shortcut

Statistical Tolerancing in Design for Six Sigma

This short cut is an adaptation of the forthcoming book
Commercializing Great Products with Design for Six Sigma
(www.prenhallprofessional.com/title/0132385996, Prentice Hall).

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BONUS: This Short Cut contains embedded files

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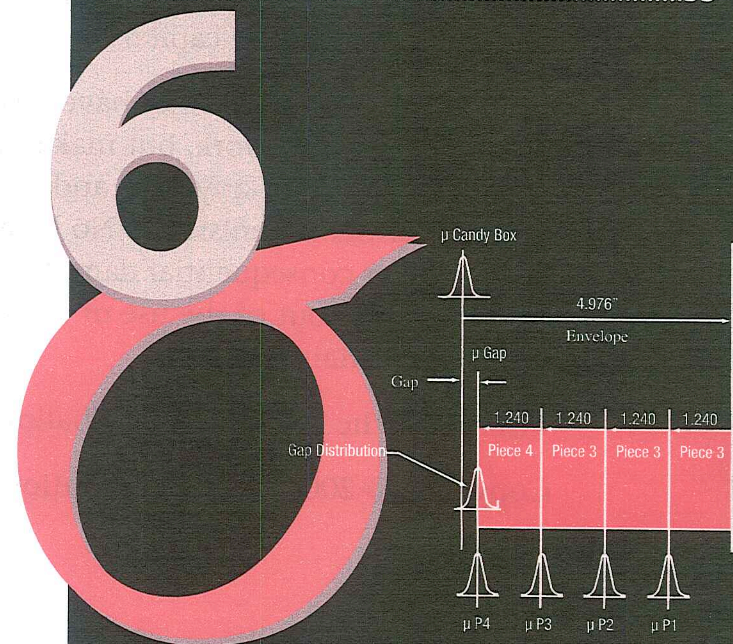

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What This Short Cut Covers

Development of a new product requires the product development team to address many complex customer requirements during the commercialization process. Consider a situation in which a new product being developed must meet specified upper and lower specification limits based on Voice of the Customer interviews. The design team must model and understand the sources of potential variation in the new product that need to be monitored and controlled if the product is to meet the identified customer needs. The process of analyzing component variation and designing a final product that meets customer tolerance requirements is known as statistical tolerancing.

In this short cut, various Design for Six Sigma techniques for determining the impact of multiple sources of variation on a final product are examined in detail. A procedure is described for using representative models for individual product components to estimate the expected overall level of variation in the performance of a final product.

Three methods of tolerance analysis are presented and the merits of each are discussed: Worst Case Analysis, Root Sum of Squares Analysis, and Six Sigma Tolerance Analysis. A detailed case study example, involving multiple sources of variation, is employed to illustrate the application of these methods. Minitab® is used to identify the best-fitting distributions from data sets for individual components. Monte Carlo Simulation with Crystal Ball® is then employed to determine the most important individual sources of variation and the overall variation of the final product. Finally, Crystal Ball's OptQuest® optimization feature is utilized to determine the required design value for each key parameter to meet final customer requirements.

Introduction

During the commercialization process, we often have to determine the impact of multiple sources of variation on our final product. As we develop representative models for individual product components, we can use this information to estimate the overall level of variation we expect to find in our final product. The process of analyzing component variation and designing a final product that meets customer tolerance requirements is known as *statistical tolerancing*.

SECTION 2

Worst Case Analysis

Files Embedded

In order to demonstrate the concept of statistical tolerancing, in this short cut we provide an Excel Case Study and Minitab data file adapted from our forthcoming book *Commercializing Great Products with Design for Six Sigma* (2007, http://www.prenhallprofessional.com/title/0132385996). In order to follow the step-by-step analysis provided in the short cut, it is also necessary that the reader have copies of of Minitab statistical software and Crystal Ball simulation software. A copy of Crystal Ball's OptQuest optimization routine is also required in order to perform the optimization analysis presented. Trial versions of Minitab (http://www.minitab.com) and Crystal Ball (http://www.decisioneering.com) may be downloaded from the Web sites provided.

Let's suppose for moment that we've been asked to make some candy for an upcoming family gathering. As shown in Figure 1, we have selected a candy box and must determine how many pieces of candy we can put in the box given its length. Initially, it seems pretty obvious that we can fit four pieces of candy lengthwise in the box. But as we see in the figure, there is a gap between the candy and the end of the box. Is the gap important? Does the gap vary? How do we estimate the size of the gap? To answer these questions, we introduce three methods of tolerance analysis:

