Minitab®

# SPC

## LESSON: Statistical Speaking: What is Six Sigma Quality?

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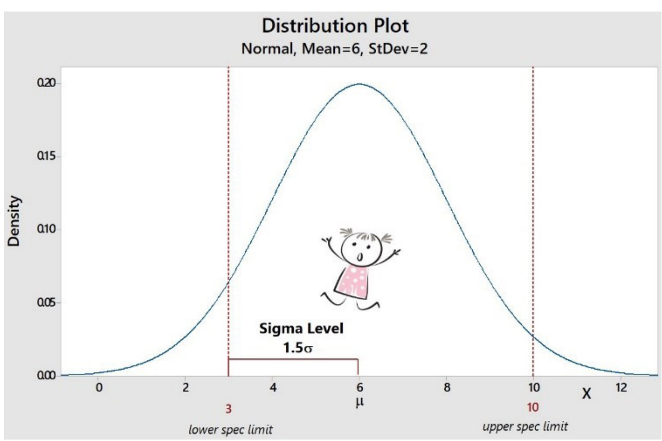
**Definition: σ** is a **Greek letter** used by statisticians to represent what with respect to a process?

1. It's center (mean)
2. It's center (median)
3. It's spread (std deviation)
4. It's spread (range)
5. It's sum

- **Sigma Level** (e.g. **6σ**) describes how well process ____________________ meets customer specifications.
- The "Sigma Level" is the number of **standard deviations** (σ's) between the **process mean** and the **nearest specification limit**.




Specification limits are set by the customer.

- In an **nσ process**, the distance between the **process mean μ** and the **nearest specification limit** is **n standard deviations**.
- **Question**: Would you rather hire a 6σ or 3σ company to do work for you?

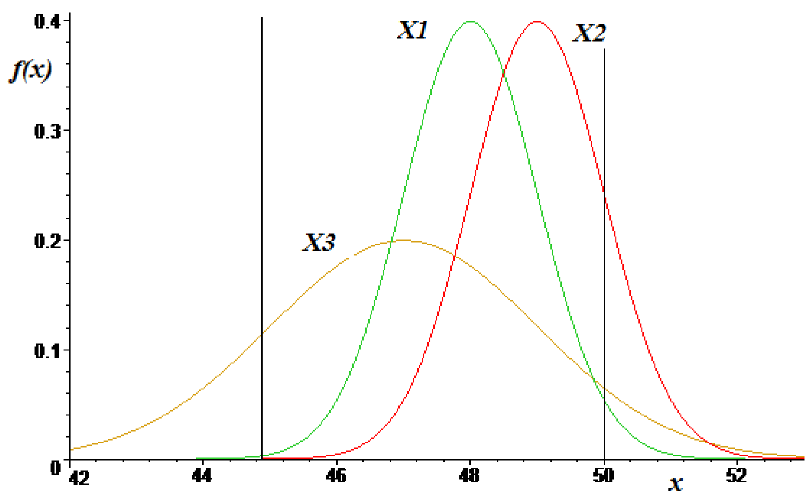
The probability density function for a normally distributed random variable X with mean μ (center of bell curve) and standard deviation σ (spread of bell curve) is given by:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \text{ for } -\infty < x < \infty.$$

**Example 1. Process Metric X**: Length of time an RHIT 50-min is in-class. Students set specifications as: Lower Specification Limit, **LSL = 45 mins** and Upper Specification Limit, **USL = 50 mins**.

Three distributions representing the length of time a 50-min class spends in class are drawn below (in minutes):

$\mathbf{X_1}$ ~ Normal (μ = 48, σ = 1), $\mathbf{X_2}$ ~ Normal (μ = 49, σ = 1), $\mathbf{X_3}$ ~ Normal (μ = 47, σ = 2)




$X_1$ is a ___________ Sigma Process. $X_2$ is a ___________ Sigma Process. $X_3$ is a ___________ Sigma Process.

**Example 2**. Using the metric "length of time an RHIT 50-min class is in-class" with the same specifications as in Example 1: **LSL = 45** and **USL = 50**, construct a **3σ process, 5σ process**, and **6σ process**:

