by
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Sequential Range Capability Test

ONE of the objectives of this department is to bring to light useful statistical techniques that have lain dormant for many years. I believe a good example is the technique for assessing the capability of a process by means of a sequential range test discussed by Wies and Burr (1964).

The word "capability" is used here to refer to the number of standard deviations in the tolerance (T). Tolerance is defined to be the difference between the upper and lower specifications. To quote from the original article, "If the process standard deviation should be as much as a sixth of the tolerance, then there will be very little room for the process level to be off from the nominal dimension. On the other hand if the process standard deviation is, say, a tenth of the tolerance or less, then the process can comfortably meet the specifications with some allowance for setup error and tendency to drift."

One might feel that, if a process is in statistical control we will have a good estimate of its standard deviation (namely, $\bar{R}/d_2$), and it would then only be necessary to decide whether T/σ were large enough. On the other hand, if a process is not in statistical control, we are not in a position to forecast its output. Wies and Burr (1964) adopt a middle ground. They argue that a series of consecutive measurements (say, 50) will reflect only short-term variability and that, over a longer time, the process mean will not be held exactly at the nominal level with the result that the overall process variation will be increased. To take this into account, they suggest that the short-term standard deviation be small enough that the tolerance is at least 8σ. If T/σ is as low as six, then very good statistical control of the process average will be required to avoid items outside of the specifications. If it is as much as ten, then there will be ample room for tool wear, inaccurate setup, and so on.

KEY WORDS: Process Capability, Sequential Tests

The Test

With these considerations in mind, the test was set up as follows.

$$\text{Rejectable condition: } \sigma = T/6 \text{ (or more)}$$

$$\text{Acceptable condition: } \sigma = T/10 \text{ (or less)}$$

Two sets of risks were used: $\alpha = \beta = 0.05$ and $\alpha = \beta = 0.01$ for the conditions listed above. Table 1 gives acceptance and rejection values for these two sequential tests. Samples of size eight are used throughout. A normal distribution is assumed.

The test is carried out as follows. The range R_1 of the first sample of eight is compared to the acceptance and rejection multiples of T obtained using the factors in row one of Table 1. If it is less than $0.19T$, accept the process. If it is greater than $0.54T$, reject the process. Otherwise take a second sample of eight and find its range R_2. If $R_1 + R_2 \leq 0.55T$, accept the process. If $R_1 + R_2 > 0.90T$, reject the process. Otherwise take a third sample of eight and check $R_1 + R_2 + R_3$ against $0.92T$ and $1.26T$, and so on, until a decision is reached. Notice that, for $\alpha = \beta = 0.05$, a decision will be reached by the eighth sample. Similarly, for $\alpha = \beta = 0.01$, the test is truncated at the twelfth sample.

A useful property of this test is that the acceptable and rejectable values do not have to be $T/10$ and $T/6$, respectively, but can be $T/(10K)$ and $T/(6K)$ with any finite, positive value K. For example, if $T/8.33$ is regarded as acceptable and $T/5$ is regarded as rejectable, it is only necessary to multiply the acceptance and rejection values by $5T/6$. Note that the same factor K (here 5/6) is applied to both the acceptance and rejection values. The error rates α and β are unchanged. Wies and Burr (1964) provide additional discussion and cite Cox (1949) as a basic reference.

Example (from Wies and Burr)

A machine is to be tested for its capability of meeting a tolerance of $T = 0.002''$. A tenth of this, $0.0002''$, would be acceptable for a short-time stan-

TABLE 1. Tolerance Multipliers for Comparison with Cumulative Range of Samples of n = 8. Acceptable σ = T/10, Rejectable σ = T/6.

No. of Ranges in Sum	Cumulative Sample Size	Tolerance Multipliers $\alpha = \beta = 0.05$ Acc. Value	$\alpha = \beta = 0.05$ Rej. Value	$\alpha = \beta = 0.01$ Acc. Value	$\alpha = \beta = 0.01$ Rej. Value
1	8	0.19	0.54	0.09	0.63
2	16	0.55	0.90	0.46	1.00
3	24	0.92	1.26	0.82	1.36
4	32	1.28	1.63	1.18	1.73
5	40	1.64	1.99	1.55	2.09
6	48	2.01	2.36	1.91	2.45
7	56	2.37	2.72	2.27	2.82
8	64	2.91	2.91	2.64	3.18
9	72			3.00	3.54
10	80			3.36	3.91
11	88			3.73	4.27
12	96			4.36	4.36

dard deviation; whereas a sixth of it, 0.0003″ would be rejectable. Suppose a scale reading in units of 0.0001″ is used. Then $T = 20$ in these units. Acceptance and rejection values will, therefore, be 20 times the values given in Table 1. Risks of $\alpha = \beta = 0.05$ were chosen. Samples of size $n = 8$ were drawn from a process with $\sigma = 3.4$ with the results shown in Table 2.

TABLE 2. An example for which True σ = 3.4. (Acceptance and Rejection Based on σ = 2.00 and σ = 3.33, respectively.) T = 20, $\alpha = \beta = 0.05$.

Sample No.	Accept Value	R	Sum of R	Reject Value
1	3.8	7	7	10.8
2	11.0	6	13	18.0
3	18.4	8	21	25.2
4	25.6	6	27	32.6
5	32.8	13	40 Reject	39.8

The cumulative sum of the ranges exceeded the rejection value on the fifth sample and the process was (correctly) rejected.

Advantages

In common with most sequential tests, this test will give an early decision if the process is very poor or very good while still providing the required protection against wrong decisions. Further, its use of ranges gives it a computational advantage over the sequential use of variances. Formulas for the operating characteristic curve and the average sample number are available in closed form.

References

Cox, D. R. (1949). "The Use of Ranges in Sequential Analysis". *Journal of the Royal Statistical Society*, Series B, 11, pp. 101–114.

Wies, Jr., H. M. and Burr, I. W. (1964). "Simple Capability Analysis Acceptance Test". *Industrial Quality Control* 21, pp. 266–268.