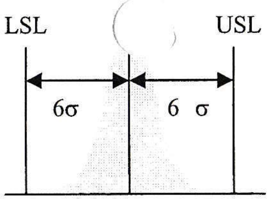
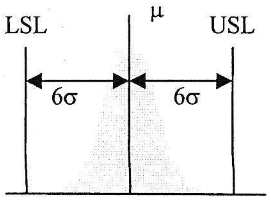


The Tech Group

Fractional Factorial Design Of Experiments

Lean Sigma Black Belt Training



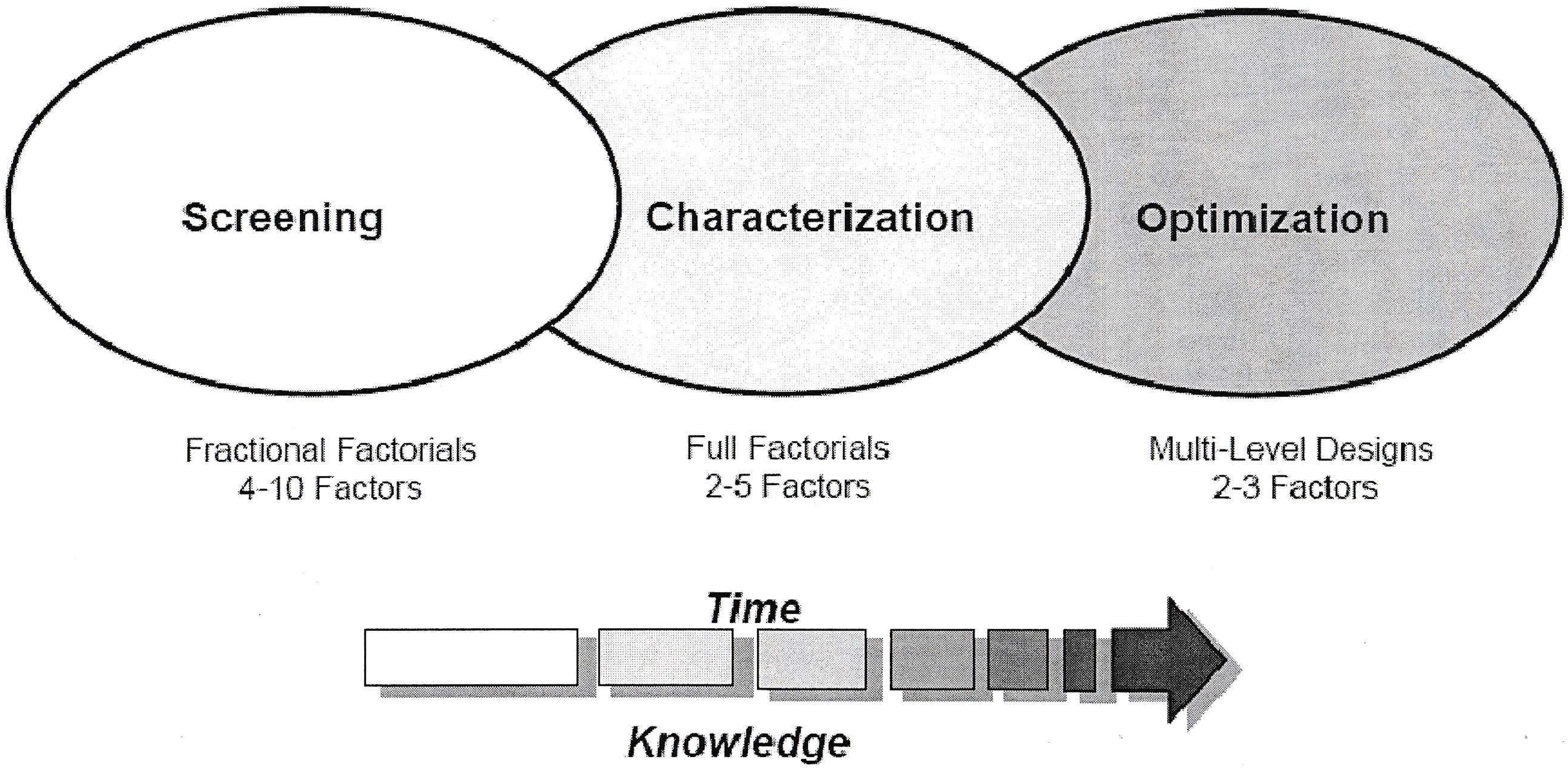
Module Objectives

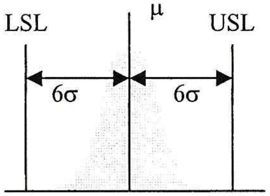
The Tech Group

Chapter Objectives:

- Understand how to create a fractional factorial design.
- Understand when to apply screening experiments.
- Determine how to design and analyze fractional factorials.
- Apply concepts to actual problems.

Families of Designs





The Tech Group

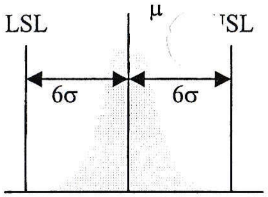
Factors vs. Runs
Full Factorial

As the number of factors increase, it quickly becomes expensive and impractical to conduct a full factorial.

A full factorial experiment provides complete information on main effects and interactions, but is all of this knowledge really needed?

Factors	Symbol	# of Runs
2	2^2	4
3	2^3	8
4	2^4	16
5	2^5	32
6	2^6	64
7	2^7	128
8	2^8	256
9	2^9	512
10	2^{10}	1024
11	2^{11}	2048
12	2^{12}	4096
13	2^{13}	8192
14	2^{14}	16384
15	2^{15}	32768

Note: 2 level designs



Fractional Factorial Designed Experiment

The Tech Group

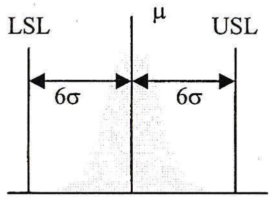
When process knowledge is low we typically have many possible factors to consider. We want to eliminate insignificant factors so we can focus on the critical few.

