NHS National Services Scotland

Tutorial Guide

Statistical

Monitoring Quality in Healthcare

Process

Clinical Indicators Support Team

Control

Contents

Introduction

NHSScotland routinely collects a vast array of data from healthcare processes. The analysis of these data can provide invaluable insight into the behaviour of these healthcare processes.

Statistical Process Control (SPC) techniques, when applied to measurement data, can be used to highlight areas that would benefit from further investigation. These techniques enable the user to identify variation within their process. Understanding this variation is the first step towards quality improvement.

There are many different SPC techniques that can be applied to data. The simplest SPC techniques to implement are the run and control charts. The purpose of these techniques is to identify when the process is displaying unusual behaviour.

The purpose of this guide is to provide an introduction to the application of run charts and control charts for identifying unusual behaviour in healthcare processes. SPC techniques are a tool for highlighting this unusual behaviour. However, these techniques do not necessarily indicate that the process is either right or wrong – they merely indicate areas of the process that could merit further investigation.

History of SPC

1928 saw the introduction of the first Statistical Process Control (SPC) Charts. Commissioned by Bell Laboratories to improve the quality of telephones manufactured, Walter Shewhart developed a simple graphical method – the first of a growing range of SPC Charts.

Understanding the causes of variation within an industrial process proved indispensable as actions could be taken to improve process and output. In the 1950's, with the effective use of SPC, Deming converted post war Japan into the world leader of manufacturing excellence.

This approach is increasingly being applied in healthcare by thinking of healthcare systems as processes. As well as providing a basis for quality improvement within healthcare, SPC Charts also offer alternative methods of displaying data.

1. Understanding Variation

1.1 Types of Variation

Variation exists in all processes around us. For example:

- Every person is different
- No two snowflakes are identical
- Each fingerprint is unique

The two types of variation that we are interested in are ‘common cause’ and ‘special cause’ variation.

Common Cause
All processes have random variation - known as ‘common cause variation’. A process is said to be ‘in control’ if it exhibits only common cause variation i.e. the process is completely stable and predictable.

Special Cause
Unexpected events/unplanned situations can result in ‘special cause variation’. A process is said to be ‘out of control’ if it exhibits special cause variation i.e. the process is unstable.

SPC charts are a good way to identify between these types of variation, as we will see later. SPC charts can be applied to both dynamic processes and static processes.

Dynamic Processes
A process that is observed across time is known as a dynamic process. An SPC chart for a dynamic process is often referred to as a ‘time-series’ or a ‘longitudinal’ SPC chart.

Static Processes
A process that is observed at a particular point in time is known as a static process. An SPC chart for a static process is often referred to as a ‘cross-sectional’ SPC chart.

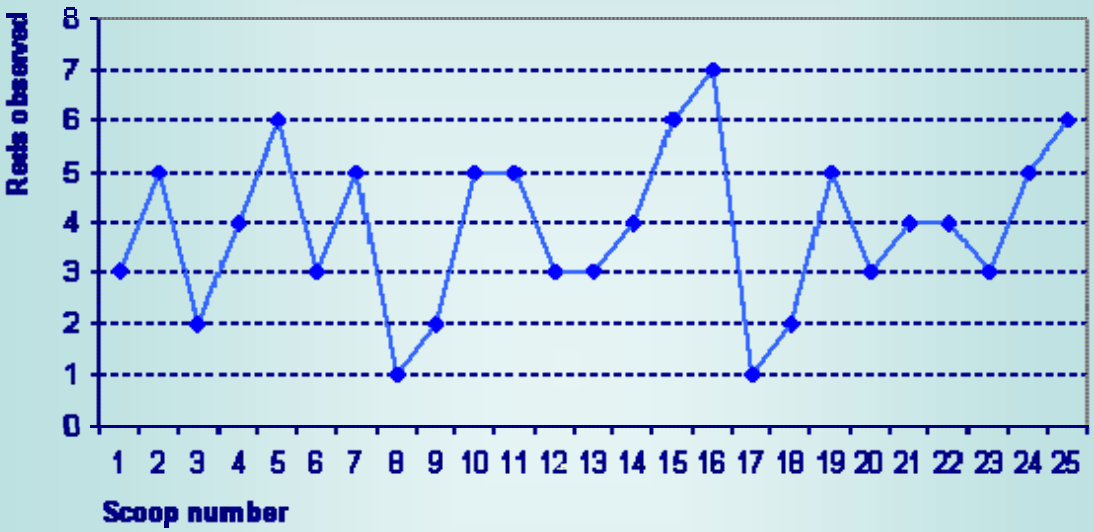
A cross-sectional SPC chart is a good way to compare different institutions. For example, hospitals or health boards can be compared as an alternative to league tables as we will see later.

Example 1

Coloured beads pulled from a bag – a dynamic process

A bag contains 100 beads that are identical - except for colour. Twenty of the beads are red and 80 are blue. Scoopfuls of 20 are repeatedly drawn out, with replacement, and the number of red beads in each scoop is observed. Figure 1 shows the result of 25 scoops.

Figure 1 Number of red beads observed in 25 scoops



Twenty of the 100 beads in the bag are red, which means that the proportion of red beads in the bag is 1/5. Therefore, if a sample of 20 is drawn each time, we expect four of the beads in the sample to be red, on average.