- Worst Case Analysis
- Root Sum of Squares (RSS)
- Six Sigma Tolerance Analysis

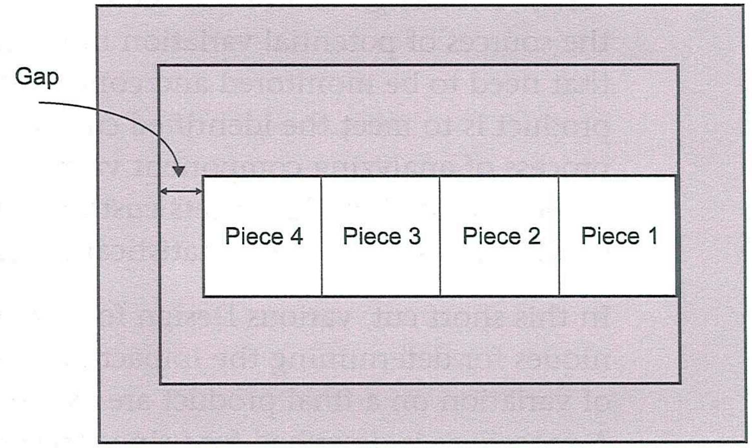


FIGURE 1 Candy box top view

Worst Case Analysis

The first type of tolerance analysis we consider is *worst case analysis*. In worst case tolerancing analysis, we must have additional information about the dimensions of the candy pieces and the candy box being used. As we see in Figure 2, each piece of candy has a nominal length of 1.240 inches, while the candy box is expected to have a length of 4.976 inches. Using this information, we can determine the number of candy pieces that will fit within the length of the box. We can also develop an initial estimate of the gap dimension, as shown in Figure 3. We expect that we will be able to put four pieces of candy in each row of the box given the box length dimension. In addition, we estimate that the nominal value of the gap will be 0.016 inches. But are things really this simple? As we also see in Figure 2, both the candy pieces and the box have variation of +/–0.003 inches around their nominal dimensions.

Using the nominal dimensions and variation estimates for the candy pieces and the box, we can estimate the nominal and worst case values for the gap. As shown in Table 1, the minimum worst case estimate of the gap is calculated by taking the lowest value estimated for the box dimension and subtracting the highest values estimated for the candy pieces. Using this set of worst case assumptions, we determine that the minimum worst case gap is estimated to be 0.001 inches.

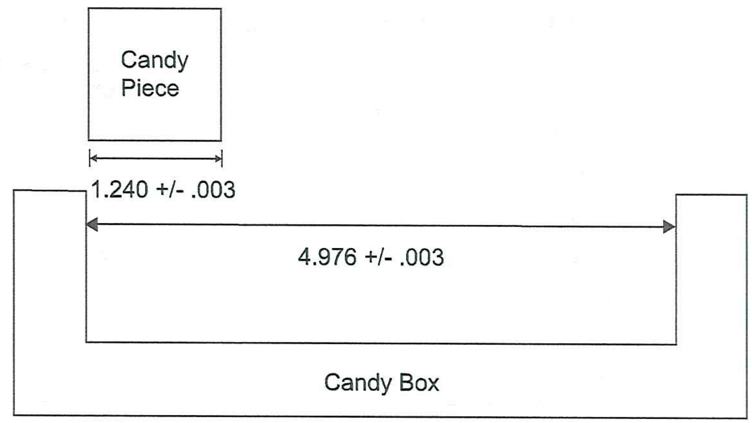


FIGURE 2 Candy and box dimensions: side view

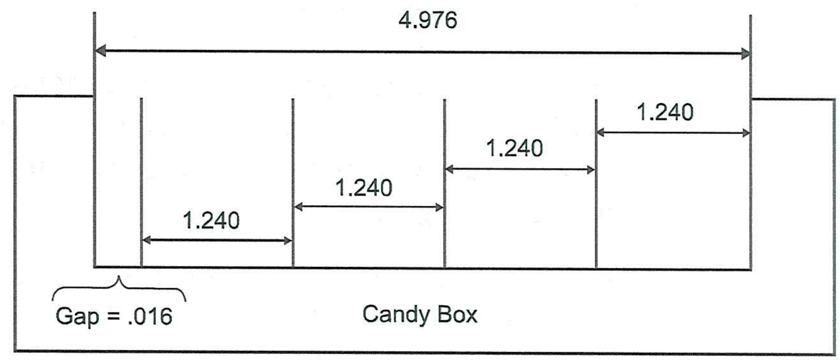


FIGURE 3 Nominal gap value

SECTION 2

Worst Case Analysis

TABLE 1 Worst Case Gap Calculations

	Nominal	Nominal Candy Dimensions	Minimum	Maximum Candy Dimensions	Maximum	Minimum Candy Dimensions
Candy Box	4.976		4.973		4.979	
Piece 1		–1.24		–1.243		–1.237
Piece 2		–1.24		–1.243		–1.237
Piece 3		–1.24		–1.243		–1.237
Piece 4		–1.24		–1.243		–1.237
Total Candy	–4.960		–4.972		–4.948	
Gap	0.016		0.001		0.031	

Similarly, the maximum gap can be estimated by using the maximum box dimension and the lowest candy dimensions. Using the given candy and box dimensions, we can estimate the worst case maximum gap to be 0.031 inches. A diagram representing the minimum and maximum worst case tolerance scenarios is presented in Figure 4.