The experiment we run under these circumstances is called a **screening DOE**. A screening DOE is always a fractional factorial DOE.

Knowledge about some interactions is lost, but a fractional factorial experiment can be done with far fewer runs.

Factors	Symbol for Full	Full	1/2 Fract.	1/4 Fract.	1/8 Fract.	1/16 Fract.	1/32 Fract.	1/64 Fract.	1/128 Fract.
		# of Runs							
2	2^2	4							
3	2^3	8	4						
4	2^4	16	8	4					
5	2^5	32	16	8	4				
6	2^6	64	32	16	8	4			
7	2^7	128	64	32	16	8	4		
8	2^8	256	128	64	32	16	8	4	
9	2^9	512	256	128	64	32	16	8	4
10	2^{10}	1024	512	256	128	64	32	16	8
11	2^{11}	2048	1024	512	256	128	64	32	16
12	2^{12}	4096	2048	1024	512	256	128	64	32
13	2^{13}	8192	4096	2048	1024	512	256	128	64
14	2^{14}	16384	8192	4096	2048	1024	512	256	128
15	2^{15}	32768	16384	8192	4096	2048	1024	512	256

Note: 2 level designs



The Tech Group

Full vs. Fractional Factorial Experiments

Full factorial experiments work best when:

1. Only have a few factors of interest.
2. Knowledge of process is high so few factor needed.
3. Cost & time to conduct many runs is low.
4. Higher order interaction suspected.

Fractional factorial experiments work best when:

1. Many factors of interest.
2. Cost & time to conduct many runs is high.
3. Higher order interaction not suspected.
4. Knowledge of process is low so need to explore many factors (screening).