$X_3$ ~ $X_5$ ~ $X_6$ ~

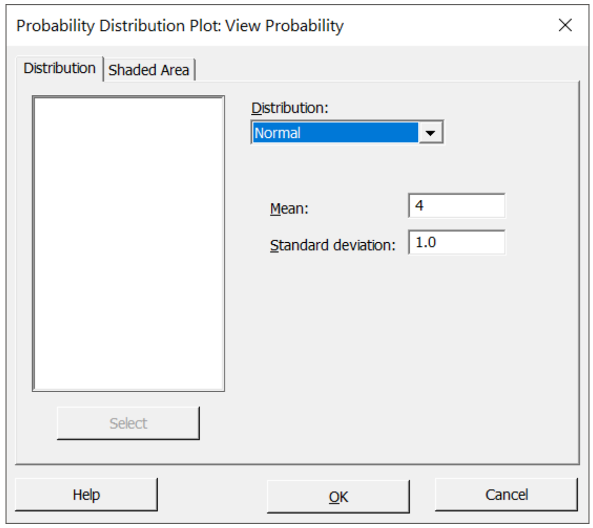
**Example 3**. Customer specifications for waiting in RHIT's Chauncey's lunch line are LSL = 0 and USL = 8 min. If the actual wait time for the process is **normally distributed** with mean **μ = 4 min** and standard deviation **σ = 1 min**, then

1. What is the "Sigma Level" of the process?
2. If the mean shifts to μ = 5 min (and the std dev stays at σ = 1), what is the corresponding Sigma Level?
3. If Chauncey's wants wait times to be a 6σ process, what will the standard deviation need to be when μ = 4?
4. If Chauncey's wants wait times to be a 10σ process, what will the standard deviation need to be when μ = 4?
5. Still assuming that wait times are Normal (μ = 4, σ = 1) min, what proportion of customer wait times

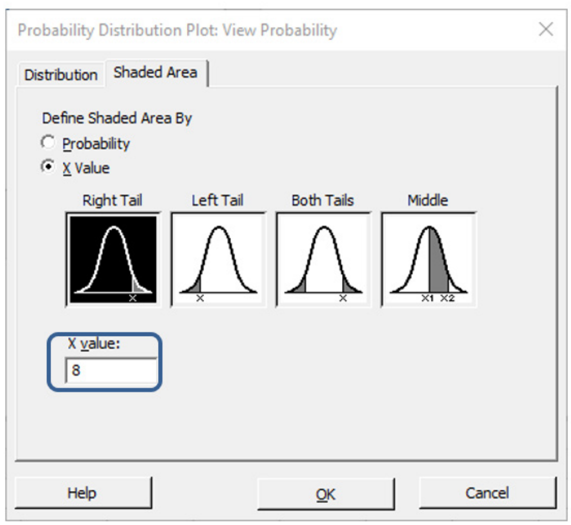
will exceed the USL = 8 min?

## Minitab desktop (20 or higher):

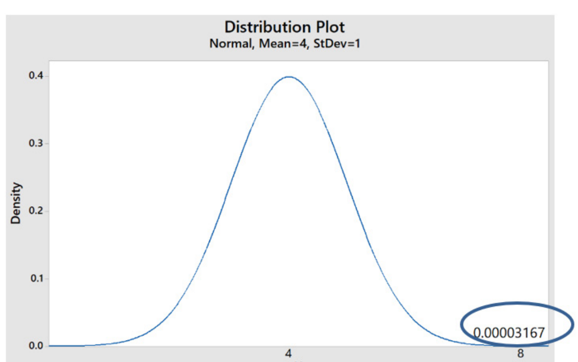
1. Choose **Graph** > **Probability Distribution Plot** > **View Probability.** Select distribution and enter parameters.




2. Click **Shaded Area** and complete the dialog box as shown.




3. Click **OK**.



## Minitab web app:

1. Choose **Graph** > **Probability Distribution Plot**. Under **One Curve**, select **View Probability**.
2. Select **Normal**. In **Mean**, enter 4. In **Standard Deviation**, enter 1.
3. Click **Options**. Select **A specified x value** and select **Right Tail**.
4. In **X value**, enter 8.
5. Click **OK** in each dialog box.

This is the same as integrating f(x) for a Normal (μ = 4, σ = 1) from x = 8 to infinity.

$$\int_{8}^{\infty} \frac{\sqrt{2}\, e^{-\frac{(x-4)^2}{2}}}{2\sqrt{\pi}}\, dx$$

## Sigma Levels and Defective Parts per Million (DPM)

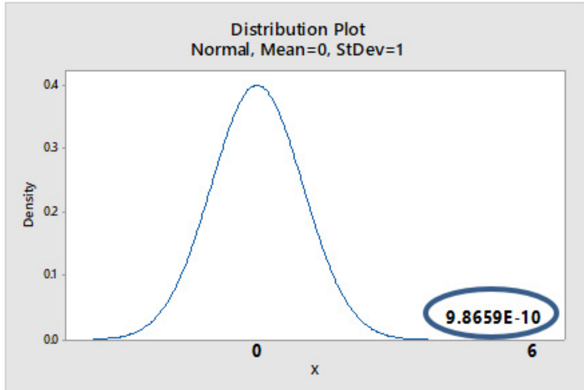
**Notation**: Throughout the course, the random variable **Z** will represent a **standard normal random variable**. A standard normal random variable has mean **μ = 0** and standard deviation **σ = 1**; that is **Z ~ Normal (μ = 0, σ = 1)**.

