TECHNICAL AIDS

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What Do Low *F* Ratios Tell You?

MORE precisely the title of this article should be "In the Analysis of a Designed Experiment What Do Significantly Low F Ratios Tell You?". There are three ways in which significantly low F ratios can come about: chance, improper randomization, and using the wrong model.

We can dismiss the chance cause from this discussion because nothing can be done about it. If the assumption of normality of error is correct, randomization is properly carried out, and the correct model is used in the analysis of variance (ANOVA), we can, however, know the frequency with which the chance cause occurs. The remaining two causes are considered in detail.

Improper Randomization

Consider an experiment involving the comparison of B batches of chemicals by randomly sampling each batch N times and analyzing for a particular ingredient. There will be a total of $B \times N$ trials. Suppose $B = 6$ and $N = 4$.

B_1	B_2	B_3	B_4	B_5	B_6
—	—	—	—	—	—
—	—	—	—	—	—
—	—	—	—	—	—
—	—	—	—	—	—

Imagine that there are, say, four analysts who differ in the results they obtain with this analytical method (that is, they have different biases). Now if a particular analyst analyzes *most* of the samples in a given row, the biases of the analysts will contribute *mostly* to the within column variation. The larger this contribution, the smaller the F ratio for testing batches will become. It could become significantly small (tested by seeing if the inverted F ratio, having interchanged degrees of freedom, is too large.)

Randomization of the assignments of samples to analysts will, on the average, spread the biases around so that the net effect is not a selective contribution to the within variation. Of course a better design would include analysts as a factor. The point is that randomization tends to keep effects (both known and unknown) that are external to the design from influencing the factors being studied.

Incorrect Model

Every experimental design has a model associated with it that corresponds to the physical manner in which the experiment was carried out and dictates the calculation procedure. In industrial experimentation it is not uncommon to find the split-plot design. Quite frequently this design is not recognized as such with the result that an incorrect analysis is applied. The tipoff that this has occurred is often a too small F ratio as we shall see in the following example. A fine discussion of this topic is given by Wooding (1973).

Suppose that two Furnaces (F_1, F_2) are available. Four objects, each having received a different one of four Treatments (T_1, T_2, T_3, T_4), are placed together in each Furnace. This is then repeated; that is, a second Replication is carried out. Note that in total four Furnace runs were made involving four sets of four objects. Experimental results (artificial) are given in Table 1.

If this had been carried out as a completely randomized, replicated 2×4 factorial experiment, the correct model would be

$$X_{ijk} = \mu + F_i + T_j + (FT)_{ij} + \epsilon_{(ij)k}.$$

The ANOVA using this model gives the results shown in Table 2. One would conclude that the Furnaces

TABLE 1. Data from a Replicated Split-Plot Experiment

	Replication 1		Replication 2	
	F_1	F_2	F_1	F_2
T_1	9	25	23	39
T_2	8	23	21	36
T_3	10	26	21	38
T_4	11	26	22	40

TABLE 2. Incorrect Analysis of Data in Table 1

Source	DF	SS	MS	F	P
Furnaces (F)	1	1024.00	1024.00	12.5	0.008
Treatments (T)	3	16.25	5.42	0.066	0.976
$F \times T$	3	1.50	0.50	0.006	0.999
Error (ϵ)	8	656.00	82.00		
	15				

TABLE 3. Correct Analysis of Data in Table 1

Source	DF	SS	MS	F	P
Furnaces (F)	1	1024.00	1024.00	3.14	0.22
Replication of F (R)	2	651.25	325.62		
Treatments (T)	3	16.25	5.42	6.86	0.02
$F \times T$	3	1.50	0.50	0.63	0.62
$R \times T$	6	4.75	0.79		
	15				

were highly significantly (0.01 level) different. The F ratios for Treatments and the Furnace × Treatment interaction are both significantly too small. This arises from basing the analysis on the wrong model.

According to the way the experiment was actually done, the correct model is

$$X_{ijk} = \mu + F_i + R_{(i)j} + T_k + (FT)_{ik} + (RT)_{(i)jk}.$$

The ANOVA using this model is given in Table 3. We now conclude that: (1) there is no significant difference between Furnaces, (2) the Furnace × Treatment interaction is not significant, and (3) Treatments show an effect significant at the 0.02 level. Notice that each of these conclusions is different from that arrived at using the wrong model. A simpler example would be the analysis of a paired test as though it were an unpaired test. However, this is a mistake that is very rarely made.

Reference

WOODING, W. M. (1973). "The Split-Plot Design". *Journal of Quality Technology* 5, pp. 16–33.

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