We now understand that under the worst case assumption, the gap between the candy and the box is expected to be 0.016 inches, but we also understand that it could range from a minimum of 0.001 inches to a maximum of 0.031 inches. If we are in the process of commercializing a new candy product, should we proceed to production scale-up using these estimates?

What are the chances that we will really have a worst case condition in a given box of candy? If we use the tolerance information of +/–0.003 inches as an estimate of +/–3 standard deviations for normally distributed data, the probability of receiving an abnormally long or abnormally short piece of candy at the extreme values of the range would be 1–0.9973 or 0.0027. For all four pieces of candy and the box to be at an extreme value the probability would be $(0.0027)^5$ or 0.000000000000143. This is certainly a very unlikely event! Given the very low probability that either worst case scenario will occur, we turn to a more practical and useful tolerancing method, the root sum of squares analysis.

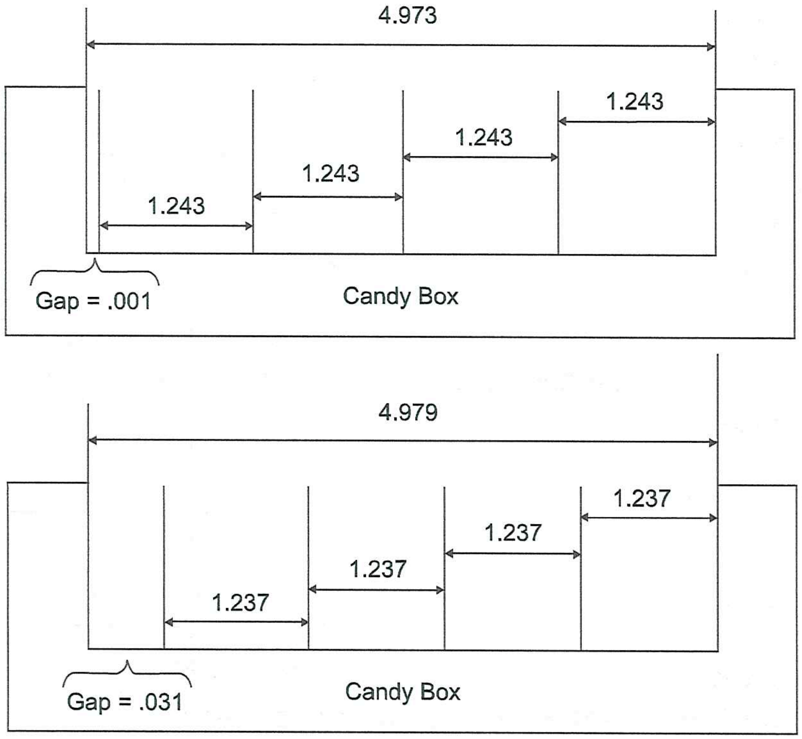


FIGURE 4 Worst case gap estimates

Root Sum of Squares Analysis

As we have seen, worst case tolerance analysis is interesting but the probability that a worst case scenario will actually occur is very low. To develop a more realistic tolerancing analysis we turn to the root sum of squares (RSS) technique. To demonstrate the RSS technique, let's again examine the candy packaging arrangement presented in Figure 5.

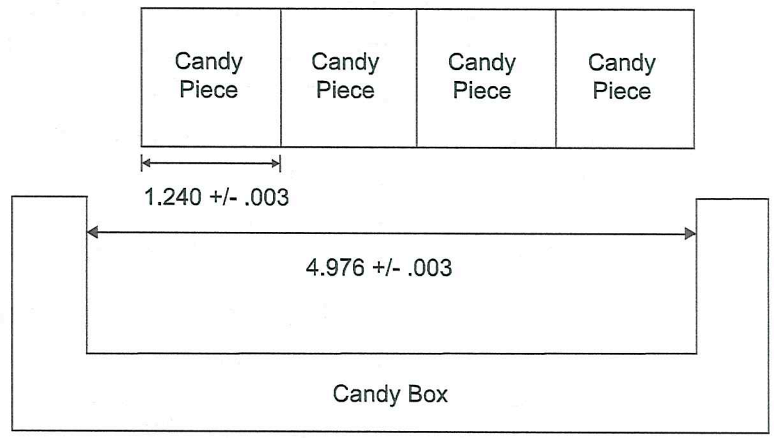


FIGURE 5 Root sum of squares candy arrangement

As stated earlier, a worst case tolerance analysis occurs when each piece of candy and the box are each at their maximum levels of variation from the nominal. For our candy packaging example, the worst case value is calculated as:

$$\begin{aligned}\text{Worst Case Variation} &= \text{Piece}_1 + \text{Piece}_2 + \text{Piece}_3 + \text{Piece}_4 + \text{Piece}_5 + \text{Box}\\ &= 0.003 + 0.003 + 0.003 + 0.003 + 0.003\\ &= 0.015 \text{ inches}\end{aligned}$$