The Tech Group

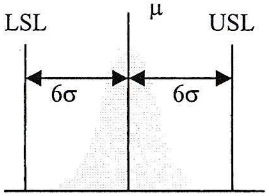
Fractional Factorials

With a full factorial DOE we can estimate main effects and effects from all possible interactions. How many effects exist with two factors? Three factors? Four factors?

<u>Example #1:</u> 2^2 Full Factorial (Factors A, B)
Main Effects: A & B = 2
Interactions: AB (2-way) = 1

<u>Example #2:</u> 2^3 Full Factorial (Factors A,B,C)
Main Effects: A, B & C = 3
Interactions: AB, AC, BC (2-way) & ABC (3-way) = 4

<u>Example #3:</u> 2^4 Full Factorial (Factors A,B, C, D),
Main Effects: A, B C & D = 4
Interactions: AB, AC, AD, BC, BD, CD (2-way)
ABC, ABD, BCD, ACD (3-way) & ABCD (4-way)= 11

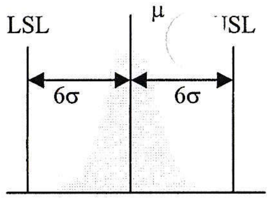


Higher Order Interactions

How likely is it a 3-way or higher interaction will be significant?

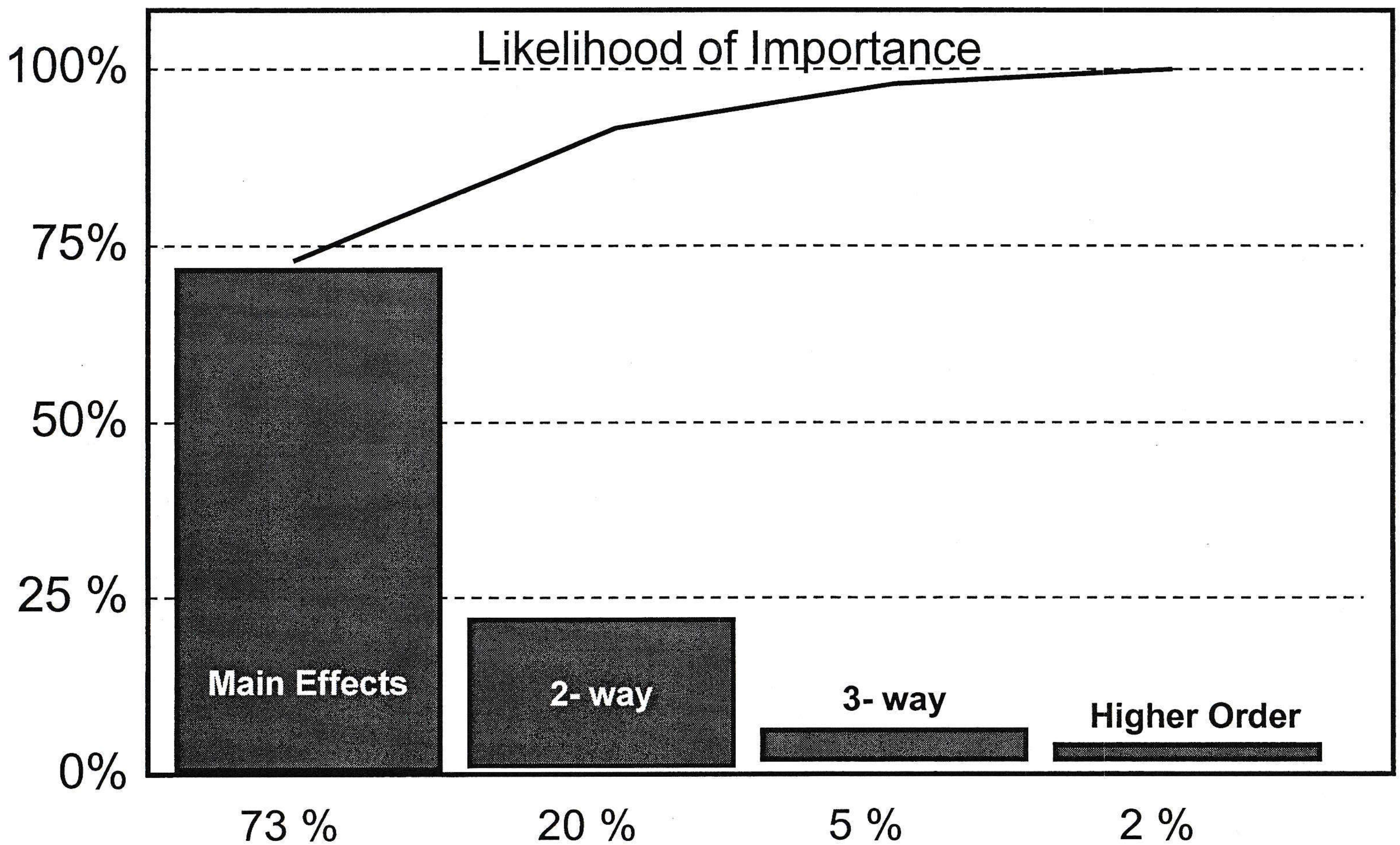
Design	# of Factors	Effects Estimated in Full Fractional DOE							Total 3-way & higher Interactions
		Main Effects	2-Way	3-Way	4-Way	5-Way	6-Way	7-Way	
2^2	2	2	1						0
2^3	3	3	3	1					1
2^4	4	4	6	4	1				5
2^5	5	5	10	10	5	1			16
2^6	6	6	15	20	15	6	1		42
2^7	7	7	21	35	35	21	7	1	99