In figure 1 the plotted points oscillate around four.

In general, every time a sample of 20 is drawn you won't necessarily observe four reds. The number that you observe will vary due to random variation. The random variation that you see in the graph above is common cause variation as there is no unusual behaviour in this process.

If a sample of 20 beads were drawn from the bag and 10 or more red beads were consistently being observed then this would indicate something unusual in the process i.e. special cause variation which may require further investigation.

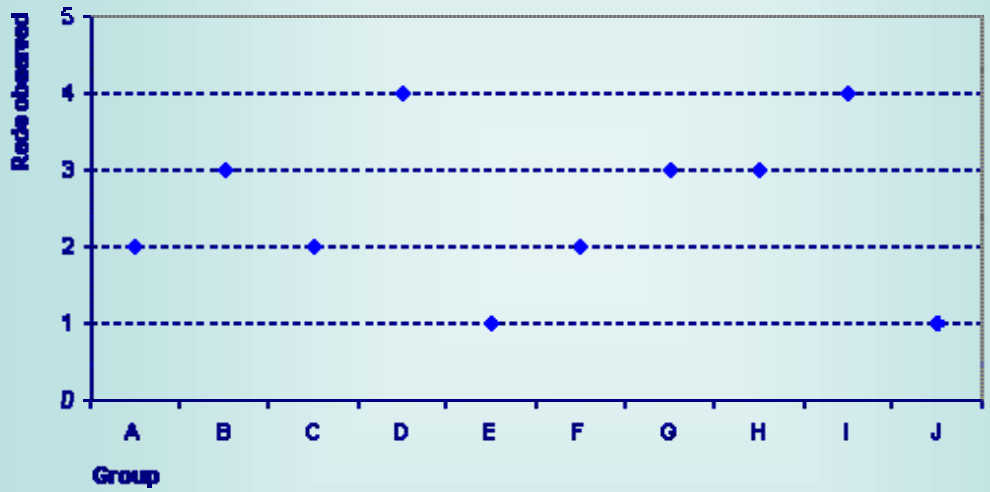
The example above is a simplification of Deming's red bead experiment where the red beads represent an undesired outcome of the process. This process is not dissimilar to the many situations that often occur in healthcare processes. This is how data, which is collected over time, is typically presented and it shows the behaviour and evolution of a dynamic process.

Example 2

Coloured beads pulled from a bag – a static process

There are 10 groups in a room and each group has a bag that contains 20 beads – four of these beads are red. Each group is required to draw out 10 beads and the number of red beads in each groups' scoop is observed. Figure 2 shows the result from the 10 groups.

Figure 2 Number of red beads observed in each groups' scoop



The proportion of red beads in the bag is again 1/5. Therefore, if each group draws out a sample of 10, we expect two of the beads in the sample to be red, on average.

In figure 2 the plotted points oscillate around two.

The variation in this sample is again random variation (common cause variation).

This example illustrates how data is typically presented at a single point in time and it is an example of a static process. This situation arises when data is analysed across units. For example, NHS boards, GP practices, surgical units etc and is known as a cross-sectional chart.

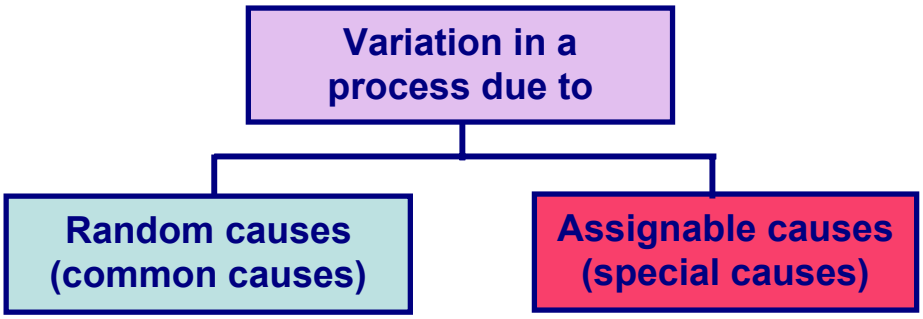
1.2 Sources of Variation

Variation in a process can occur through a number of different sources. For example:

- People - Every person is different
- Materials - Each piece of material/item/tool is unique
- Methods - Signatures for example
- Measurement - Samples from certain areas etc can bias results
- Environment - The effect of seasonality on hospital admissions

1.3 Causes of Variation

W. A. Shewhart recognised that a process can contain two types of variation. Variation contributable to random causes and/or to assignable causes.



W. E. Deming later derived the expressions 'common cause variation' (variation due to random causes) and 'special cause variation' (variation due to assignable causes).

Common cause variation is an inherent part of every process. Generally, the effect of this type of variation is minimal and results from the regular rhythm of the process.

Special cause variation is not an inherent part of the process. This type of variation highlights something unusual occurring within the process and is created by factors that were not part of the process' design. However, these causes are assignable and in most cases can be eliminated.

If common cause is the only type of variation that exists in the process then the process is said to be 'in control' and stable. It is also predictable within set limits i.e. the probability of any future outcome falling within the limits can be stated approximately. Conversely, if special cause variation exists within the process then the process is described as being 'out of control' and unstable.