SECTION 3

Root Sum of Squares Analysis

A more realistic estimate of variation occurs when we take advantage of the statistical fact that variances are additive. Let's now suppose that +/–0.003 inches represents +/–3 standard deviations of variation for each piece of candy and for the box. In this case, the standard deviation for each component is 0.001 inches. Using the additive property of variances, we can now estimate the standard deviation of the candy box assembly gap shown in Figure 6 as:

$$\sigma = \sqrt{(0.001)^2 + (0.001)^2 + (0.001)^2 + (0.001)^2 + (0.001)^2} = 0.0022 \text{ inches}$$

If the tolerance for the gap is plus or minus 3 standard deviations, it can be calculated as:

$$3\sigma = \sqrt{(0.003)^2 + (0.003)^2 + (0.003)^2 + (0.003)^2 + (0.003)^2} = 0.0067 \text{ inches}$$

Using our new value for root sum of squares tolerance, we can develop a new expected tolerance range for the gap, as seen in Figure 7. The nominal value for the gap is still 0.016 inches, as calculated for our worst case tolerance analysis. The root sum of squares tolerance value of 0.0067 inches is now used to calculate a minimum gap estimate of 0.0093 and a maximum estimate of 0.0227 inches.

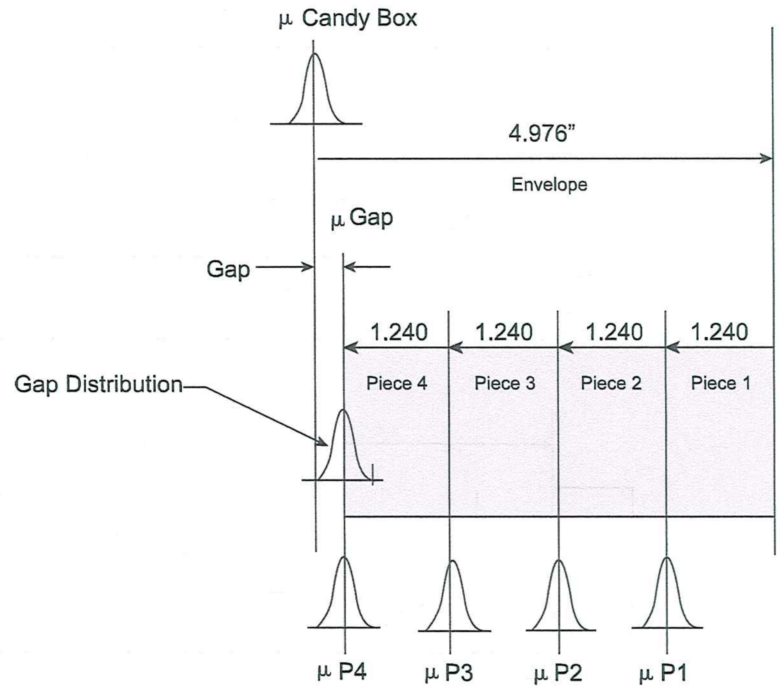
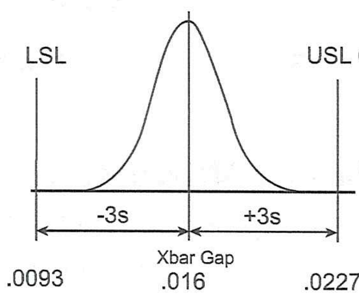


FIGURE 6 Candy box gap variation

In Figure 8, we see that because these "tolerance" ranges have been set at +/–3 standard deviations, we would expect the gap to exceed the new minimum or maximum gap estimates 0.27% of the time.

Box Nominal	4.976	Tolerance	$.003^2$
Piece 1 Nominal	-1.240	Tolerance	$.003^2$
Piece 2 Nominal	-1.240	Tolerance	$.003^2$
Piece 3 Nominal	-1.240	Tolerance	$.003^2$
Piece 4 Nominal	-1.240	Tolerance	$.003^2$
Nomianl Gap	.016		$\sqrt{.000045}$
Minimum Gap	.0093	Total Tolerance +/-.0087	
Maximum Gap	.0227		

FIGURE 7 Root sum of squares tolerance estimation



Root-Sum-Squares

$$\text{USL Gap}\,\sigma = \sqrt{\sigma_T^2+\sigma_1^2+\sigma_2^2+\sigma_3^2+\sigma_4^2}$$

$$\text{Gap}\,\sigma = \sqrt{(.001)^2+(.001)^2+(.001)^2+(.001)^2+(.001)^2}$$

$$\text{Gap}\,\sigma = .0022$$

% "Defective" = 27%

FIGURE 8 Root sum of squares percent defective

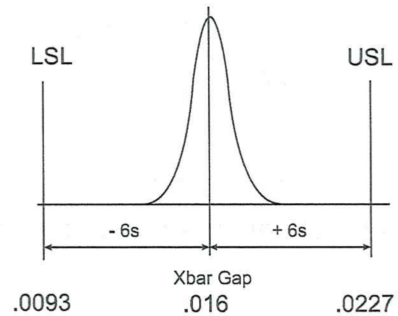
The root sum of squares estimate of tolerance range is much tighter than that calculated using the worst case analysis approach. For this reason, we see that the probability of exceeding the minimum and maximum values of the gap is also much more realistic.