- How do **Sigma Levels** correspond to **Defective Parts per Million (DPM)**?
- **6σ quality** is associated with **3.4 DPM**; **5σ quality** is associated with **233 DPM**. What do these mean?

The probability that Z > 6 standard deviations from the mean is approximately $0.9865876450 \cdot 10^{-9}$, which results in **0.0009865876450 DPM**. This is NOT 3.4 DPM! Here is how **0.0009865876450 DPM** is computed:

**Minitab desktop: Graph** > **Probability Distribution Plot** >**View Probability**; Select **Normal**. Enter 0 for the **Mean** and 1 for the **Standard Deviation**; Click **Shaded Area** > Check **X value** > Select **Right Tail** > Input **X value as 6**

**Minitab web app: Graph** > **Probability Distribution Plot** > **One Curve** > **View Probability**; Select **Normal**. Enter 0 for the **Mean** and 1 for the **Standard Deviation**; Click **Options** > Select **A specified x value** > Select **Right Tail** > Input **X value as 6**




Using integration with the standard normal distribution's probability density function with μ = 0, σ = 1:

$$10^6 \cdot P(Z > 6) = 10^6 \cdot \int_{z=6}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \cong 10^6 \cdot 0.9865876450 \ x \ 10^{-9} \cong 0.0009865876450 \text{ DPM}$$

Why do we say **6σ quality** represents **3.4 DPM**, and not **0.0009866 DPM**?

- **Bill Smith (Motorola)**, the "**Father of Six Sigma,**" noticed that defect rates for actual processes at his company were greater than what he expected according to the normal distribution calculation above.
- Smith reasoned that a **shift of 1.5σ** in the process mean over time would explain the difference.
- **Where does 1.5σ come from**? Bill Smith! **Should it be something else**? Maybe, but it is standard practice now and no one wants to change it.

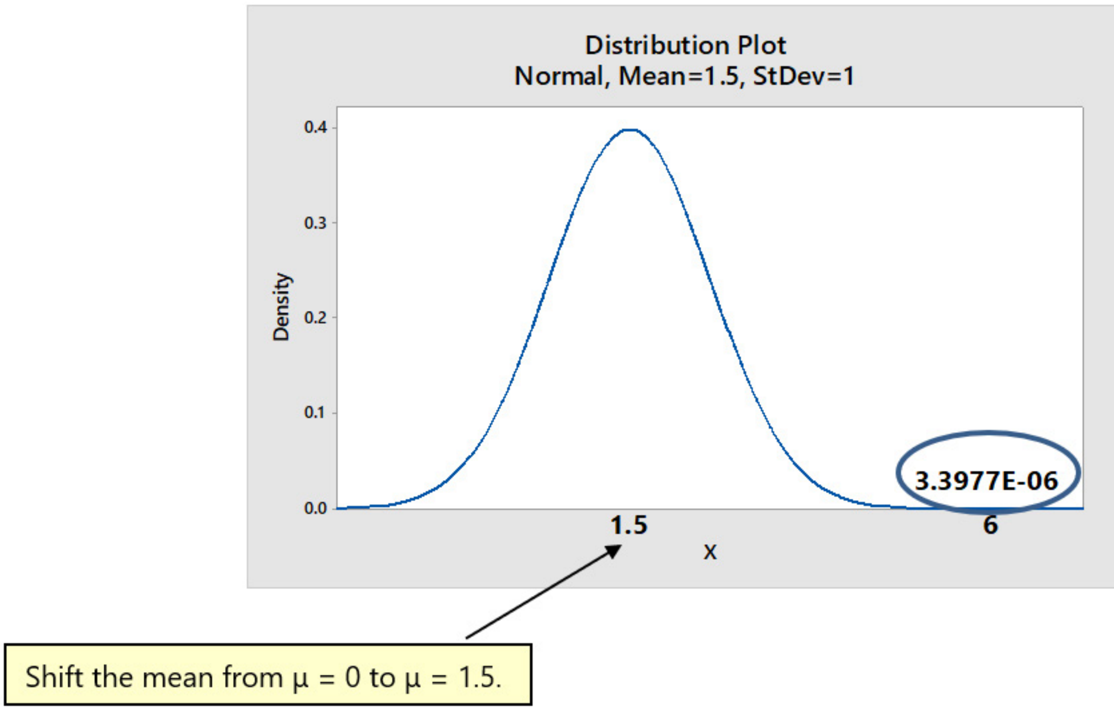
**Example 4**. Determine the DPM of a 6σ process with shift in the mean of 1.5σ. It's easiest to show this using a normal curve with mean μ = 0 and a standard deviation of σ = 1. Then shift μ = 0 by a (1.5 • 1) = 1.5 to the right of 0.