Do we really need to know the effects of higher order interactions?



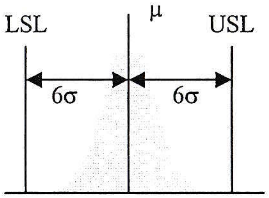
The Tech Group

Effects from Higher Order Interactions



Sparsity of Effects Principle:

Processes are likely to be driven by a few main effects and low order interactions



The Tech Group

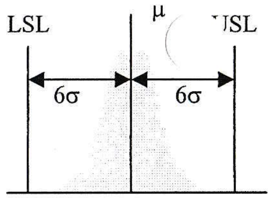
Fractional Factorials

Recall the Full Factorial Statapult Experiment?

Run	Stop Pin	Launch Angle	Tension Pin	Distance
1	- (3)	- (155)	- (2)	
2	+ (5)	- (155)	- (2)	
3	- (3)	+ (180)	- (2)	
4	+ (5)	+ (180)	- (2)	
5	- (3)	- (155)	+ (4)	
6	+ (5)	- (155)	+ (4)	
7	- (3)	+ (180)	+ (4)	
8	+ (5)	+ (180)	+ (4)	

Suppose instead of 3 factors we wanted to test 4 factors (adding hook position) but still using 8 runs.

How can the Sparsity of Effects principle be used to create a new 8 run design with 4 factors?



The Tech Group

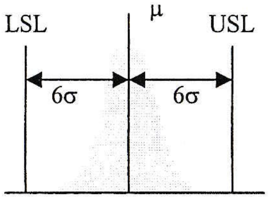
Creating a Fractional Factorial from a Full Factorial

3 factors at 2 levels (Stop Pin-A, Launch Angle-B & Tension Pin-C)

#	A	B	C	AB	AC	BC	ABC
1	-	-	-	+	+	+	−
2	+	-	-	−	−	+	+
3	-	+	-	−	+	−	+
4	+	+	-	+	−	−	−
5	-	-	+	+	−	−	+
6	+	-	+	−	+	−	−
7	-	+	+	−	−	+	−
8	+	+	+	+	+	+	+

Complete the interaction columns above.

- Which interaction is most unlikely to occur?
- Which column of + and – would you use for the added factor D?