But should we set a tolerance range based on what our product can currently achieve in performance? The tolerance range should in fact be determined by the difference between the upper and lower specifications required to meet customer requirements. If we cannot meet those customer requirements consistently, we may be forced to seek relief from those tight specifications from the customer. In this case, the root sum of squares analysis gives us guidance to help in those specification negotiations. But our goal in design is to create Six Sigma products and processes. At best, our current product using the RSS approach is a 3 sigma level product unless we negotiate less stringent specification limits with the customer. Let's next examine how we can use the candy box assembly information we have to design a Six Sigma product.

Six Sigma Tolerance Analysis

To develop a Six Sigma product without changing the product specs, we will have to reduce product variation. In this section, we demonstrate how the root sum of squares method can be used to design for Six Sigma product performance. We also introduce the use of Monte Carlo simulation for design in more complex tolerancing situations.

As described in our discussion of process capability in Chapter 28 of our forthcoming book, a Six Sigma process is defined as a process that has six short-term standard deviations between the process operating point and the closest specification limit. Using this definition, we can calculate the gap standard deviation required to make our candy box example a Six Sigma product. As shown in Figure 9, we calculate the required gap standard deviation by simply subtracting the lower spec limit from the upper spec limit and dividing the result by 12. The resulting gap standard deviation required for Six Sigma performance is 0.001117 inches. We can also estimate the standard deviation required for individual candy pieces and for the box itself by using the root sum of squares calculation. If the standard deviations of all components are equal, the root sum of squares calculation indicates that the standard deviation for each candy piece and for the box must be 0.0005 inches. In the short term, this Six Sigma process will produce out-of-spec conditions only 0.001 times out of a million. In the long term, assuming a 1.5 standard deviation process shift, the process will produce product outside of the tolerance range 3.4 times out of a million opportunities.



Six Sigma Standard Deviation

$$\text{Gap}\,\sigma = (\text{USL} - \text{LSL})/\,12$$
$$\text{Gap}\,\sigma = (.0227 - .0093)/\,12$$
$$\text{Gap}\,\sigma = .001117$$

Root-Sum-Squares

$$\text{Gap}\,\sigma = .001117 = \sqrt{\sigma_T^2 + \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2}$$
$$.001117 = \sqrt{5 * (\sigma_t)^2}$$
$$\sigma_t = .0005 \text{ inches}$$

FIGURE 9 Producing a Six Sigma product with root sum of squares Analysis

Different Levels of Variation

While calculating the required standard deviation using root sum of squares analysis is relatively straightforward, we typically do not have equal standard deviations for each system component. In situations in which we have components with different levels of variation, tolerance analysis becomes more complex.

Let's consider our candy box example again with a slight modification. Consider the situation in which variation is occurring in each piece of candy, the box, and the film used to wrap the candy. During product development, the Candy Wrapper Film design team has determined that a wrapper consisting of three layers of film produces the best candy taste for the customer. As shown in Figure 10, the new wrapper design consists of three film layers that unfortunately have different characteristics of variation. The design team has gathered data for the variation of each film layer's thickness, and the data are presented in the Minitab worksheet *Film Variation Data.MTW* in the project file *Statistical Tolerancing.MPJ*.