## Minitab desktop (20 or higher):

1. Choose **Graph** > **Probability Distribution Plot** > **View Probability**.
2. Select **Normal**. In **Mean**, enter **1.5.** In **Standard Deviation**, enter 1.
3. Click **Shaded Area**. Check **X value** and select **Right Tail**.
4. In **X value**, enter 6.
5. Click **OK**.

## Minitab web app:

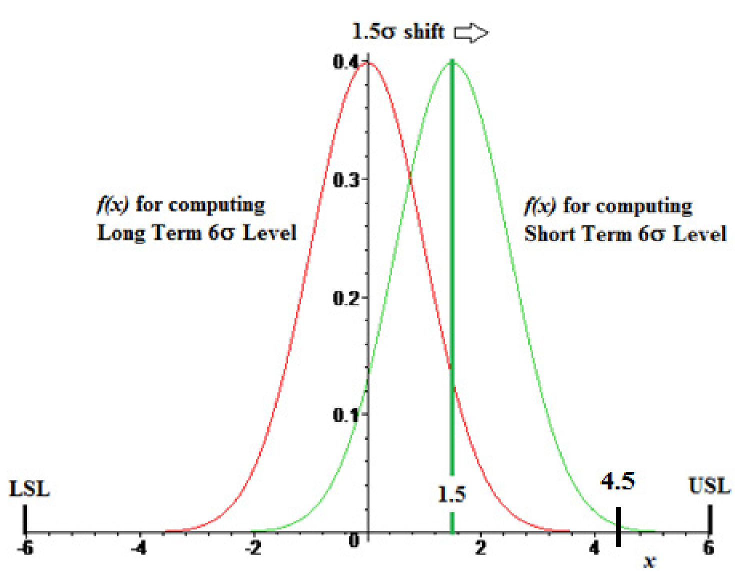
1. Choose **Graph** > **Probability Distribution Plot**. Under **One Curve**, select **View Probability**.
2. Select **Normal**. In **Mean**, enter 1.5. In **Standard Deviation**, enter 1.
3. Click **Options**. Select **A specified x value** and select **Right Tail**.
4. In **X value**, enter 6.
5. Click **OK** in each dialog box.




Using integration with a probability density function of a normal curve centered at mean μ = 1.5 and standard deviation σ = 1.

$$10^6 \cdot \int_{x=6}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-1.5)^2}{2}} dx \cong 10^6 \cdot 0.3398\, x\, 10^{-5} \cong \mathbf{3.4\ DPM}$$

The graph with and without the shift is shown below.



## There are two types of Sigma Levels – Short term and Long term.

**Definition: Short Term** Sigma Level assumes there is a shift in the process mean μ by 1.5σ in one direction from the process center.

**Definition: Long Term** Sigma Level assumes no shift in the process mean μ.

The **relationship** between **Short Term** and **Long Term** Sigma Levels is:

**Short Term** Sigma Level **nσ = Long Term** Sigma Level **(n – 1.5)σ**.

**Examples:**

- Short Term 6σ = Long Term 4.5σ
- Short Term 7.5σ = Long Term 6σ
- Short Term 2σ = Long Term 0.5σ

**Example 4**. Fill in the table below using the "**long term**" and "**short term**" accepted definitions of Sigma Levels for computing defect rates.