Summary

1. Variation exists everywhere
2. Processes displaying only common cause variation are predictable within statistical limits
3. Special cause variation should be eliminated if possible

1.4 Tools for Identifying Process Variation

Now we know that variation exists in all processes we can proceed to identify which type of variation is present. One method of identifying the type of variation present is by using SPC charts. Originally developed for use in manufacturing, many applications are now involving healthcare processes for quality improvement purposes. The following section explains the fundamentals of SPC in more detail.

2. SPC Charts – Dynamic Processes

Statistical Process Control (SPC) Charts are essentially:

- Simple graphical tools that enable process performance monitoring
- Designed to identify which type of variation exists within the process
- Designed to highlight areas that may require further investigation
- Easy to construct and interpret

Two of the most popular SPC tools in use today are the run chart and the control chart. They are easy to construct, as no specialist software is required. They are easy to interpret, as there are only a few basic rules to apply in order to identify the variation type without the need to worry too much about the underlying statistical theory.

The following sections step through the construction and interpretation of run charts and control charts.

2.1 Constructing a Run Chart

Run Chart
A time ordered sequence of data, with a centreline drawn horizontally through the chart. A run chart enables the monitoring of the process level and identification of the type of variation in the process over time.

The centreline of a run chart consists of either the mean or median. The mean is used in most cases unless the data is discrete.

Discrete Data
Where the observations can only take certain numerical values. Almost all counts of events e.g. number of patients, number of operations etc

Continuous Data
These data are usually obtained by a form of measurement where the observations are not restricted to certain values. For example - height, age, weight, blood pressure etc.

Steps to create a Run Chart

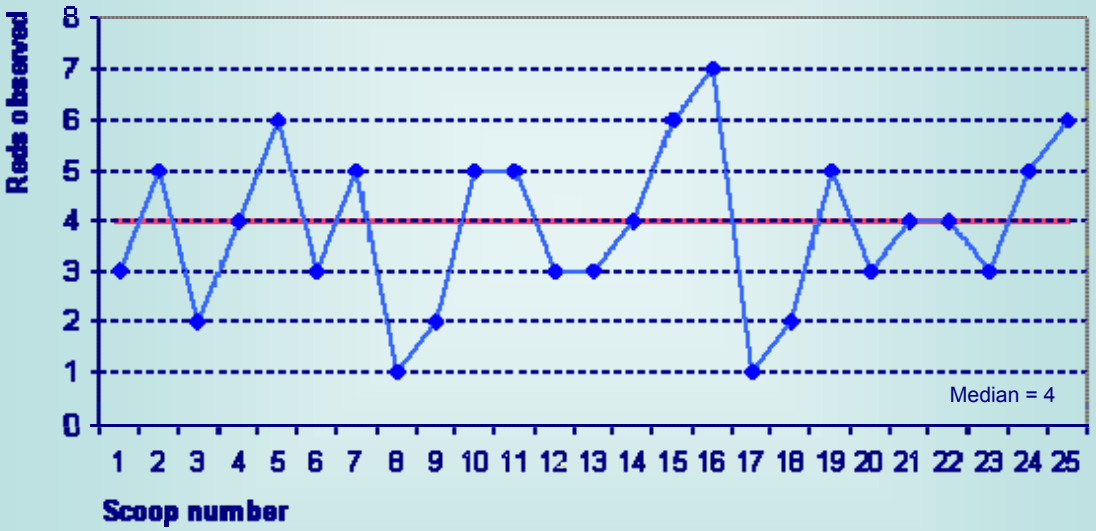
1 Ideally, there should be a minimum of 15 data points.
2 Draw a horizontal line (the x-axis), and label it with the unit of time.
3 Draw a vertical line (the y-axis), and scale it to cover the current data, plus sufficient room to accommodate future data points. Label it with the outcome.
4 Plot the data on the graph in time order and join adjacent points with a solid line.
5 Calculate the mean or median of the data (the centreline) and draw this on the graph.

Example 1 (continued)

Coloured beads pulled from a bag – a dynamic process

The run chart for this data is shown in figure 3.

Figure 3 Run chart for the number of red beads observed in 25 scoops



It is a good idea to state which measure has been used for the centreline.

As the data for the above example (number of red beads observed) is discrete, the median has been used to construct the centreline.

The following definitions are useful before proceeding onto the rules for detecting special variation within run charts and later, control charts.

Useful Observations
Those observations that do not fall directly on the centreline are known as 'useful observations'. The number of useful observations in a sample is equal to the total number of observations minus the number of observations falling on the centreline.

In the above example, four observations fall on the centreline. Therefore, there are 25 – 4 = 21 useful observations in the sample.
N.B. If the mean (=3.88) had been used for the calculation of the centreline, as no observations would have fallen on the centreline, the number of useful observations would have been 25 (the number of observations in the sample).