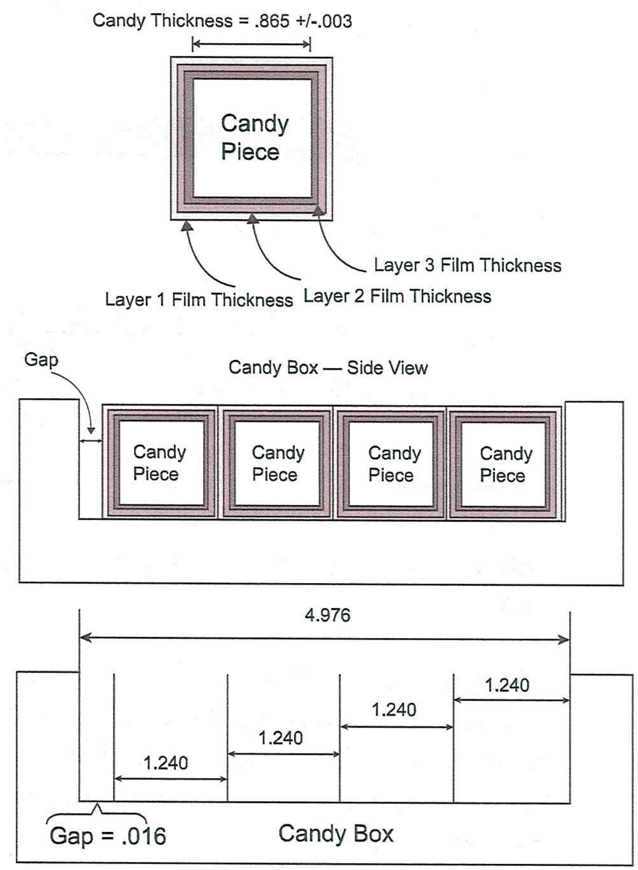


FIGURE 10 Tolerance analysis with objects of differing variation

Identifying the Best Distribution Fit with Minitab

To evaluate the gap between the box and the candy with the new wrapper design, we must identify the best distribution fit for our film data. We take advantage of Minitab's distribution identification tool and click **Stat > Reliability / Survival > Distribution Analysis (Right Censoring) > Distribution ID Plot**, as shown in Figure 11.

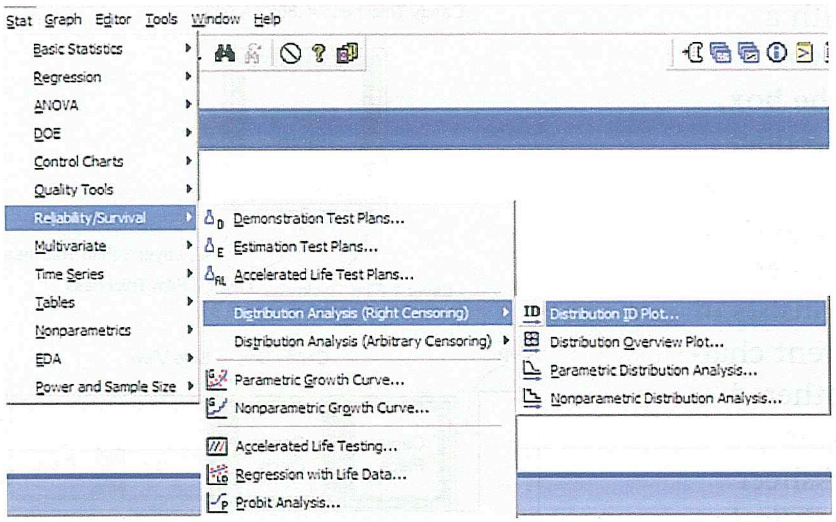


FIGURE 11 Identifying the best distribution fit with Minitab

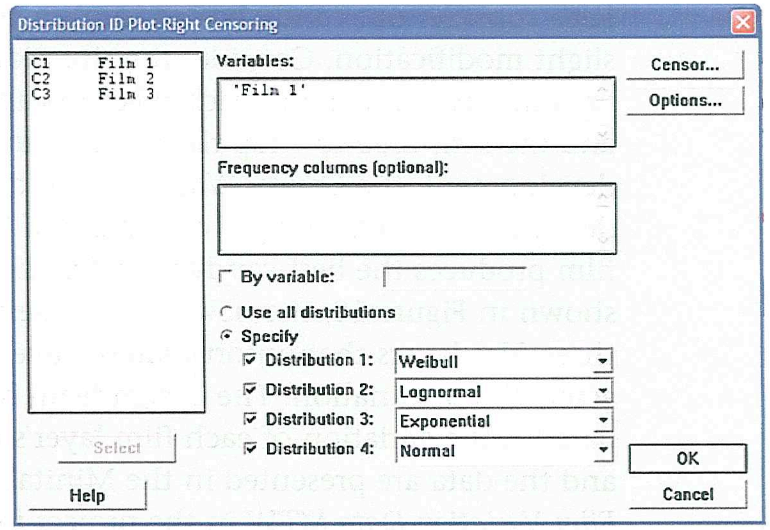


FIGURE 12 Film 1 distribution ID setup

We initially analyze the Film 1 data using the Weibull, Lognormal, Exponential, and Normal distributions, as described in Figure 12. Minitab offers additional distributions that we can use to further evaluate the data, if needed.

In Figure 13, we see Minitab's output from the distribution ID analysis. The best distribution fit is identified by examining both the graphical and analytical results presented. We see through visual inspection of the graphs developed by Minitab that the straight line for the Weibull distribution appears to fit the data best. Analytically, the Weibull distribution also has the highest value for the correlation coefficient, giving us verification that the Weibull distribution provides the best fit to the Film 1 data. To define the parameters for the Weibull distribution, we further analyze the data with Minitab's Distribution Overview Analysis by clicking **Stat > Reliability / Survival > Distribution Analysis (Right Censoring) > Distribution Overview Plot**, as shown in Figure 14.