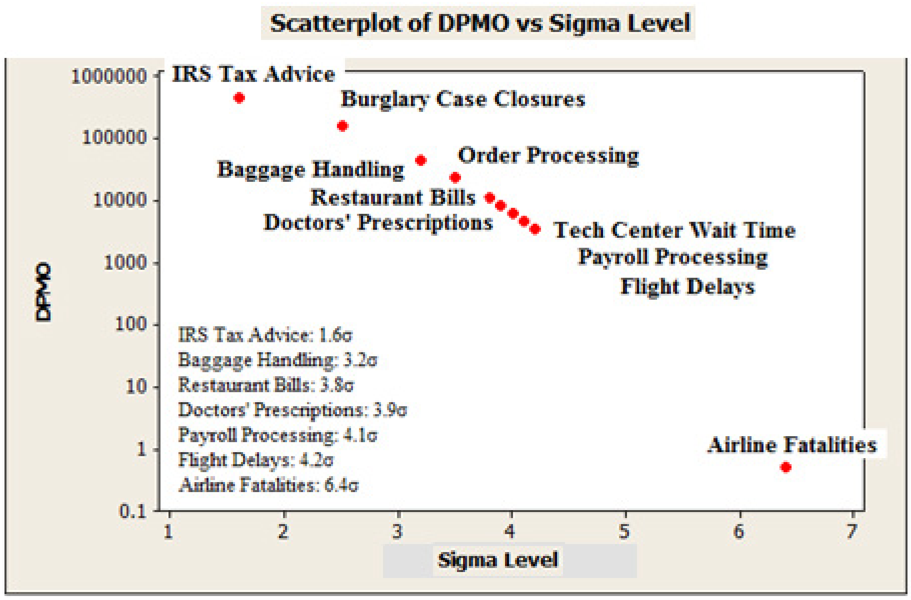
| Long Term Sigma Level: no shift in mean μ | Short Term Sigma Level: shift in mean μ | DPM for Short Term: nσ and Long Term: (n – 1.5)σ |
|---|---|---|
| -0.5σ | 1σ | ~691,462 |
| 0.5σ | 2σ | |
| 1.5σ | 3σ | ~66,807 |
| 2.5σ | 4σ | ~6,210 |

**Sigma Conversion Chart**

| Long Term Sigma Level | Short Term Sigma Level | Defects Per Million | | Long Term Sigma Level | Short Term Sigma Level | Defects Per Million |
|---|---|---|---|---|---|---|
| 4.5 | 6 | 3 | | 1.9 | 3.4 | 28,717 |
| 4.4 | 5.9 | 5 | | 1.8 | 3.3 | 35,930 |
| 4.3 | 5.8 | 9 | | 1.7 | 3.2 | 44,565 |
| 4.2 | 5.7 | 13 | | 1.6 | 3.1 | 54,799 |
| 4.1 | 5.6 | 21 | | 1.5 | 3 | 66,807 |
| 4 | 5.5 | 32 | | 1.4 | 2.9 | 80,757 |
| 3.9 | 5.4 | 48 | | 1.3 | 2.8 | 96,800 |
| 3.8 | 5.3 | 72 | | 1.2 | 2.7 | 115,070 |
| 3.7 | 5.2 | 108 | | 1.1 | 2.6 | 135,666 |
| 3.6 | 5.1 | 159 | | 1 | 2.5 | 158,655 |
| 3.5 | 5 | 233 | | 0.9 | 2.4 | 184,060 |
| 3.4 | 4.9 | 337 | | 0.8 | 2.3 | 211,855 |
| 3.3 | 4.8 | 483 | | 0.7 | 2.2 | 241,964 |
| 3.2 | 4.7 | 687 | | 0.6 | 2.1 | 274,253 |
| 3.1 | 4.6 | 968 | | 0.5 | 2 | 308,538 |
| 3 | 4.5 | 1,350 | | 0.4 | 1.9 | 344,578 |
| 2.9 | 4.4 | 1,866 | | 0.3 | 1.8 | 382,089 |
| 2.8 | 4.3 | 2,555 | | 0.2 | 1.7 | 420,740 |
| 2.7 | 4.2 | 3,467 | | 0.1 | 1.6 | 460,172 |
| 2.6 | 4.1 | 4,661 | | 0 | 1.5 | 500,000 |
| 2.5 | 4 | 6,210 | | -0.1 | 1.4 | 539,828 |
| 2.4 | 3.9 | 8,198 | | -0.2 | 1.3 | 579,260 |
| 2.3 | 3.8 | 10,724 | | -0.3 | 1.2 | 617,911 |
| 2.2 | 3.7 | 13,903 | | -0.4 | 1.1 | 655,422 |
| 2.1 | 3.6 | 17,864 | | -0.5 | 1 | 691,462 |
| 2 | 3.5 | 22,750 | | | | |

> In this course, I will always try to specify Short Term or Long Term Sigma Level. If neither is given, assume Short Term.

**Example 5.** The "Sights of Sigma" from Keller's "Six Sigma Deployment," 2001.




**Example 6**. The "Reality of Sigma" in the real-world where millions of processes are happening daily:

| 4σ quality: 99.38% acceptable | 6σ quality: 99.9997% acceptable |
|---|---|
| 20,000 lost articles of mail/hour | 7 lost articles of mail/hour |
| Unsafe drinking water for almost 15 minutes each day | One unsafe minute every 7 months |
| 5,000 incorrect surgical operations/ week | 1.7 incorrect operations/week |
| Two short or long landings at most major airports per day | One short or one long landing every 5 years |
| 200,000 wrong drug prescriptions/year | 68 wrong prescriptions/year |
| No electricity for almost 7 hours each month | One hour without electricity every 34 years |