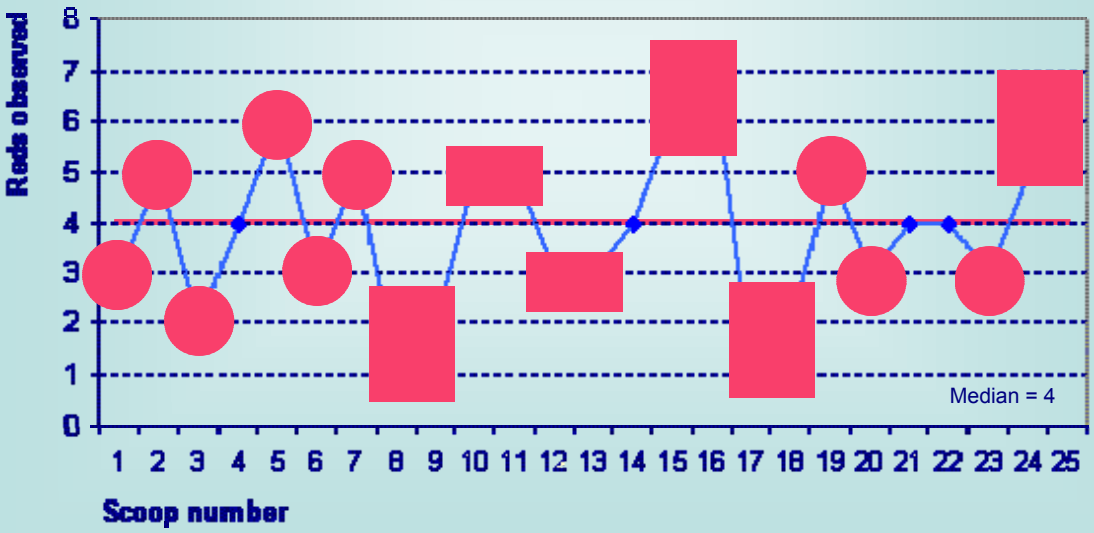
Run
A sequence of one or more consecutive useful observations on the same side of the centreline. The observations falling directly on the centreline can be ignored.

Example 1 (continued)

Coloured beads pulled from a bag – a dynamic process

The run chart for this data is shown in figure 4.

Figure 4 Run chart for the number of red beads observed in 25 scoops with runs highlighted in red



Trend
A sequence of successive increases or decreases in your observations is known as a 'trend'. An observation that falls directly on the centreline, or is the same as the preceding value is not counted.

From the run chart in example 1, the longest trend is of length 3. One of these trends occurs between observations 13 and 16 where there is an increasing sequence of length 3 (observation 14 is not counted since it falls on the centreline).

2.2 Interpreting a Run Chart

A run chart is a useful tool for identifying which type of variation exists within a process.

The following rules can be applied to the run chart for determining the type of variation in the process.

Run Chart Rules
Identifying Special Cause Variation

Number of Runs
If there are too few or too many runs in the process. The table below is a guide based on the number of useful observations in your sample.

Shift
If the number of successive useful observations, falling on the same side of the centreline, is greater than 7.

Trend
If the number of successive useful observations, either increasing or decreasing, is greater than 7.

Zig-Zag
If the number of useful observations, decreasing and increasing alternately (creating a zig-zag pattern), is greater than 14.

Wildly different
If a useful observation is deemed as wildly different from the other observations. This rule is subjective and is easier to identify when interpreting control charts.

Cyclical Pattern
If a regular pattern is occurring over time – for example a seasonality effect.

Number of useful observations	Too few runs	Too many runs
15	4	12
16	5	12
17	5	13
18	6	13
19	6	14
20	6	15
21	7	15
22	7	16
23	8	16
24	8	17
25	9	17
26	9	18
27	9	19

Number of useful observations	Too few runs	Too many runs
28	10	19
29	10	20
30	11	20
31	11	21
32	11	22
33	11	22
34	12	23
35	13	23
36	13	24
37	13	25
38	14	25
39	14	26
40	15	26

The rules listed above are purely guidelines. Some textbooks may quote different sizes of trends, shifts and zig-zags. The above are standard to the work that is carried out within the Clinical Indicators Support Team (CIST) and ISD wide but their primary intention was for applications in industry. Although SPC lends itself well to healthcare processes, healthcare processes deal with lives. With this in mind, common sense is often the best guideline - SPC charts will illustrate the variation within your process but if fewer observations in a trend, shift or zig-zag etc is unusual behaviour in your process then this is just as good an indication of special cause variation and is therefore worth investigating.

2.3 Constructing a Control Chart

Control Chart
A time ordered sequence of data, with a centreline calculated by the mean. Control charts bring the addition of control limits (and warning limits – optional). A control chart enables the monitoring of the process level and identification of the type of variation in the process over time with additional rules associated with the control (and warning) limits.

Steps to create a Control Chart

1 First, select the most appropriate control chart for your data, which is dependent on the properties of your data. See flow chart.
2 Proceed as for the run chart, using the mean as the centreline.
3 Calculate the standard deviation (sd) of the sample using the formula listed in the appendix (for appropriate chosen control chart).
4 Calculate the control limit:
centreline±(3*sd)
5 Calculate the warning limits (optional):
centreline±(2*sd)

Standard Deviation (sd)
The spread of the observations. For example, if there is a large amount of variation between observations then the sd will be bigger than the sd for observations more tightly packed together (i.e. with less variation).