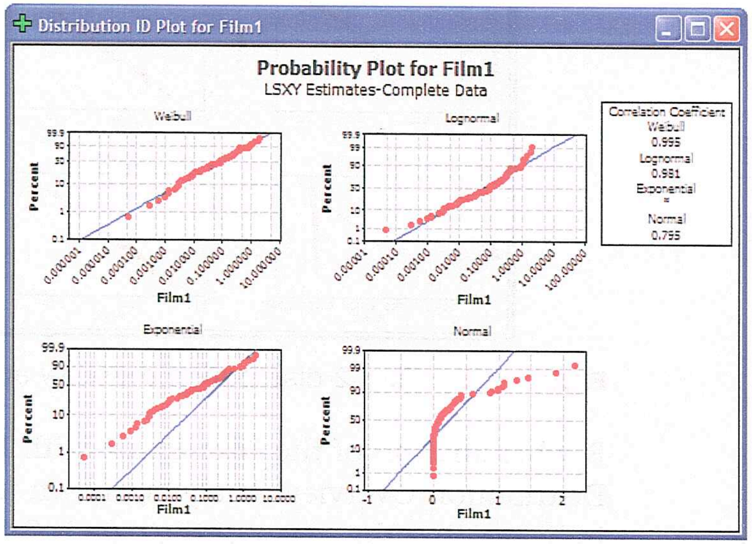


FIGURE 13 Film 1 distribution ID setup

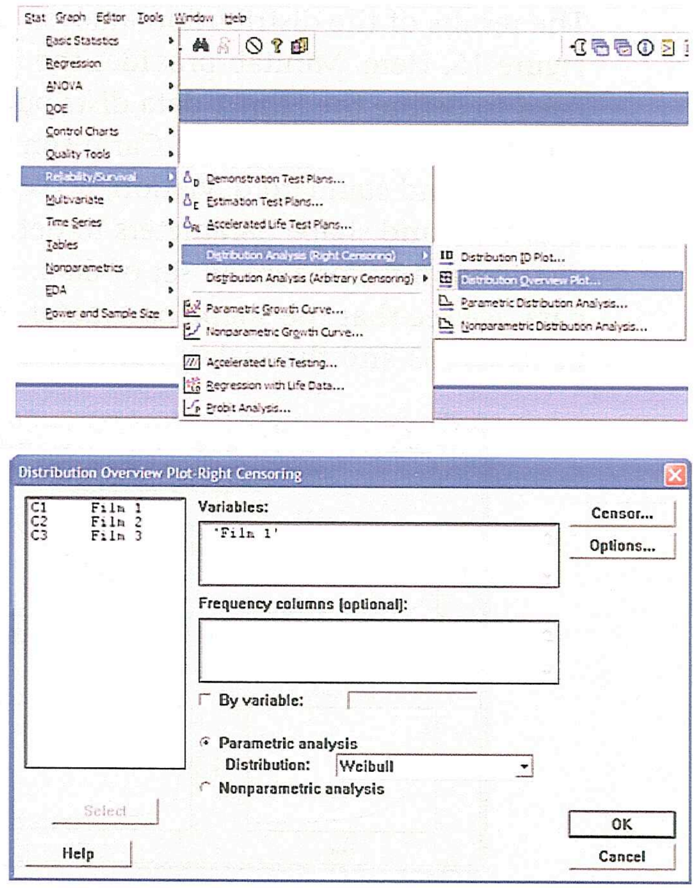


FIGURE 14 Film 1 distribution overview analysis

The results of the distribution analysis are shown in Figure 15. Here, Minitab provides the information we need to define the Film 1 data distribution as we prepare to conduct a Monte Carlo simulation. Instead of mean and standard deviation, a Weibull distribution uses scale and shape parameters to define the best distribution fit for a given set of data. For the Film 1 data, we see that the shape parameter is calculated to be 0.587863 and the scale parameter is 0.156633.

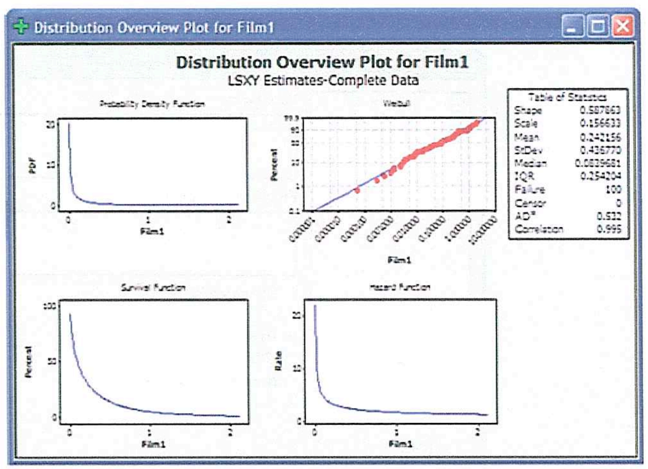


FIGURE 15 Film 1 distribution overview analysis

A similar analysis for the Film 2 data suggests that the Film 2 data are normally distributed, as seen in Figure 16.