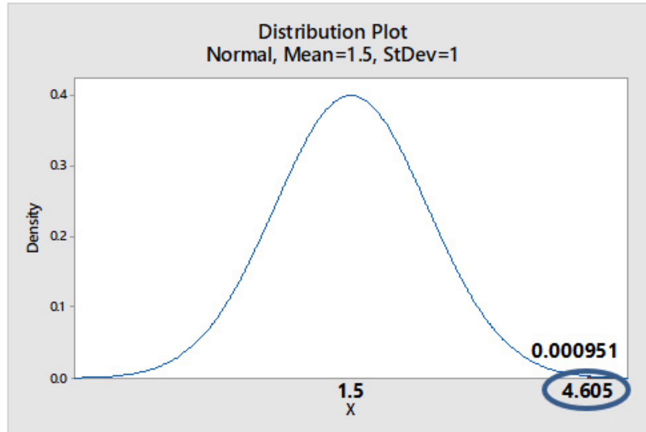
**Why are many companies still "stuck" at 3σ or 4σ levels?**

- Past successes
- Dependence on inspection/rework
- Reliance on trial and error
- Rewarding fire fighters

- Minimal focus on measurement

**Example 7.** [Sigma Levels, DPM Calculations.] Calculate the **Short Term Sigma Level** and **DPM** of a telecom network that had **500 minutes of downtime** in 2014.

In Sigma Level terminology:



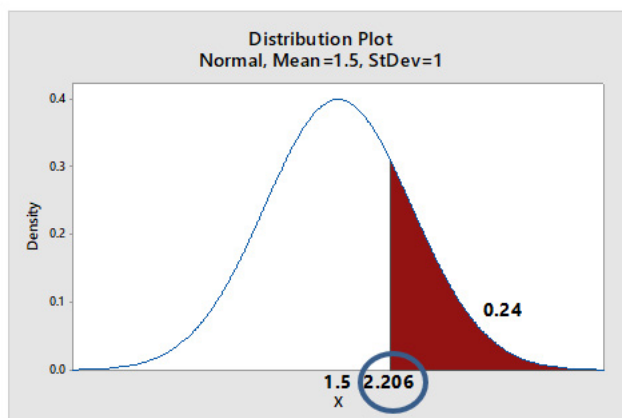

**Critical to Quality (CTQ) Metric**: Network "down time"
**CTQ Measurement Units**: Minutes
**CTQ Specifications**: No downtime (0 minutes)
**Defect measure**: One minute of network down time
**Total Defects in 2014**: 500 minutes of network down time

**Total Minutes in 1 year**: 525,600 minutes
[365 days x 24 hours x 60 minutes = 525,600 minutes]
**Proportion of down time in 2014:**
500 / 525600 = $0.9512937595 \cdot 10^{-3}$
**Defects Per Million:** ~951

**Short Term** Sigma Level: **4.605; Long Term** Sigma Level: 4.605 – 1.5 = **3.105**

**Example 8**. A Six Sigma project is focused on improving a billing process within a company. The Six Sigma team wants to have correct bills sent to the customer. They have defined one opportunity for this process – **either the bill is correct or not**. All the bills produced are approximately the same in terms of complexity. The team took a random sample of **250 bills** and found **60 defects**.

In Six Level terminology:



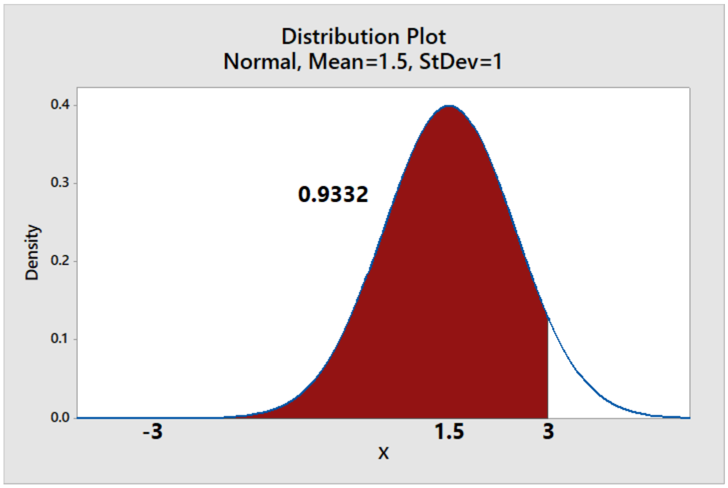

**Critical to Quality (CTQ) Metric**: Number of defective bills
**CTQ Measurement Units**: Bills
**CTQ Specifications**: No defective bills
**Defect measure**: One bill
**Proportion of Defects**: 60 / 250 = 0.24
**Defects Per Million**: 240,000 bills