Control Chart Types

Selecting an appropriate control chart for your data

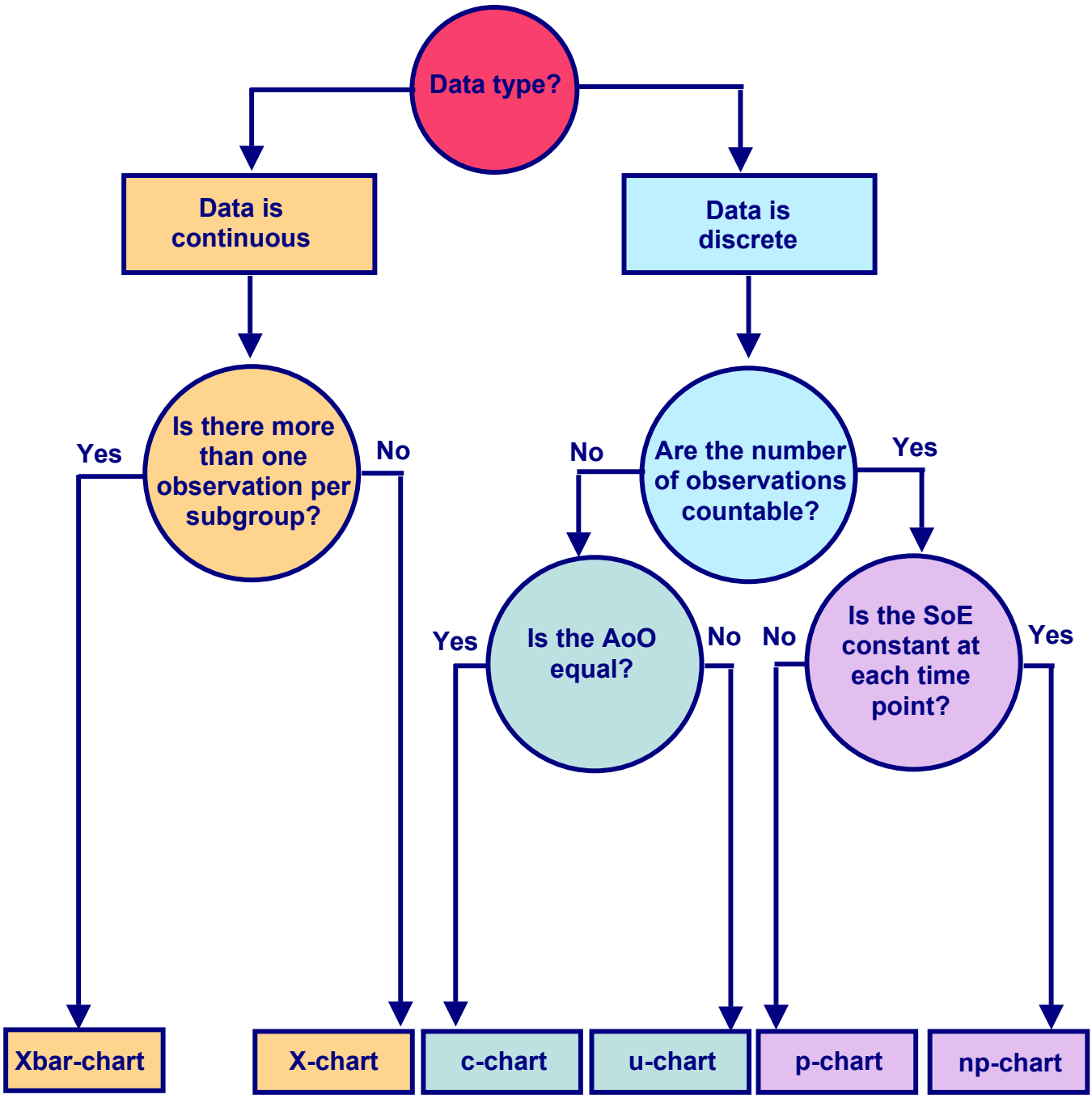
	Continuous	Poisson	Binomial
X X-bar	1 observation 1+observation	- -	- -
c u	- -	Constant AoO Heterogeneous AoO	- -
np p	- -	- -	Constant SoE Heterogeneous SoE

Area of Opportunity (AoO)
The parameter of each observation – see examples below:
Constant AoO – Calculating weekly mortality rates for a surgeon that always works a 5-day week.
Heterogeneous AoO - Calculating weekly mortality rates for a surgeon that works a variable amount of days each week.

Sum of Events (SoE)
The denominator of each observation – see examples below:
Constant SoE – Calculating the number of weekly admissions within a set population where the population does not change over a given time.
Heterogeneous SoE - Calculating the number of weekly admissions within a population that does change over time.

Poisson
Data that has a Poisson distribution is discrete and is based on events occurring over time (or space) at a fixed rate on average, but where each event occurs independently and at random. For example, the number of new hip fracture admissions.

Binomial
Data that has a Binomial distribution is discrete and is based on data with only two possibilities e.g. the probability of being dead or alive, male or female etc.



Example 1 (continued)

Coloured beads pulled from a bag – a dynamic process

The best type of control chart to use for the data below would be either an np-chart (to measure the number of red beads in the 25 scoops) or a p-chart (to measure the proportion of red beads in the 25 scoops).

Below are the steps to creating a p-chart (see appendix).