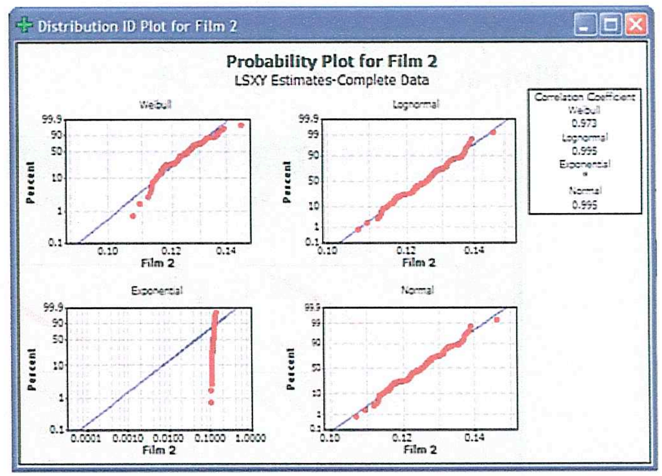


FIGURE 16 Film 2 distribution overview analysis

Further analysis of the Film 2 data with Minitab's Distribution Overview Analysis, shown in Figure 17, indicates that the mean of the data is 0.125071 inches and the standard deviation is 0.0077098 inches.

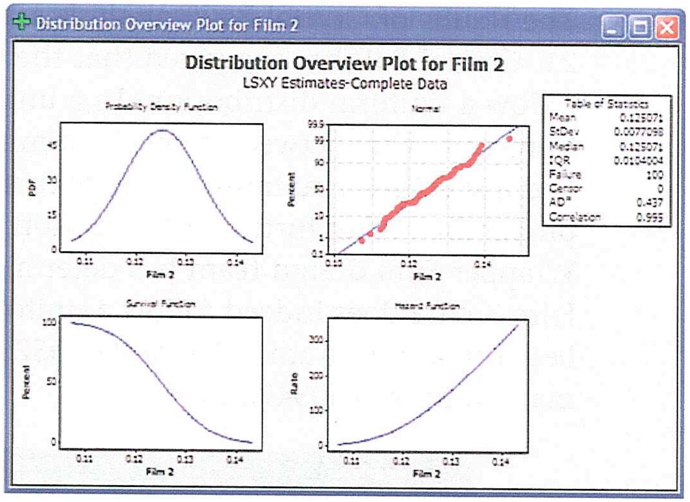


FIGURE 17 Film 2 distribution overview analysis

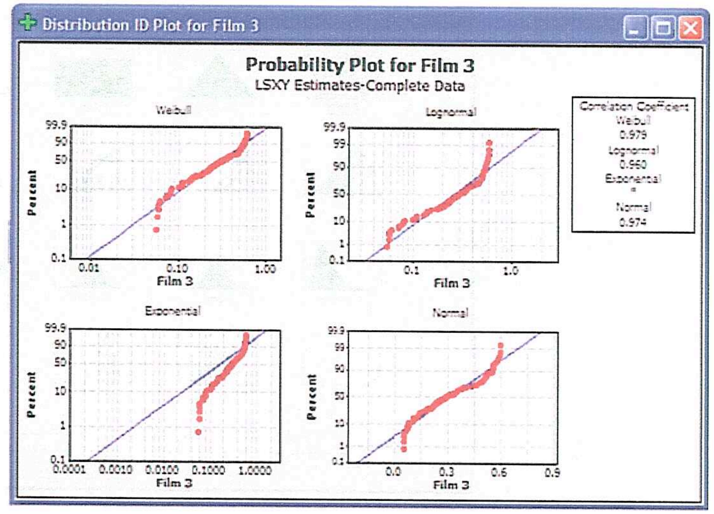


FIGURE 18 Film 3 distribution overview analysis

Identifying the Best Distribution Fit with Crystal Ball

Analyzing the Film 3 data with the Distribution ID Plot does not yield an acceptable result, as shown in Figure 18. There is a significant disagreement between the data and the straight line for each distribution. We could reanalyze the data using other distributions available in Minitab, but we instead use a similar distribution fit function that is available in Crystal Ball.

To use Crystal Ball's distribution fit capability we begin with the Distribution Gallery. At the bottom of the gallery, as shown in Figure 19, is the Fit button, which we click. We are then asked to identify the location of the data in the analysis worksheet. In Figure 20, we have indicated that the Film 3 data we wish to analyze are located in the cell range D5 to D104.