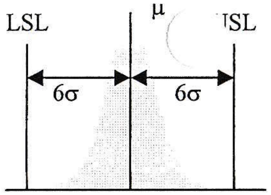
Creating a Fractional Factorial from a Full Factorial (con't)

4 factors at 2 levels

(Stop Pin-A, Launch Angle-B & Tension Pin-C with Hook-D added)

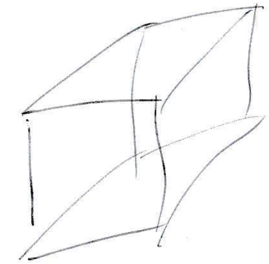
#	A	B	C	AB	AC	BC	D= ABC
1	-	-	-				-
2	+	-	-				+
3	-	+	-				+
4	+	+	-				-
5	-	-	+				+
6	+	-	+				-
7	-	+	+				-
8	+	+	+				+

- How would you calculate the effect of D?
- What happened to the ABC interaction?
- Where are the other interactions with D (AD, BD, CD, etc.)?



The Tech Group

Creating a Fractional Factorial from a Full Factorial (con't)



4 factors at 2 levels

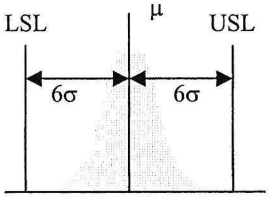
Identifying 2-way interactions

#	A	B	C	AB	AC	BC	D= ABC	AD	BD	CD
1	-	-	-	+	+	+	-	+	+	+
2	+	-	-	+	+	[illegible]	+	+	-	-
3	-	+	-	+	+	+	+	-	+	-
4	+	+	-	+	- -	-	-	-	-	+
5	-	-	+	+	+	-	+	-	-	+
6	+	-	+	-	-	-	-	-	+	-
7	-	+	+	-	+	+	-	+	-	-
8	+	+	+	+	+	+	+	+	+	+

Complete the 2-way interactions columns above?

Which 2-way interaction columns are the same?

What is the meaning of identical 2-way interaction columns?



The Tech Group

Aliased Effects

4 factors at 2 levels

Identifying similar patterns

				CD	BD	AD	D
#	A	B	C	AB	AC	BC	ABC
1	-	-	-	+	+	+	-
2	+	-	-	-	-	+	+
3	-	+	-	-	+	-	+
4	+	+	-	+	-	-	-
5	-	-	+	+	-	-	+
6	+	-	+	-	+	-	-
7	-	+	+	-	-	+	-
8	+	+	+	+	+	+	+

Aliased

The unique effect of a factor or interaction can not be separated from the unique effect of another factor or interaction because both columns have the *same* sign at each experimental run.

Expression for Fractional Factorial

The expression for fractional factorials is:

Number of Factors

$$2_R^{\,n-p}$$

Number of Levels

Size of Fraction

Resolution

EXAMPLE:

2^4 where 2 is the number of levels and 4 is the number of factors.

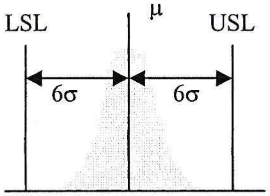
$2^4 = 16$: We want 8.

8 is 1/2 of 16, therefore we want 1/2 of 2^4.

1/2 may be written as 2^{-1}.