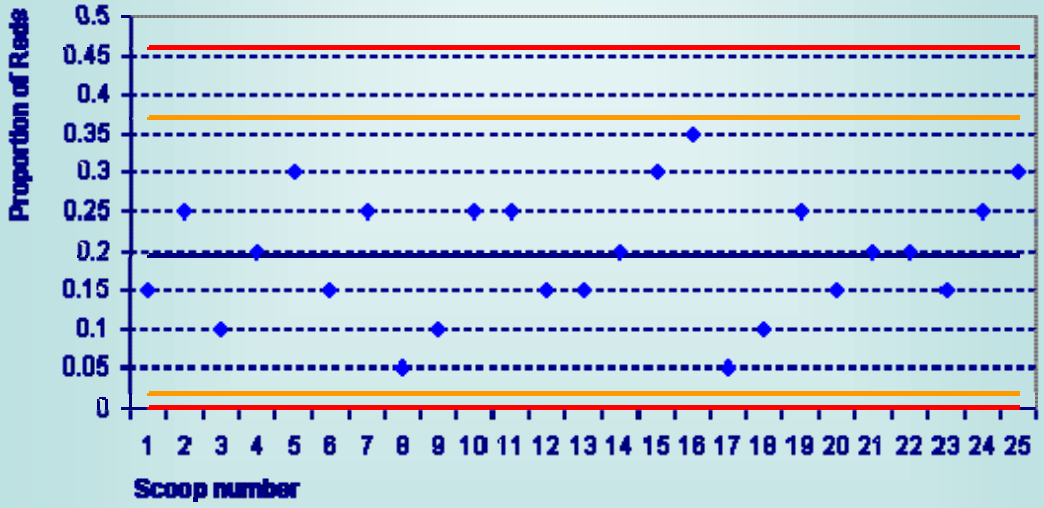
Where:

n = number of beads drawn in each scoop
p = number of red beads observed (success)

For this process:

Mean (x)	= Σ(p/n)	= 97/500
Rate (r)	= p/n	= 3/20, 5/20, 2/20, 6/20 etc
SD	= sqrt((x*(1-x))/n)	= sqrt((97/500*403/500)/20)
Control Limits	= x ± 3*SD	= 0.194 ± 3*0.088421 etc
Warning Limits	= x ± 2*SD	= 0.194 ± 2*0.088421 etc

Figure 4 p chart for the proportion of red beads observed in 25 scoops



In this case the lower control and warning limits have been set to the maximum value of either the formula (given above) and zero, as the proportion of red beads cannot take a value below zero. When dealing with percentages, in most cases, the upper control and warning limits can be dealt with in the same way (i.e. the minimum value of the formula and 100).

2.4 Interpreting a Control Chart

The same rules for identifying special cause variation in run charts also apply to control charts, with the addition of two extra rules.

Control Chart Rules
Additional rules for identifying Special Cause Variation

Control Limits
If there is one or more observation outwith the control limits.
Warning Limits
If there are two successive observations outwith the same warning limits.

The setting of control limits and warning limits are an attempt to balance the risk of committing two possible types of error:

- Type I – False positives
- Type II – False negatives

Type I
Identifying special cause variation when there is none.
As the limits are set at 3 standard deviations from the centreline, only 99.7% of our observations are expected to fall within the limits (and 95% for warning limits) if the process is stable. This means that 3 in 1000 (and 50 in 1000) observations are expected to fall outside the control limits even when the process is stable.

Type II
Not identifying special cause variation when there is.
This occurs when action is not signalled for an observation that falls within the limits when the process is actually out of control.

As mentioned earlier, the rules for identifying special cause variation are guidelines and may be altered in light of the process that is being investigated. Similarly, the boundaries of the control and warning limits can also be adjusted.

The combined risk of committing Type I and Type II errors is minimised when the control limits are set at 3 standard deviations from the centre line (Carey and Lloyd, 1995) however in some cases this may be deemed as too conservative. For example, if poor surgical performance is the process that is being investigated, in order to increase the chances of identifying possible aberrant practice, it may be beneficial to choose tighter limits.

3. SPC Charts – Static Processes

SPC Charts are most typically plotted over time for a single process. However, it is also possible to construct SPC Charts at a static point in time for a process carried out by multiple institutions (e.g. NHS Boards, Hospitals etc), which are often referred to as cross-sectional charts.

The cross-sectional chart that we are going to cover is one of the most common SPC charts for static processes and is known as a funnel chart due to the fact that the control limits take the shape of a 'funnel'.

3.1 Constructing a Funnel Chart

Funnel Chart
SPC Chart for cross-sectional data at a particular point in time. The rate of the process (e.g. mortality rate, survival rate etc) is plotted on the vertical axis and the denominator (i.e. population, number of admissions etc) is plotted on the horizontal axis. The centreline is calculated by the mean. Generally, only control limits are calculated, as the rule for warning limits does not apply to cross-sectional data.