**Short Term** Sigma Level: 2.206
**Long Term** Sigma Level: 2.206 – 1.5 = 0.706

## Sigma Level / DPM Problems from Previous Assignments (Default: Assume Short Term)

**1.** If a company is operating at a 3σ Level, what percent of its product is conforming?

A. 99.73%    B. 99.97%    C. 99.997%    **D. 93.32%**

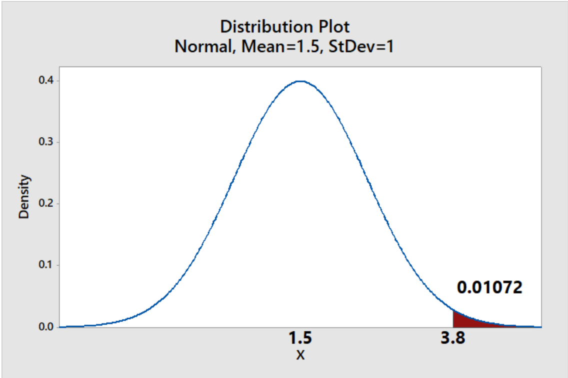



**2-3.** I have read part of the book: "Straight from the Gut" by the Quality Guru Jack Welch (former GE CEO who popularized Six Sigma in the 90's). He has the following quote on page 334: "A Black Belt team solved the problem and designed a change in the production process that gave the color and static qualities that Sony demanded. We went from **3.8σ** to **5.7σ** and earned Sony's business." Again, assume these are "Short Term" Sigma Levels (i.e. a shift of 1.5σ in the process mean as occurred).

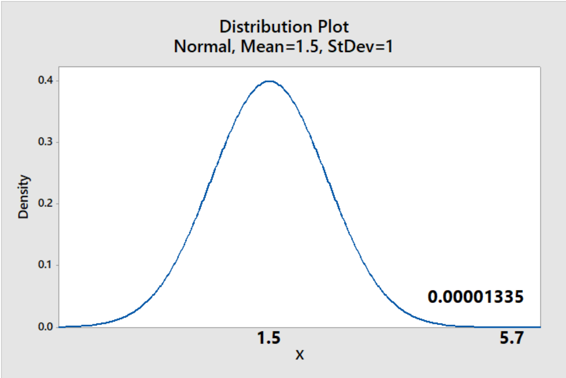
**2.** As a 3.8σ company, how many defects per million were they producing? The DPM has been rounded to the nearest integer value.

A. 72%    B. 10724%    C. 989276%    **D. 999928%**

Problem 2




Problem 3




**3.** As a 5.7σ company, how many defects per million were they producing? (SEE ABOVE)

A. 0    **B. 13**    C. 72    D. 687

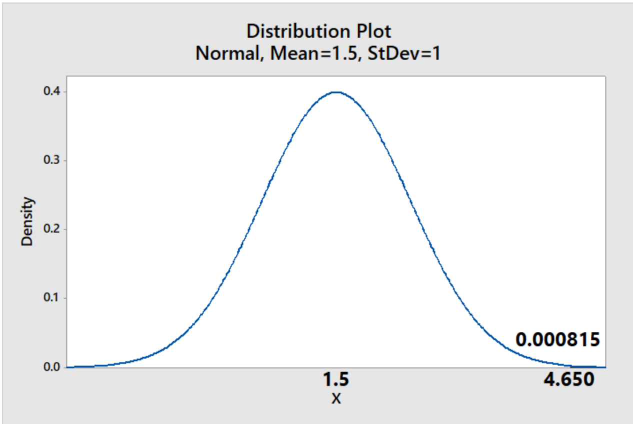
**4.** Increasing performance in a Six Sigma Corporation from 3σ to 4σ would reduce defects per million by a factor of:

A. ~2   B. ~8   **C. ~11**   D. ~16

**Solution**: 3σ quality: ~66807 DPM; 4σ quality: ~6210 DPM

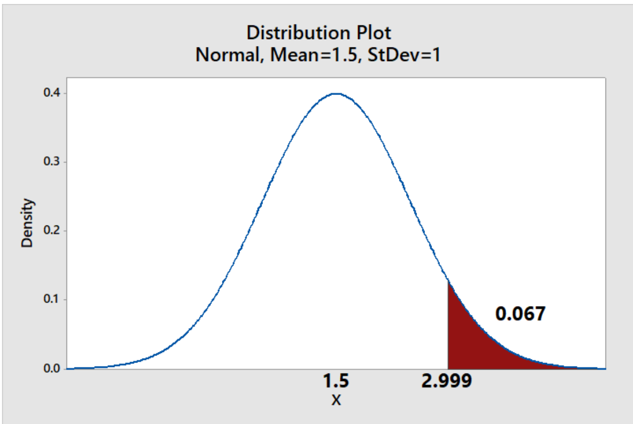
**5.** The DPM for a process is 815. What is the approximate Sigma Level of the process?