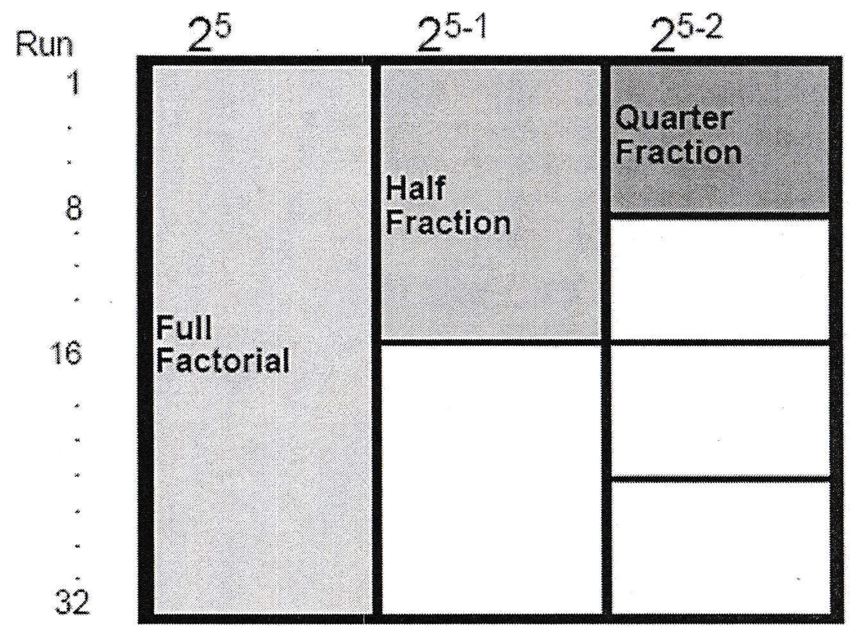
Therefore it may be rewritten as $(2^{-1})\ (2^4)$ or 2^{4-1} (add exponents). This is of the form 2^{n-p} where p is the fraction.

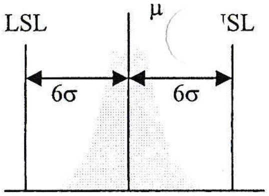
Adding the resolution designation we get $\mathbf{2_{IV}^{4-1}}$.



Expression for Fractional Factorial (con't)

- 1/2 Fraction
  - 2^{n-1} fractional factorial design
- 1/4 Fraction
  - 2^{n-2} fractional factorial design
- 1/8 Fraction
  - 2^{n-3} fractional factorial design



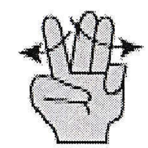


Experiment Resolution

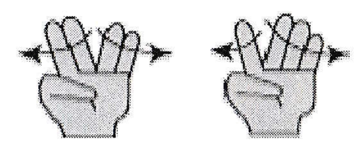
Resolution refers to the amount of information that may be obtained from a given experiment.

- The higher the resolution, the more information may be obtained from the experiment (i.e., learn about interactions and higher order terms).
- For most experiments, three resolutions are appropriate to discuss (III, IV, & V).

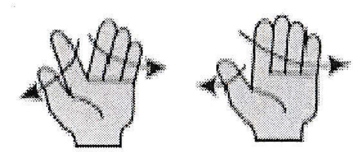
Resolution III: A design in which main effects are separated from other main effects, but not from interactions. That is, 2-way interactions & higher are confounded or aliased with main effects.

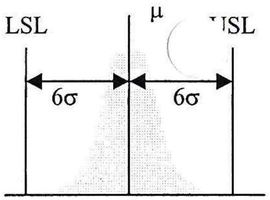


Resolution IV: A design in which main effects may be separated from other main effects and two-way interactions (two factor), but two-way interactions are confounded with other two-way and higher order interactions.



Resolution V: A design in which main effects may be separated from other main effects, and two-way interactions may be separated from other two-way interactions, but higher order interactions are confounded.





Team Exercise – Fractional Factorial Experiment

The Tech Group

Fill in the levels settings (Use 1 & 3) for the 3rd factor (Tension Pin) that has been added to the 2^2 full factorial shown.

A B C AB AC ABC BC

Stop Pin & Angle Interaction

Run	Stop Pin	Launch Angle	Tension Pin	Distance
1	- (3)	- (155)	+ (3)	
2	+ (5)	- (155)	- (1)	
3	- (3)	+ (180)	- (1)	
4	+ (5)	+ (180)	+ (3)	

~~I 10-6=4~~
~~5+3=2~~
~~2~~
2

When the effect of Tension is calculated, what else is included in this estimate of the effect? 2

5 2
3

183
33
-155

What is estimated with the Stop Pin and Angle Effect estimates?