Steps to create a Funnel Chart

1 Order the data by the denominator (d) in ascending order.
2 Calculate proportions (p) for each individual institution and an overall proportion (this will be the centreline (c)).
3 Calculate the standard deviation (sd):
$$\text{sqrt}(d^* c^*(1- c)).$$
4 Calculate the control limits (ucl and lcl):
$$c \pm (3^*sd)/d$$

Example 2 (continued)

Coloured beads pulled from a bag – a static process

Using the data from example 2 we will now assume that each bag in fact contained a variable number of red beads (the total number of all beads in a bag is still 20). The table below illustrates the data.

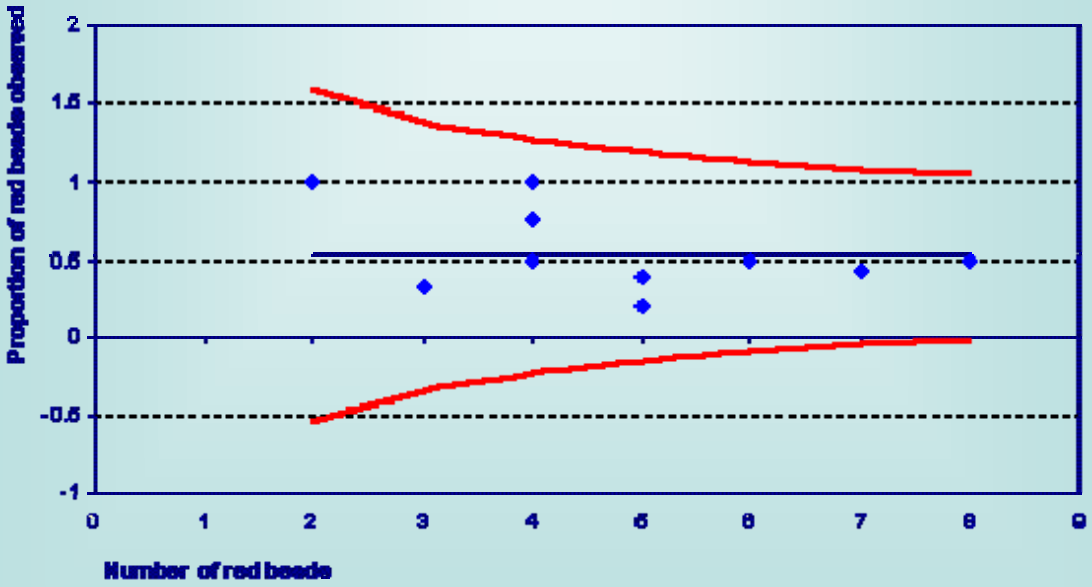
Group	Number of red beads per bag (d)	Number of red beads observed (obs)
A	4	2
B	6	3
C	5	2
D	4	4
E	3	1
F	2	2
G	4	3
H	7	3
I	8	4
J	5	1

After sorting the data by the denominator (d), in this case the numbers of red beads in each bag calculate the proportions (p) and limits (lcl and ucl).

Group	d	obs	p	c	sd	lcl	ucl
F	2	2	1	0.52	0.71	-0.54	1.58
E	3	1	0.33	0.52	0.87	-0.35	1.39
D	4	4	1	0.52	1.00	-0.23	1.27
G	4	3	0.75	0.52	1.00	-0.23	1.27
A	4	2	0.5	0.52	1.00	-0.23	1.27
C	5	2	0.4	0.52	1.12	-0.15	1.19
J	5	1	0.2	0.52	1.12	-0.15	1.19
B	6	3	0.5	0.52	1.22	-0.09	1.13
H	7	3	0.43	0.52	1.32	-0.05	1.09
I	8	4	0.5	0.52	1.41	-0.01	1.05

We then create a chart using a 'scatter plot', where the x-axis values are always plotted as the denominator (d). The only other values to plot, against the x-axis values, are the rate (r), centreline (c) and the lcl and ucl.

Figure 5 Funnel Chart for the proportion of red beads observed in each groups' scoop



The graph above indicates that all the groups are in control and do not display any cause for concern.

3.2 Interpreting a Funnel Chart

The rules for identifying special cause variation in a static process are very simple and are identified purely by an observation falling out with the control limits.

Funnel Chart Rule

If one of the observations falls outwith the control limits.

If one of the observations does fall outwith the control limits, it is often worth investigating that particular process more fully with a control chart over time (i.e. dynamic process).

4. Alternative SPC Charts

4.1 CUSUM and EWMA Charts

There are many other different types of SPC Charts that may be more appropriate for the type of investigation that your process requires. For example Cumulative Summation (CUSUM) Charts are more sensitive to small shifts than the types of SPC Charts that have been discussed so far.

Likewise, Exponentially Weighted Moving Average (EWMA) Charts, as well as being more sensitive to smaller shifts, also have the advantage of taking into account past data, which avoids biasing the process variation to the current time period.

These charts are not widely used in healthcare as they are more complex to construct and more difficult to interpret. There are also very few cases in healthcare that would require additional time and resources directed towards investigating very small shifts that are most likely an effect of common cause variation. However, when used appropriately they can provide useful, additional analysis.

4.2 g-charts

The most widely used alternative to the ‘regular’ SPC Charts within the health service has been g-charts. These charts measure the number-between specific observations and are used for processes with low frequencies. In particular, g-charts have been used extensively in monitoring Healthcare Acquired Infections (HAIs).