A. 4.60   **B. 4.65**   C. 4.70   D. 4.75




**6.** Corporation XYZ operates at a yield of 93.3%. Therefore, this corporation is operating at what Sigma Level?

A. 2σ   **B. 3σ**   C. 3.5σ   D. 4σ




**7.** A company can increase their Six Sigma level from 4.5σ to 5.4σ. How many less defects per million will they have?

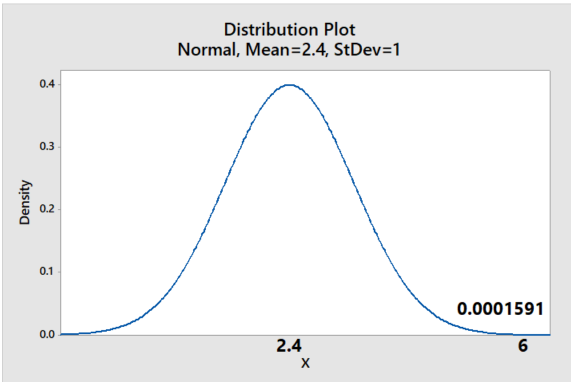
A. 1402   B. 1398   C. 1350   **D. 1302**

**Solution**: 4.5σ quality: ~1350 DPM; 5.4σ quality: ~48 DPM

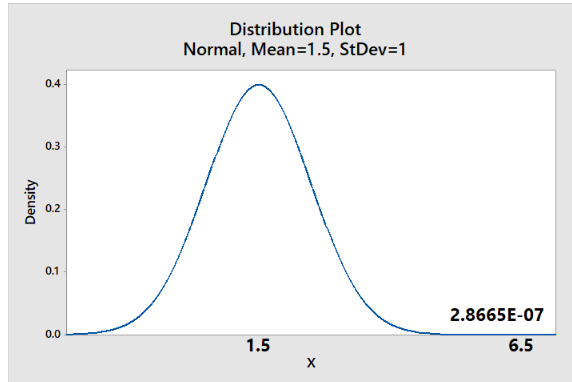
**8.** Suppose I know my company has problems with its process mean drifting from the target by as much as 2.4 standard deviations. If my specification limits are 6 standard deviations away from the target value, then in the worst case scenario, i.e., a drift of 2.4σ, how many defective parts per million should I expect?

**A. 159** B. 8198 C. 17864 D. 184,060

Problem 8




Problem 9



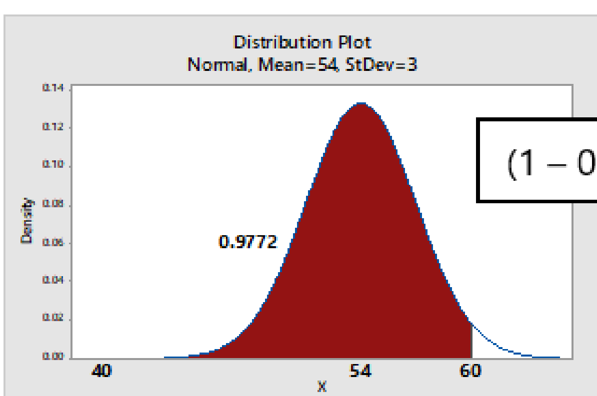

**9.** I'm going to start a new quality movement and call it 6.5σ. That is, I want the specification limits of my process to be 6.5 standard deviations away from my target value. Suppose I still know my company has problems with its process average drifting from the target by 1.5 standard deviations. If my specification limits are 6.5 standard deviations away from the target value, how many defective parts per million should I expect? (SEE ABOVE)

A. 0.0012 DPM B. 0.0024 DPM **C. 0.2867 DPM** D. 233 DPM

**10.** Suppose a certain process makes a part that has a metric of interest that is normally distributed. How many parts in a week are expected to not satisfy customer requirements given the following information on the parts: Upper specification limit: 60; Lower specification limit: 40; Mean of the process: 54; Process standard deviation: 3; Weekly production: 652 parts. Note: We are not talking about Sigma Levels here, so there is NO shift in the mean.

A. 0.33 **B. 15** C. 44 D. 196 E. 300

**Note**: We are not talking about Sigma Levels here, so there is NO shift in the mean.




$$(1 - 0.9772) \cdot 652 \cong 15$$