Perhaps best described through the billboards in construction sites that state ‘x days since the last accident’, g-charts simply look at the number of days, patients or catheters etc since the last event of interest e.g. infection.

g-charts are interpreted in the same way as the control charts that we have already seen i.e. special cause variation is identified when an observation falls outwith the control limits and/or when two successive points fall out with the warning limits. These type of charts are only applicable with dynamic processes but have already proved invaluable throughout hospital wards in Scotland that have implemented g-charts to monitor the success of specific care bundles.

Contacts

For further information on SPC charts visit our website:

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or subscribe to our bulletin (placing ‘add me’ in the subject field)
email: QIcom@isd.csa.scot.nhs.uk

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Appendix

In this section you will find the formulae required for constructing a control chart for your data, assuming that you have made the most appropriate choice of chart.

X-chart

Assume you have m observations, Xi, i =1, 2,…, m.

Calculate the process average,

$$\overline{X} = \sum_{i=1}^{m} X_{i.}$$

Calculate the absolute moving ranges (MRs) between adjacent observations, where

$$MR\{i,i+1\} = |X_i - X_{i+1}|,\ i = 1, 2, \ldots, m-1.$$

Calculate the mean range, $\overline{R}$, as

$$\overline{R} = \frac{1}{m-1}\sum_{i=1}^{m-1} MR_{\{i,i+1\}}.$$

Set the control limits at

$$\overline{X} \pm (2.66 \times \overline{R}).$$

c-chart

Assume that you have m observations from a Poisson(μ) distribution, i.e.

$$X_i \sim \text{Poisson}(\mu),\quad i = 1,2,...,m,$$

where:

Xi is the number of occurrences for observation i, and
μ is the process average.

Since the true process average, μ, is not known, we replace its value by the observed process average, which is given by

$$\overline{X} = \sum_{i=1}^{m} X_{i.}$$

Since it is the number of occurrences that is plotted, i.e. the sequence of values, {X1, X2, …, Xm}, we're required to calculate that standard deviation, s, for each Xi, i=1,2,…m.

Since the area of opportunity is constant for all i, s is simply calculated as

$$s = \sqrt{\overline{X}}.$$

It is this s that is used for the calculation of the control (and warning) limits.

u-chart

As with the c-chart, assume that you have m observations from a Poisson(μ) distribution, i.e.

$$X_i \sim \text{Poisson}(\mu), \quad i = 1,2,...,\text{m}.$$

Since it is the proportion of occurrences that is plotted, i.e. the sequence of values, {Y1, Y2, …, Ym}, where $Y_i = X_i / n_i \sim$ Poisson(μ/ni) and ni is simply a scaling constant that allows for the heterogeneity of the area of opportunity, we're required to calculate that standard deviation, s, for each Yi, i=1,2,…m. This is given by

$$s(Y_i) = \sqrt{\overline{X}/n_i}\ .$$

It is this s that is used for the calculation of the control (and warning) limits.

Note that since a different s is required to be calculated for each proportion, Yi, i=1,2,…, m, different control (and warning) limits are required to be calculated, too.

np-chart

Assume you have m observations from a Bin(ni ,p) distribution, i.e.

$$Xi \sim \text{Binomial}(ni\ , p), \quad i = 1, 2, \ldots, m,$$

where:

Xi is the number of non-conforming units for observation i,
ni is the number of units for observation i, and
p is the probability of "success".

In addition, let

$N = \sum_{i=1}^{m} n_i$ be the sum of the units,

$X = \sum_{i=1}^{m} X_i$ be the sum of non - conforming units,

$\overline{X} = \frac{1}{m}\sum_{i=1}^{m} X_i$ be the mean number of non - conforming units, and

$\hat{p} = X / N$ be the observed probability of obtaining a non - conforming unit.

Since it is the number of non-conforming units that is plotted, i.e. the sequence of values, {X1, X2, …, Xm}, we're required to calculate the standard deviation, s, for each Xi, i=1,2,…m. This is given by

$$s = \sqrt{n_i \hat{p}(1-\hat{p})}.$$

However, since $n = n_1 = n_2 = \cdots n_m$, s is simply

$$s = \sqrt{n\hat{p}(1-\hat{p})}.$$

It is this s that is used for the calculation of the control (and warning) limits.

p-chart

As with the np-chart, assume that we have m observations from a Bin(ni ,p) distribution, i.e.

$$X_i \sim \text{Binomial}(n_i, p), \quad i = 1, 2, \ldots, m,$$

Since it is the proportion of non-conforming units that is plotted, i.e. the sequence of values, {Y1, Y2, …, Ym}, where $Y_i = X_i / n_i$, we're required to calculate the standard deviation, s, for each Yi, i=1,2,…, m. This is given by

$$\begin{aligned} s(Y_i) &= s(X_i / n) \\ &= \frac{1}{\text{n}} s(X_i) \\ &= \frac{1}{n_i} \sqrt{n_i \hat{p}(1-\hat{p})} \\ &= \sqrt{\hat{p}(1-\hat{p})/n_i}, \quad i = 1,2,\ldots, m. \end{aligned}$$

It is this s that is used for the calculation of the control (and warning) limits.

Note that since a different s is required to be calculated for each proportion, Yi, i=1,2,…, m, different control (and warning) limits are required to be calculated, too.